2-1 Relations and Functions

KeyConcept Functions

Example 1 Domain and Range

State the domain and range of each relation. Then determine whether ea is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, o

a. {(−6, −1), (−5, −9), (−3, −7), (−1, 7), (6, −9)}

Domain: {−6, −5, −3, −1, 6} Range: {−9, −7, −1, 7}

function: Yes, because each element of the domain is paired with one of the range.

one-to-one: No, because each element of the domain is not paired with element of the range.

onto: Yes, because each element of the range corresponds to an e of the domain.

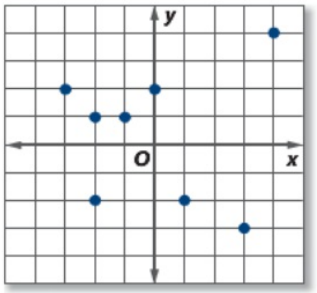
GuidedPractice

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.

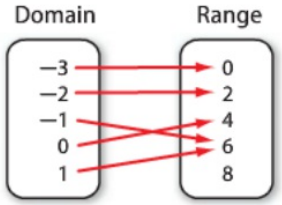
1A. D = {−3, −2, −1, 0, 1, 3, 4}, R = {−3, −2, 1, 2, 4}; not a function

1B. D = {−3, −2, −1, 0, 1}, R = {0, 2, 4, 6, 8}; function; not one-to-one, not onto

1A.



1B.



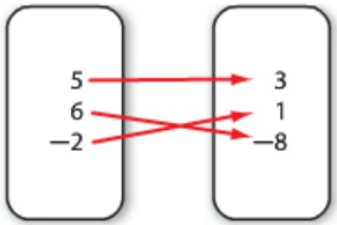
Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 CCSS STRUCTURE State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.

D = {−2, 5, 6}, R = {−8, 1, 3}; function; both

1.



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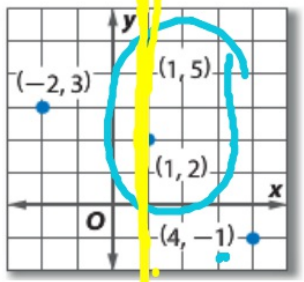


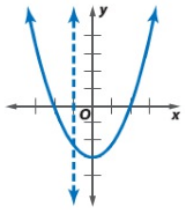
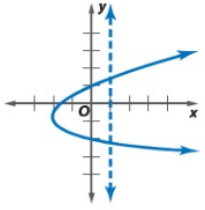
3

x	y
−2	−4
1	−4
4	−2
8	6

2. D = {−2, 1, 4}, R = {−1, 2, 3, 5}; not a function

3. D = {−2, 1, 4, 8}, R = {−4, −2, 6}; function; onto

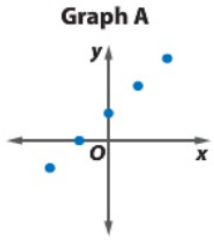
KeyConcept Vertical Line Test

Words	If no vertical line intersects a graph in more than one point, the graph represents a function.	If a vertical line intersects a graph in two or more points, the graph does not represent a function.
Models		

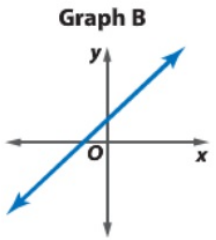
Additional Answer

4d. yes

which the domain is a set of individual points, like the relation in Graph A, **discrete relation**. Notice that its graph consists of points that are not hen the domain of a relation has an infinite number of elements and the e graphed with a line or smooth curve, the relation is a **continuous relation**.



discrete relation



continuous relation

crete and continuous graphs, you can use the **vertical line test** to determine elation is a function.

Example 2

4. BASKETBALL The table shows the average points per game for Dwayne Wade of the Miami Heat for four seasons.

Season	Dwayne Wade's Age	Average Points Per Game
2005–2006	24	27.2
2006–2007	25	27.4
2007–2008	26	24.6
2008–2009	27	30.2

Source: *Basketball-Reference*

a. Assume that the ages are the domain. Identify the domain and range.

4a. D = {24, 25, 26, 27}, R = {24.6, 27.2, 27.4, 30.2}

b. Write a relation of ordered pairs for the data.

4b. {(24, 27.2), (25, 27.4), (26, 24.6), (27, 30.2)}

c. State whether the relation is *discrete* or *continuous*.

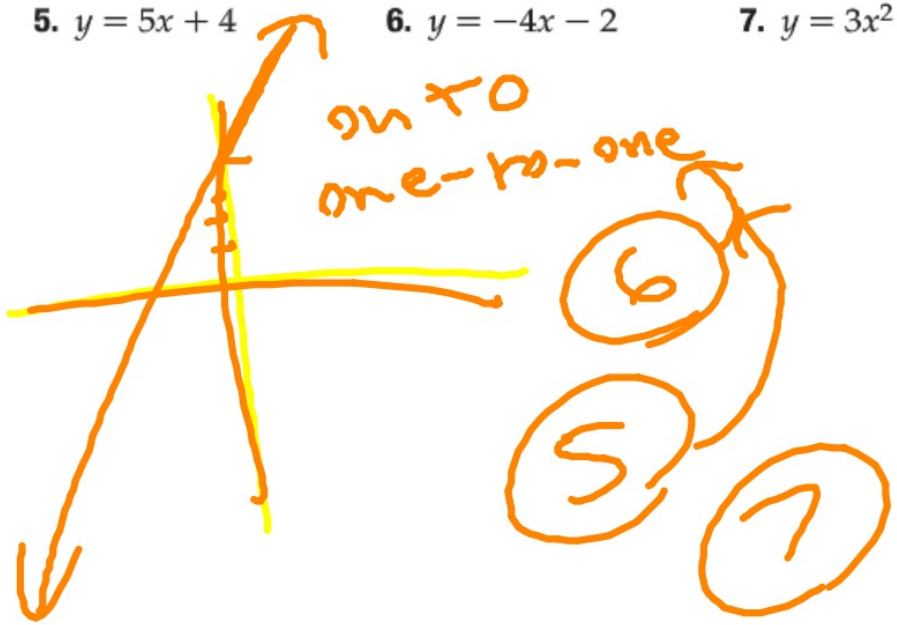
discrete

d. Graph the relation. Is this relation a function?

See margin.

Example 3 Graph each equation, and determine the domain and range. Determine whether the equation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. Then state whether it is *discrete* or *continuous*. **5–8. See Chapter 2 Answer Appendix.**

5. $y = 5x + 4$ **6.** $y = -4x - 2$ **7.** $y = 3x^2$ **8.** $x = 7$



Example 4 Evaluate a Function

Given $f(x) = 2x^2 - 8$, find each value.

a. $f(6)$

$f(x) = 2x^2 - 8$	Original function
$f(6) = 2(6)^2 - 8$	Substitute.
$= 2(36) - 8$	Evaluate 6^2.
$= 72 - 8$ or 64	Simplify.

b. $f(2y)$

$f(x) = 2x^2 - 8$	Original function
$f(2y) = 2(2y)^2 - 8$	Substitute.
$= 2(4y^2) - 8$	$(2y)^2 = 2^2y^2$
$= 8y^2 - 8$	Simplify.

Example 4 **Evaluate each function.**

9. $f(-3)$ if $f(x) = -4x - 8$ **4**

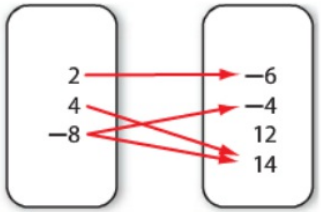
10. $g(5)$ if $g(x) = -2x^2 - 4x + 1$ **−69**

Example 1 **State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both*, or *neither*.**

11.

x	y
−0.3	−6
0.4	−3
1.2	−1
1.2	4

12.



13. {(−3, −4), (−1, 0), (3, 0), (5, 3)}

See margin.

12. D = {−8, 2, 4}, R = {−6, −4, 14}; not a function

11. D = {−0.3, 0.4, 1.2}, R = {−6, −3, −1, 4}; not a function

Example 2 **14. POLITICS** The table below shows the population of several states and the number of U.S. representatives from those states.

a. Make a graph of the data with population on the horizontal axis and representatives on the vertical axis. **See margin.**

b. Identify the domain and range.

c. Is the relation *discrete* or *continuous*? **discrete**

d. Does the graph represent a function? Explain your reasoning.

14b. D = {8.07, 12.44, 16.03, 19.00, 20.90, 33.93}, R = {13, 19, 25, 29, 32, 53}

14d. Yes; each domain value is paired with only one range value so the relation is a function.

State	Population (millions)	Number of Representatives
California	33.93	53
Florida	16.03	25
Illinois	12.44	19
New York	19.00	29
North Carolina	8.07	13
Texas	20.90	32

Source: U.S. Bureau of the Census

Example 3 CCSS **STRUCTURE** Graph each equation, and determine the domain and range. Determine whether the equation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. Then state whether it is *discrete* or *continuous*. **15–20. See Chapter 2 Answer Appendix.**

15. $y = -3x + 2$ **16.** $y = 0.5x - 3$ **17.** $y = 2x^2$

18. $y = -5x^2$ **19.** $y = 4x^2 - 8$ **20.** $y = -3x^3 - 1$

Example 4 **Evaluate each function.**

21 $f(-8)$ if $f(x) = 5x^3 + 1$ **−2559** **22.** $f(2.5)$ if $f(x) = 16x^2$ **100**

23. DIVING The table below shows the pressure on a diver at various depths. **a–d. See margin.**

Depth (ft)	0	20	40	60	80	100
Pressure (atm)	1	1.6	2.2	2.8	3.4	4

a. Write a relation to represent the data.

b. Graph the relation.

c. Identify the domain and range. Is the relation *discrete* or *continuous*?

d. Is the relation a function? Explain your reasoning.

Find each value if $f(x) = 3x + 2$, $g(x) = -2x^2$, and $h(x) = -4x^2 - 2x + 5$.

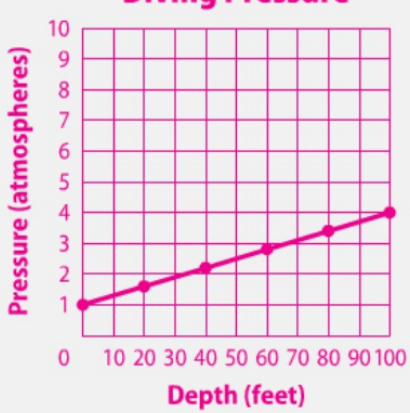
24. $f(-5)$ **−13** **25.** $f(9)$ **29** **26.** $g(-3)$ **−18**

27. $g(-6)$ **−72** **28.** $h(3)$ **−37** **29.** $h(8)$ **−267**

30. $f\left(\frac{2}{3}\right)$ **4** **31.** $g\left(\frac{3}{2}\right)$ **−4.5** **32.** $h\left(\frac{1}{5}\right)$ $\frac{111}{25}$

23a. {(0, 1), (20, 1.6), (40, 2.2), (60, 2.8), (80, 3.4), (100, 4)}

23b.



23c. D = {$x \mid x \geq 0$}; R = {$y \mid y \geq 1$}; continuous

23d. Yes; each domain value is paired with only one range value so the relation is a function.