

Glencoe

ALGEBRA 1



LESSON 11-3 Simplifying Rational Expressions

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Example 1 Find Excluded Values

State the excluded values for each rational expression.

a. $\frac{-8}{r^2 - 36}$

Exclude the values for which $r^2 - 36 = 0$.

$$r^2 - 36 = 0$$
 The denominator cannot be zero.

$$(r - 6)(r + 6) = 0$$
 Factor.

$$r - 6 = 0 \text{ or } r + 6 = 0$$
 Zero Product Property

$$r = 6 \quad r = -6$$
 Therefore, r cannot equal 6 or -6 .

b. $\frac{n^2}{n^2 + 4n - 5}$ p.690

Exclude the values for which $n^2 + 4n - 5 = 0$.

$$n^2 + 4n - 5 = 0$$
 The denominator cannot be zero.

$$(n - 1)(n + 5) = 0$$
 Factor.

$$n - 1 = 0 \text{ or } n + 5 = 0$$
 Zero Product Property

$$n = 1 \quad n = -5$$
 Therefore, n cannot equal 1 or -5 .

Check Your Understanding = Step-by-Step Solutions begin on page R13.**Example 1** State the excluded values for each rational expression.

1. $\frac{8}{x^2 - 16}$ 4, -4

2. $\frac{3m}{m^2 - 6m + 5}$ 1, 5

$$\textcircled{1} \frac{8}{(x+4)(x-4)}$$

$$\textcircled{2} \frac{3m}{(m-1)(m-5)}$$

KeyConcept Simplifying Rational Expressions

Words Let a , b , and c , be polynomials with $a \neq 0$ and $c \neq 0$.

Symbols $\frac{ba}{ca} = \frac{b \cdot a}{c \cdot a} = \frac{b}{c}$

Example $\frac{3x - 9}{4x - 12} = \frac{3(x - 3)}{4(x - 3)} = \frac{3}{4}$

Standardized Test Example 3 Use GCF to Simplify an Expression

Which expression is equivalent to $\frac{(-3x^2)(4x^5)}{9x^6}$?

A $\frac{4}{3}x$

B $\frac{4}{3x}$

C $-\frac{4}{3x}$

D $-\frac{4}{3}x$

Read the Test Item The expression represents the product of two monomials and the division of that product by another monomial.

Solve the Test Item

Step 1 Factor the numerator and denominator, using their GCF.

$$\frac{(3x^6)(-4x)}{(3x^6)(3)}$$

Step 2 Simplify. The correct answer is D.

$$\frac{\cancel{(3x^6)}(-4x)}{\cancel{(3x^6)}(3)} \text{ or } -\frac{4}{3}x$$

Handwritten work:

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{2 \cdot 1}{2 \cdot 2} = \frac{1}{2}$$

Examples 3–5 Simplify each expression. State the excluded values of the variables.

4. $\frac{28ab^3}{16a^2b} \cdot \frac{7b^2}{4a}; a \neq 0, b \neq 0$

6. $\frac{5d + 15}{d^2 - d - 12} \cdot \frac{5}{d - 4}; 4, -3$

8. $\frac{2r - 12}{r^2 - 36} \cdot \frac{2}{r + 6}; 6, -6$

5. $\frac{(-3r)(10r^4)}{6r^5} \cdot -5; 0$

7. $\frac{x^2 + 11x + 28}{x + 4} \cdot x + 7; -4$

9. $\frac{3y - 27}{81 - y^2} \cdot \frac{3}{9 + y}; -9, 9$

Handwritten work for Example 8:

$$\frac{2(r-6)}{(r+6)(r-6)}$$

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Example 4 Simplify Rational Expressions

Simplify $\frac{2r + 18}{r^2 + 8r - 9}$. State the excluded values of r .

$$\frac{2r + 18}{r^2 + 8r - 9} = \frac{2(r + 9)}{(r + 9)(r - 1)}$$

Factor.

$$= \frac{2\cancel{(r + 9)}}{\cancel{(r + 9)}(r - 1)} \text{ or } \frac{2}{r - 1}$$

Divide the numerator and denominator by the GCF, $r + 9$.

Exclude the values for which $r^2 + 8r - 9$ equals 0.

$$r^2 + 8r - 9 = 0$$

The denominator cannot equal zero.

$$(r + 9)(r - 1) = 0$$

Factor.

$$r = -9 \text{ or } r = 1$$

Zero Product Property

So, $r \neq -9$ and $r \neq 1$.

(4) $\frac{7 \cdot \cancel{4} \cdot \cancel{a} \cdot b \cdot b \cdot b}{4 \cdot \cancel{4} \cdot \cancel{a} \cdot \cancel{a} \cdot b}$

$$\frac{7b^2}{4a}$$

$\swarrow -30$

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8. $\frac{2r - 12}{r^2 - 36} \cdot \frac{2}{r + 6}; 6, -6$

5. $\frac{(-3r)(10r^4)}{6r^5} \cdot -5; 0$

7. $\frac{x^2 + 11x + 28}{x + 4} \cdot x + 7; -4$

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Example 5 Recognize Opposites

Simplify $\frac{36 - t^2}{5t - 30}$. State the excluded values of t .

$$\frac{36 - t^2}{5t - 30} = \frac{(6 - t)(6 + t)}{5(t - 6)}$$

Factor.

$$= \frac{-1(t - 6)(6 + t)}{5(t - 6)}$$

Rewrite $6 - t$ as $-1(t - 6)$.

$$= \frac{-1(\cancel{t-6}^1)(6 + t)}{5(\cancel{t-6}^1)} \text{ or } -\frac{6 + t}{5}$$

Divide out the common factor, $t - 6$.

Handwritten work for Example 5:

6

~~$5(x+3)$~~

~~$(x+3)(x-4)$~~

~~$\begin{array}{r} 2 \\ -4 \end{array} \begin{array}{r} 3 \\ -1 \end{array}$~~

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9. $\frac{3y - 27}{81 - y^2} -\frac{3}{9 + y}; -9, 9$

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Example 6 Rational Functions

Find the zeros of $f(x) = \frac{x^2 + 3x - 18}{x - 3}$.

$$f(x) = \frac{x^2 + 3x - 18}{x - 3} \quad \text{Original function}$$

$$0 = \frac{x^2 + 3x - 18}{x - 3} \quad f(x) = 0$$

$$0 = \frac{(x + 6)(x - 3)}{x - 3} \quad \text{Factor.}$$

$$0 = \frac{(x + 6)\cancel{(x - 3)}}{\cancel{x - 3}} \quad \text{Divide out common factors.}$$

$$0 = x + 6 \quad \text{Simplify.}$$

When $x = -6$, the numerator becomes 0, so $f(x) = 0$. Therefore, the zero of the function is -6 .

Example 6 Find the zeros of each function.

10. $f(x) = \frac{x^2 - x - 12}{x - 2}$

11. $f(x) = \frac{x^2 - x - 6}{x^2 + 8x + 12}$

Example 1 State the excluded values for each rational expression.

12. $\frac{-n}{n^2 - 49}$ **7, -7**

13. $\frac{5x + 1}{x^2 - 1}$ **1, -1**

14. $\frac{12a}{a^2 - 3a - 10}$ **5, -2**

15. $\frac{k^2 - 4}{k^2 + 5k - 24}$ **3, -8**

Example 2 16. **GEOMETRY** The volume of a rectangular prism is $3x^3 + 34x^2 + 72x - 64$. If the height is $x + 4$, what is the area of the base of the prism? **$3x^2 + 22x - 16$**

17. **GEOMETRY** Use the circle at the right to write the ratio $\frac{\text{circumference}}{\text{area}}$. Then simplify. State the excluded value of the variable. **$\frac{2\pi(5f)}{\pi(5f)^2} = \frac{2}{5f}$; 0**



Examples 3–5 Simplify each expression. State the excluded values of the variables.

18. $\frac{15x^4y^2}{40x^3y^3}$ **$\frac{3x}{8y}$; $x \neq 0, y \neq 0$**

19. $\frac{32n^2p}{2n^4p}$ **$\frac{16}{n^2}$; $n \neq 0, p \neq 0$**

20. $\frac{(4t^3)(2t)}{20t^2}$ **$\frac{2t^2}{5}$; 0**

21. $\frac{(7c^2)(-6c^3)}{21c^4}$ **$-2c$; 0**

22. $\frac{4x - 24}{x^2 - 12x + 36}$ **$\frac{4}{x - 6}$; 6**

23. $\frac{a^2 + 3a}{a^2 - 3a - 18}$ **$\frac{a}{a - 6}$; 6, -3**

24. $\frac{n^2 + 7n - 18}{n - 2}$ **$n + 9$; 2**

25. $\frac{x^2 + 4x - 32}{x + 8}$ **$x - 4$; -8**

26. $\frac{x^2 - 25}{x^2 + 5x}$ **$\frac{x - 5}{x}$; 0, -5**

27. $\frac{2p^2 - 14p}{p^2 - 49}$ **$\frac{2p}{p + 7}$; 7, -7**

28. $\frac{2x - 10}{25 - x^2}$ **$\frac{2}{-(x + 5)}$; 5, -5**

29. $\frac{64 - c^2}{c^2 - 7c - 8}$ **$\frac{-(8 + c)}{c + 1}$; 8, -1**

Example 6 Find the zeros of each function.

30. $f(x) = \frac{x^2 - x - 12}{x^2 + 2x - 35}$ **-3, 4**

31. $f(x) = \frac{x^2 + 3x - 4}{x^2 + 9x + 20}$ **1**

32. $f(x) = \frac{2x^2 + 11x - 40}{2x + 5}$ **-8, $\frac{5}{2}$**

33. $f(x) = \frac{3x^2 - 18x + 24}{x - 6}$ **2, 4**

34. $f(x) = \frac{x^3 + x^2 - 6x}{x - 1}$ **0, -3, 2**

35. $f(x) = \frac{x^3 - 4x^2 - 12x}{x + 2}$ **0, 6**