

2-3 Rate of Change and Slope

1 Rate of Change **Rate of change** is a ratio that compares how much one quantity changes, on average, relative to the change in another quantity. If x is the independent variable and y is the dependent variable, then rate of change = $\frac{\text{change in } y}{\text{change in } x}$. This is sometimes referred to as $\frac{\Delta y}{\Delta x}$.


Real-World Example 1 Constant Rate of Change

CHEMISTRY The table shows the temperature of a solution after it has been removed from a heat source. Find the rate of change in temperature for the solution.

Use the ordered pairs (2, 139.4) and (5, 133.1).

$$\begin{aligned} \text{rate of change} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{\text{change in temperature}}{\text{change in time}} \\ &= \frac{133.1 - 139.4}{5 - 2} \\ &= \frac{-6.3}{3} \text{ or } -2.1 \end{aligned}$$

The rate of change is -2.1 . This means that the temperature is decreasing by 2.1°C each minute.



Time (min)	Temperature ($^\circ\text{C}$)
0	143.6
2	139.4
5	133.1
8	126.8
12	118.4

Example 1

CCSS REGULARITY Find the rate of change for each set of data.

1.

Time (min)	2	4	6	8	10
Distance (ft)	12	24	36	48	60

6 feet/min

2.

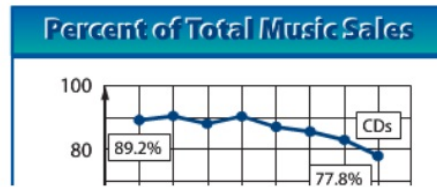
Time (sec)	5	10	15	20	25
Volume (cm^3)	16	32	48	64	80

3.2 cm^3/sec



Real-World Example 2 Average Rate of Change

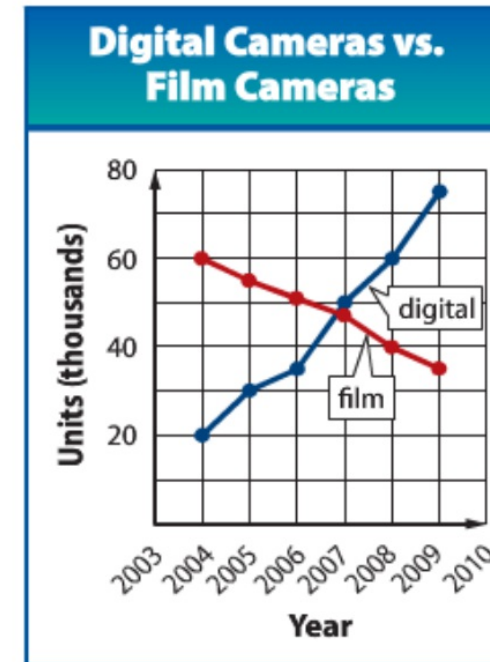
MUSIC Refer to the graph at the right. Find the average rate of change of the percent of total music sales for both CDs and downloads from 2001 to 2008. Compare the rates.



Example 2

3. CAMERAS The graph shows the number of digital still cameras and film cameras sold by Yellow Camera Stores in recent years.

- Find the average rate of change of the number of digital cameras sold from 2004 to 2009. **about 11,000 per year**
- Find the average rate of change of the number of film cameras sold from 2004 to 2009. **about -5000 per year**
- What do the signs of each rate of change represent?
The positive rate in part a represents an increase in sales of digital cameras. The negative rate in part b represents a decrease in sales of film cameras.



Example 3 Find Slope Using Coordinates

Find the slope of the line that passes through $(-4, 3)$ and $(2, 5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope Formula

$$= \frac{5 - 3}{2 - (-4)}$$

$(x_1, y_1) = (-4, 3), (x_2, y_2) = (2, 5)$

$$= \frac{2}{6} \text{ or } \frac{1}{3}$$

Simplify.

Example 3

Find the slope of the line that passes through each pair of points.

4. $(3, 2), (8, 12)$ **2**



5. $(-1, 4), (3, -8)$ **-3**



6. $(-2, -5), (-7, 10)$ **-3**



Example 4 Find Slope Using a Graph

Find the slope of the line shown at the right.

The line passes through $(-2, 0)$ and $(0, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

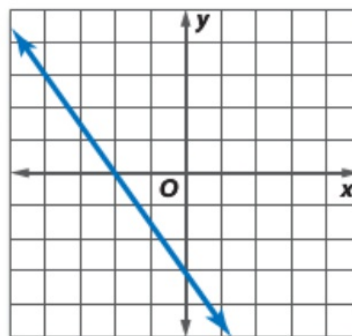
Slope Formula

$$= \frac{-3 - 0}{0 - (-2)}$$

$(x_1, y_1) = (-2, 0), (x_2, y_2) = (0, -3)$

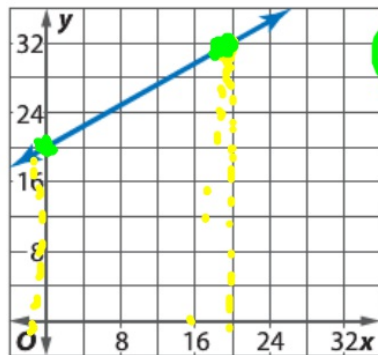
$$= \frac{-3}{2} \text{ or } -\frac{3}{2}$$

Simplify.



Example 4 Determine the rate of change of each graph.

7.

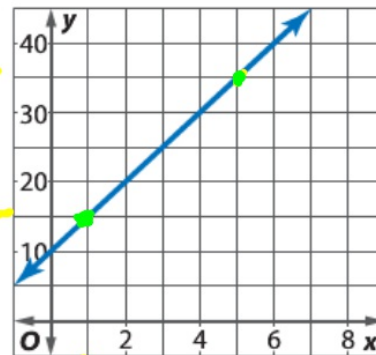


$\frac{3}{5}$

$$\frac{12}{20}$$

20

8.



5

4

$$\Delta y = 12$$

$$\frac{\Delta y}{\Delta x} = \frac{12}{20} = \frac{3}{5}$$

$$\Delta x = 20$$

Practice and Problem Solving

Example 1

Find the rate of change for each set of data.

9

Time (day)	3	6	9	12	15
Height (mm)	20	40	60	80	100

$$\frac{20}{3} \text{ mm/day}$$

10.

Weight (lb)	11	22	33	44	55
Cost (\$)	8	16	24	32	40

$$\frac{8}{11} \text{ \$/lb}$$



Example 2

11. **HEALTH** The table below shows Lisa's temperature during an illness over a 3-day period.

Day	Monday		Tuesday		Wednesday	
Time	8:00 A.M.	8:00 P.M.	8:00 A.M.	8:00 P.M.	8:00 A.M.	8:00 P.M.
Temp (°F)	100.5	102.3	103.1	100.7	99.9	98.6

11b. $-0.125^\circ/\text{h}$;
Yes; the number should be negative because her temperature is dropping.

- What was the average rate of change in Lisa's temperature from 8:00 A.M. on Monday to 8:00 P.M. on Monday? **$0.15^\circ/\text{h}$**
- What was the average rate of change in Lisa's temperature from 8:00 A.M. on Tuesday to 8:00 P.M. on Wednesday? Is your answer reasonable? What does the sign of the rate mean?
- During which 12-hour period was the average rate of change in Lisa's temperature the greatest? **Tuesday 8:00 A.M.–Tuesday 8:00 P.M.**

Example 3

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.

12. $(-2, 11), (5, 6)$ $-\frac{5}{7}$

13. $(-9, -11), (6, 3)$ $\frac{14}{15}$

14. $(-1.5, 3.5), (4.5, 6)$ $\frac{5}{12}$

15. $(-4.5, 9.5), (-1, 2.5)$ -2

16. $(-8, -0.5), (-4, 5)$ $\frac{11}{8}$

17. $(-6, -2), (-1.5, 5.5)$ $\frac{5}{3}$

Example 4

Determine the rate of change of each graph.

