2-2 Linear Relations and Functions

1 Linear Relations and Functions The points on the graph above lie along a straight line. Relations that have straight line graphs are called **linear relations**. Relations that are not linear are called **nonlinear relations**.

An equation such as $x + y = 5$ is called a linear equation. A **linear equation** has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is always a line.

Linear equations	Nonlinear equations
$4x - 5y = 16$	$2x + 6y^2 = -25$
$x = 10$	$y = \sqrt{x} + 2$
$y = -\frac{2}{3}x - 1$	$x + xy = -\frac{5}{8}$
$y = \frac{1}{2}x$	$y = \frac{1}{x}$

A **linear function** is a function with ordered pairs that satisfy a linear equation. Any linear function can be written in the form $f(x) = mx + b$, where m and b are real numbers.

b. $f(x) = \frac{2}{x}$

No; the expression includes division by the variable.

c. $g(x, y) = 3xy - 4$

No; the two variables are multiplied together.

GuidedPractice **1A. No; the variable appears in the denominator.**

1A. $f(x) = \frac{5}{x+6}$

1B. $g(x) = -\frac{3}{2}x + \frac{1}{3}$ **yes; $m = -\frac{3}{2}$, $b = \frac{1}{3}$**

① $f(x) = \frac{x}{5} + \frac{12}{5}$

Check Your Understanding = Step-by-Step Solutions begin on page R14.

Example 1 State whether each function is a linear function. Write *yes* or *no*. Explain.

1. $f(x) = \frac{x+12}{5}$ **2.** $g(x) = \frac{7-x}{x}$ **3.** $p(x) = 3x^2 - 4$ **4.** $q(x) = -8x - 21$

2. No; it cannot be written as $f(x) = mx + b$. **3. No; x has an exponent that is not 1.**

4. Yes; it is written in $f(x) = mx + b$ form.

PLANTS **The growth rate of a sample of Bermuda grass is given by the function $f(x) = 5.9x + 3.25$, where $f(x)$ is the total height in inches x days after an initial measurement.**

a. How tall is the sample after 3 days?

$f(x) = 5.9x + 3.25$ Original function

$f(3) = 5.9(3) + 3.25$ Substitute 3 for x.

$= 20.95$ Simplify.

The height of the sample after 3 days is 20.95 inches.

b. The term 3.25 in the function represents the height of the grass when it was initially measured. The sample is how many times as tall after 3 days?

Divide the height after 3 days by the initial height. $\frac{20.95}{3.25} \approx 6.4$

The height after 3 days is about 6.4 times as great as the initial height.

Example 2

5 **RECREATION** You want to make sure that you have enough music for a car trip. If each CD is an average of 45 minutes long, the linear function $m(x) = 0.75x$ could be used to find out how many CDs you need to bring.

a. How many hours of music are there on 4 CDs? **3 hours**

b. If the trip you are taking is 6 hours, how many CDs should you bring? **8 CDs**

KeyConcept Standard Form of a Linear Equation

Words	The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers with a greatest common factor of 1, $A \geq 0$, and A and B are not both zero.
Example	$3x + 5y = 12$; $A = 3$, $B = 5$, and $C = 12$

$-(-4x - y = 7)$

$4x + y = -7$

(6) $y = -4x - 7$

$+4x \quad +4x$

$4x + y = -7$

(11) $\frac{4x}{2} - \frac{6y}{2} = \frac{24}{2}$

$2x - 3y = 12$

$Ax + By = C$

Example 3 CCSS **STRUCTURE** Write each equation in standard form. Identify A, B, and C.

6. $y = -4x - 7$ **7.** $y = 6x + 5$ **8.** $3x = -2y - 1$

9. $-8x = 9y - 6$ **10.** $12y = 4x + 8$ **11.** $4x - 6y = 24$

(10) $\frac{-4x}{-4} + \frac{12y}{-4} = \frac{8}{-4}$

$x - 3y = -2$

Example 4 Use Intercepts to Graph a Line

Find the x-intercept and the y-intercept of the graph of $2x - 3y + 8 = 0$. Then graph the equation.

The x-intercept is the value of x when $y = 0$.

$2x - 3y + 8 = 0$	Original equation
$2x - 3(0) + 8 = 0$	Substitute 0 for y.
$2x = -8$	Subtract 8 from each side.
$x = -4$	Divide each side by 2.

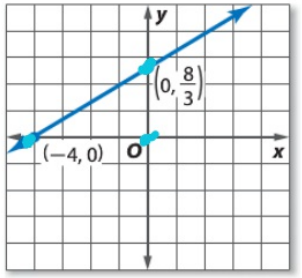
The x-intercept is -4.

Likewise, the y-intercept is the value of y when $x = 0$.

$2x - 3y + 8 = 0$	Original equation
$2(0) - 3y + 8 = 0$	Substitute 0 for x.
$-3y = -8$	Subtract 8 from each side.
$y = \frac{8}{3}$	Divide each side by -3.

The y-intercept is $\frac{8}{3}$.

Use these ordered pairs to graph the equation.



12

x-int.

$0 = 5x + 12$

$x = -12/5$

$(-12/5, 0)$

y-int.

$y = 5(0) + 12$

$y = 12$

$(0, 12)$

Example 4 **Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation using the intercepts.** **12–15. See Chapter 2 Answer Appendix.**

12. $y = 5x + 12$ **13.** $y = 4x - 10$ **14.** $2x + 3y = 12$ **15.** $3x - 4y - 6 = 15$

Example 1 **State whether each equation or function is a linear function. Write *yes* or *no*. Explain.**

16–24. See margin.

16. $3y - 4x = 20$ **17.** $y = x^2 - 6$ **18.** $h(x) = 6$

19. $j(x) = 2x^2 + 4x + 1$ **20.** $g(x) = 5 + \frac{6}{x}$ **21.** $f(x) = \sqrt{7 - x}$

22. $4x + \sqrt{y} = 12$ **23.** $\frac{1}{x} + \frac{1}{y} = 1$ **24.** $f(x) = \frac{4x}{5} + \frac{8}{3}$

Example 2

25. ROLLER COASTERS The speed of the Steel Dragon 2000 roller coaster in Mie Prefecture, Japan, can be modeled by $y = 10.4x$, where y is the distance traveled in meters in x seconds.

a. How far does the coaster travel in 25 seconds? **260 m**

b. The speed of the Kingda Ka roller coaster in Jackson, New Jersey, can be described by $y = 33.9x$. Which coaster travels faster? Explain your reasoning. **See margin.**

Example 3 **Write each equation in standard form. Identify A, B, and C.** **26–34. See margin.**

26. $-7x - 5y = 35$ **27.** $8x + 3y + 6 = 0$ **28.** $10y - 3x + 6 = 11$

29. $-6x - 3y - 12 = 21$ **30.** $3y = 9x - 12$ **31** $2.4y = -14.4x$

32. $\frac{2}{3}y - \frac{3}{4}x + \frac{1}{6} = 0$ **33.** $\frac{4}{5}y + \frac{1}{8}x = 4$ **34.** $-0.08x = 1.24y - 3.12$

Example 4 **Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation using the intercepts.** **35–40. See Chapter 2 Answer Appendix.**

35. $y = -8x - 4$ **36.** $5y = 15x - 90$ **37.** $-4y + 6x = -42$

38. $-9x - 7y = -30$ **39.** $\frac{1}{3}x - \frac{2}{9}y = 4$ **40.** $\frac{3}{4}y - \frac{2}{3}x = 12$

Lesson 2-2

6. $4x + y = -7$; $A = 4$, $B = 1$, $C = -7$

7. $6x - y = -5$; $A = 6$, $B = -1$, $C = -5$

8. $3x + 2y = -1$; $A = 3$, $B = 2$, $C = -1$

9. $8x + 9y = 6$; $A = 8$, $B = 9$, $C = 6$

10. $x - 3y = -2$; $A = 1$, $B = -3$, $C = -2$

11. $2x - 3y = 12$; $A = 2$, $B = -3$, $C = 12$

Additional Answers

16. Yes; it can be written in $f(x) = mx + b$ form, where $m = \frac{4}{3}$ and $b = \frac{20}{3}$.

17. No; x has an exponent other than 1.

18. Yes, it can be written in $f(x) = mx + b$ form, where $m = 0$ and $b = 6$.

19. No; x has an exponent other than 1.

20. No; it cannot be written in $f(x) = mx + b$ form.

21. No; it cannot be written in $f(x) = mx + b$ form.

22. No; it cannot be written in $f(x) = mx + b$ form.

23. No; it cannot be written in $f(x) = mx + b$ form; There is an xy term.

24. Yes; it can be written in $f(x) = mx + b$ form, where $m = \frac{4}{5}$ and $b = \frac{8}{3}$.

25b. Kingda Ka; Sample answer: The Kingda Ka travels 847.5 meters in 25 seconds, so it travels a greater distance in the same amount of time.

26. $7x + 5y = -35$; $A = 7$, $B = 5$, $C = -35$

27. $8x + 3y = -6$; $A = 8$, $B = 3$, $C = -6$

28. $3x - 10y = -5$; $A = 3$, $B = -10$, $C = -5$

29. $2x + y = -11$; $A = 2$, $B = 1$, $C = -11$

30. $3x - y = 4$; $A = 3$, $B = -1$, $C = 4$

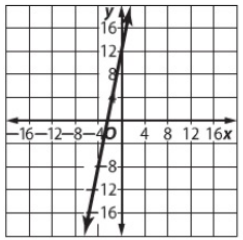
31. $6x + y = 0$; $A = 6$, $B = 1$, $C = 0$

32. $9x - 8y = 2$; $A = 9$, $B = -8$, $C = 2$

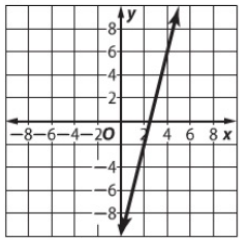
33. $5x + 32y = 160$; $A = 5$, $B = 32$, $C = 160$

34. $2x + 31y = 78$; $A = 2$, $B = 31$, $C = 78$

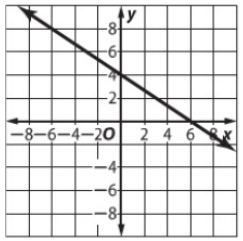
12. $-\frac{12}{5}$; 12



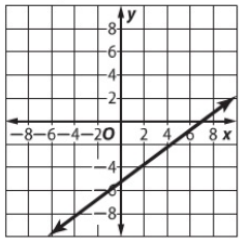
13. $\frac{5}{2}$; -10



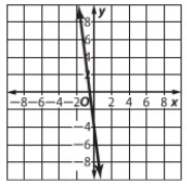
14. 6; 4



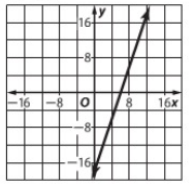
15. 7; $-\frac{21}{4}$



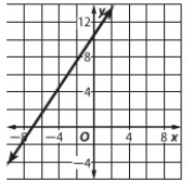
35. -0.5; -4



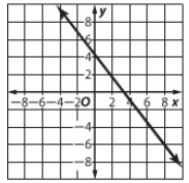
36. 6; -18



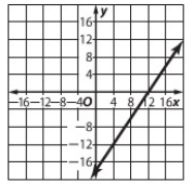
37. -7; 10.5



38. $\frac{10}{3}$, $\frac{30}{7}$



39. 12; -18



40. -18; 16