LESSON 10–2 Simplifying Radical Expressions

KeyConcept Product Property of Square Roots

Words	For any nonnegative real numbers *a* and *b*, the square root of *ab* is equal to the square root of *a* times the square root of *b*.
Symbols	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, if $a \geq 0$ and $b \geq 0$
Examples	$\sqrt{4 \cdot 9} = \sqrt{36}$ or 6 $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3$ or 6

Check Your Understanding

● = Step-by-Step Solutions begin on page R13.

Examples 1–3 Simplify each expression.

1. $\sqrt{24}$ **$2\sqrt{6}$**
2. $3\sqrt{16}$ **12**
3. $2\sqrt{25}$ **10**
4. $\sqrt{10} \cdot \sqrt{14}$ **$2\sqrt{35}$**
5. $\sqrt{3} \cdot \sqrt{18}$ **$3\sqrt{6}$**
6. $3\sqrt{10} \cdot 4\sqrt{10}$ **120**
7. $\sqrt{60x^4y^7}$ **$2x^2y^3\sqrt{15y}$**
8. $\sqrt{88m^3p^2r^5}$ **$2m|p|r^2\sqrt{22mr}$**
9. $\sqrt{99ab^5c^2}$ **$3b^2|c|\sqrt{11ab}$**

10–2 Simplifying Radical Expressions

KeyConcept Quotient Property of Square Roots

Words	For any real numbers a and b, where $a \geq 0$ and $b > 0$, the square root of $\frac{a}{b}$ is equal to the square root of a divided by the square root of b.
Symbols	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

We must get rid of all roots in the denominator to simplify.

mples 4–5 Simplify each expression.

37. $\sqrt{\frac{32}{t^4}}$

38. $\sqrt{\frac{27}{m^5}}$

39. $\frac{\sqrt{68ac^3}}{\sqrt{27a^2}}$

40. $\frac{\sqrt{h^3}}{\sqrt{8}}$

41. $\sqrt{\frac{3}{16}} \cdot \sqrt{\frac{9}{5}}$

42. $\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}}$

rationalizing the denominator

Think of the difference of squares: $(a + b)(a - b) = a^2 - b^2$

Example 5 **Simplify each expression.**

11. $\frac{3}{3+\sqrt{5}}$ $\frac{9-3\sqrt{5}}{4}$

12. $\frac{5}{2-\sqrt{6}}$

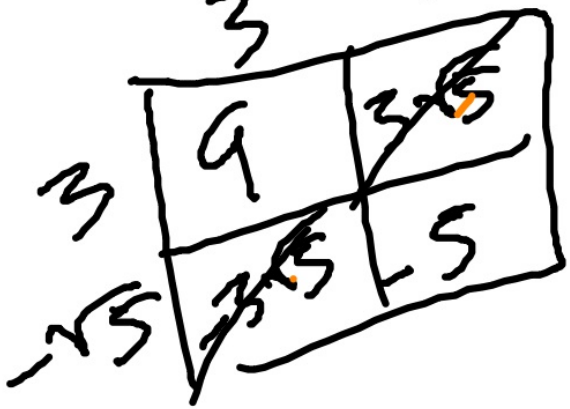
13. $\frac{2}{1-\sqrt{10}}$ $\frac{2+2\sqrt{10}}{-9}$

14. $\frac{1}{4+\sqrt{12}}$

15. $\frac{4}{6-\sqrt{7}}$ $\frac{24+4\sqrt{7}}{29}$

16. $\frac{6}{5+\sqrt{11}}$

(11) $\frac{3(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \frac{9-3\sqrt{5}}{9-5} = \frac{9-3\sqrt{5}}{4}$



(13) $\frac{2(1+\sqrt{10})}{(1-\sqrt{10})(1+\sqrt{10})} = \frac{2+2\sqrt{10}}{1-10}$

15. $\dfrac{4}{6-\sqrt{7}}$ $\dfrac{24+4\sqrt{7}}{29}$

$(a-b)(a+b) = a^2 - b^2$

$$\frac{4}{(6-\sqrt{7})}\cdot\frac{(6+\sqrt{7})}{(6+\sqrt{7})} = \frac{24+4\sqrt{7}}{36-7}$$

$6-\sqrt{7}$

conjugate

	6	$-\sqrt{7}$
6	36	$-6\sqrt{7}$
$\sqrt{7}$	$6\sqrt{7}$	-7

Examples 1–3 Simplify each expression.

1. $\sqrt{24}$ $2\sqrt{6}$
2. $3\sqrt{16}$ 12
3. $2\sqrt{25}$ 10
4. $\sqrt{10} \cdot \sqrt{14}$ $2\sqrt{35}$
5. $\sqrt{3} \cdot \sqrt{18}$ $3\sqrt{6}$
6. $3\sqrt{10} \cdot 4\sqrt{10}$ 120
7. $\sqrt{60x^4y^7}$ $2x^2y^3\sqrt{15y}$
8. $\sqrt{88m^3p^2r^5}$ $2m|p|r^2\sqrt{22mr}$
9. $\sqrt{99ab^5c^2}$ $3b^2|c|\sqrt{11ab}$

Example 4

10. **MULTIPLE CHOICE** Which expression is equivalent to $\sqrt{\frac{45}{10}}$? **D**

A $\frac{5\sqrt{2}}{10}$ B $\frac{\sqrt{45}}{10}$ C $\frac{\sqrt{50}}{10}$ D $\frac{3\sqrt{2}}{2}$

Example 5 Simplify each expression.

11. $\frac{3}{3+\sqrt{5}}$ $\frac{9-3\sqrt{5}}{4}$
12. $\frac{5}{2-\sqrt{6}}$ $\frac{10+5\sqrt{6}}{-2}$
13. $\frac{2}{1-\sqrt{10}}$ $\frac{2+2\sqrt{10}}{-9}$
14. $\frac{1}{4+\sqrt{12}}$ $\frac{2-\sqrt{3}}{2}$
15. $\frac{4}{6-\sqrt{7}}$ $\frac{24+4\sqrt{7}}{29}$
16. $\frac{6}{5+\sqrt{11}}$ $\frac{15-3\sqrt{11}}{7}$

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Practice and Problem Solving

Extra Practice is on page R10.

Examples 1–3 Simplify each expression.

17. $\sqrt{52}$ $2\sqrt{13}$

18. $\sqrt{56}$ $2\sqrt{14}$

19. $\sqrt{72}$ $6\sqrt{2}$

20. $3\sqrt{18}$ $9\sqrt{2}$

21. $\sqrt{243}$ $9\sqrt{3}$

22. $\sqrt{245}$ $7\sqrt{5}$

23. $\sqrt{5} \cdot \sqrt{10}$ $5\sqrt{2}$

24. $\sqrt{10} \cdot \sqrt{20}$ $10\sqrt{2}$

25. $3\sqrt{8} \cdot 2\sqrt{7}$ $12\sqrt{14}$

26. $4\sqrt{2} \cdot 5\sqrt{8}$ 80

27. $3\sqrt{25t^2}$ $15|t|$

28. $5\sqrt{81q^5}$ $45q^2\sqrt{q}$

29. $\sqrt{28a^2b^3}$ $2|a|b\sqrt{7b}$

30. $\sqrt{75qr^3}$ $5r\sqrt{3qr}$

31. $7\sqrt{63m^3p}$ $21m\sqrt{7mp}$

32. $4\sqrt{66g^2h^4}$ $4|g|h^2\sqrt{66}$

33. $\sqrt{2ab^2} \cdot \sqrt{10a^5b}$ $2a^3b\sqrt{5b}$

34. $\sqrt{4c^3d^3} \cdot \sqrt{8c^3d}$ $4c^3d^2\sqrt{2}$

mples 4–5 Simplify each expression.

37. $\sqrt{\frac{32}{t^4}}$ $\frac{4\sqrt{2}}{t^2}$

38. $\sqrt{\frac{27}{m^5}}$ $\frac{3\sqrt{3m}}{m^3}$

39. $\frac{\sqrt{68ac^3}}{\sqrt{27a^2}}$ $\frac{2c\sqrt{51ac}}{9|a|}$

40. $\frac{\sqrt{h^3}}{\sqrt{8}}$ $\frac{h\sqrt{2h}}{4}$

41. $\sqrt{\frac{3}{16}} \cdot \sqrt{\frac{9}{5}}$ $\frac{3\sqrt{15}}{20}$

42. $\sqrt{\frac{7}{2}} \cdot \sqrt{\frac{5}{3}}$ $\frac{\sqrt{210}}{6}$

43. $\frac{7}{5+\sqrt{3}}$ $\frac{35-7\sqrt{3}}{22}$

44. $\frac{9}{6-\sqrt{8}}$ $\frac{27+9\sqrt{2}}{14}$

45. $\frac{3\sqrt{3}}{-2+\sqrt{6}}$ $\frac{6\sqrt{3}+9\sqrt{2}}{2}$

46. $\frac{3}{\sqrt{7}-\sqrt{2}}$ $\frac{3\sqrt{7}+3\sqrt{2}}{5}$

47. $\frac{5}{\sqrt{6}+\sqrt{3}}$ $\frac{5\sqrt{6}-5\sqrt{3}}{3}$

48. $\frac{2\sqrt{5}}{2\sqrt{7}+3\sqrt{3}}$ $4\sqrt{35}-6\sqrt{15}$