2-6 Special Functions

Example 1 Piecewise-Define

Graph $f(x) = \begin{cases} x - 2 \text{ if } x < - \\ x + 3 \text{ if } x \geq - \end{cases}$

Step 1 Graph $f(x) = x - 2$ f

$f(x) = x - 2$
$= (-1) - 2$
$= -3$

Because -1 does no
begin with a circle a

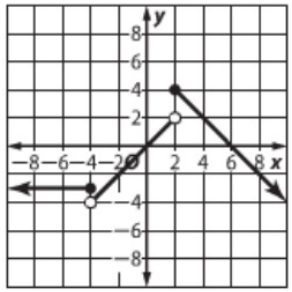
Step 2 Graph $f(x) = x + 3$ f

$f(x) = x + 3$
$= (-1) + 3$
$= 2$

Because -1 satisfies the inequality, begin with a dot at $(-1, 2)$.

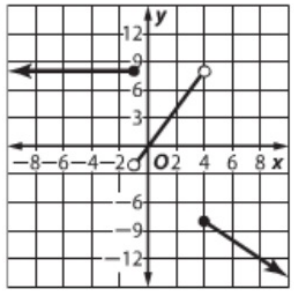
Lesson 2-6

1.



D = {all real numbers};
R = {$y \mid y \leq 4$}

2.



D = {all real numbers};
R = {$y \mid 8 \geq y > -2$ or $y \leq -8$}

Example 1 **Graph each function. Identify the domain and range.** **1, 2. See Chapter 2 Answer Appendix.**

1. $g(x) = \begin{cases} -3 \text{ if } x \leq -4 \\ x \text{ if } -4 < x < 2 \\ -x + 6 \text{ if } x \geq 2 \end{cases}$

2. $f(x) = \begin{cases} 8 \text{ if } x \leq -1 \\ 2x \text{ if } -1 < x < 4 \\ -4 - x \text{ if } x \geq 4 \end{cases}$

1. $g(x) = \begin{cases} -3 \text{ if } x \leq -4 \\ x \text{ if } -4 < x < 2 \\ -x + 6 \text{ if } x \geq 2 \end{cases}$

$g(x) = -3$

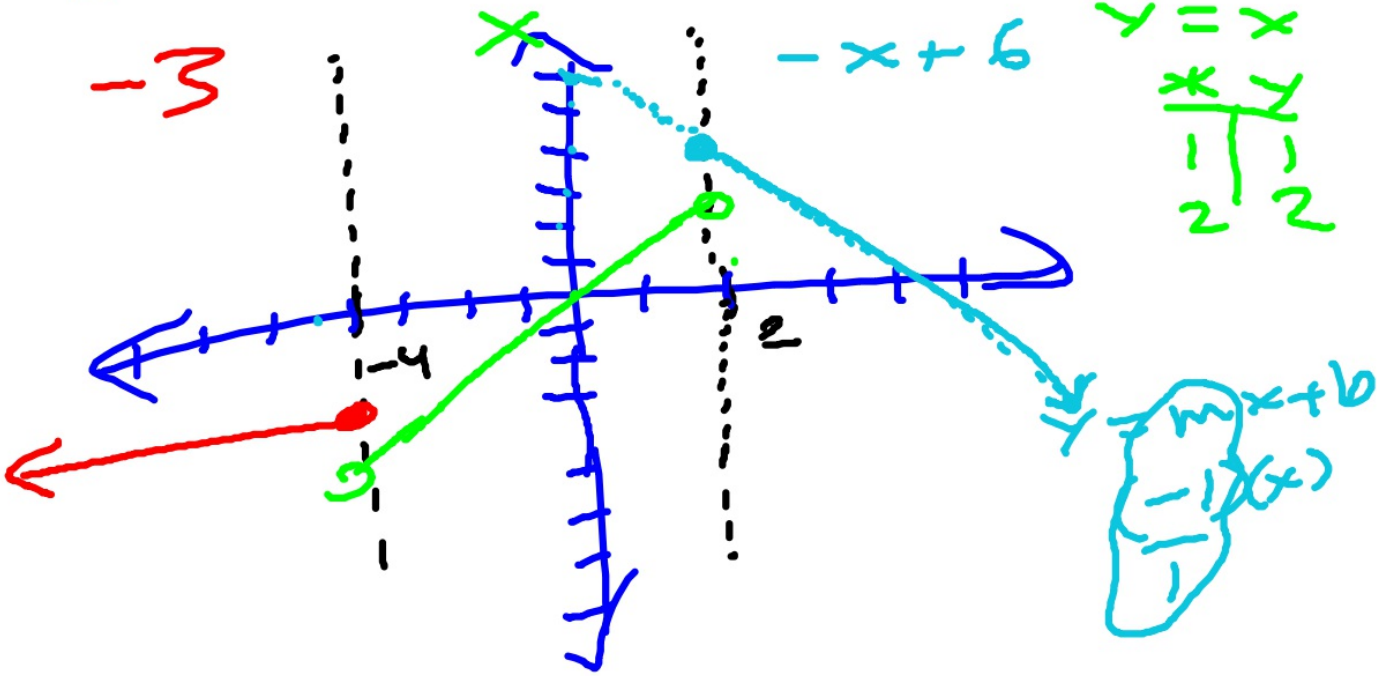
$y = -3$

$g(x) = x$

$y = x$

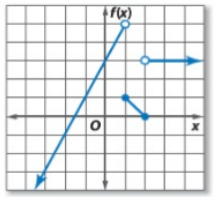
x	y
1	1
2	2

$-x + 6$



Example 2 Write a Piecewise-Defined Function

Write the piecewise-defined function shown in the graph.



Examine and write a function for each portion of the graph.

The left portion of the graph is the graph of $f(x) = 2x + 3$. There is a circle at (1, 5), so the linear function is defined for $\{x|x < 1\}$.

The center portion of the graph is the graph of $f(x) = -x + 2$. There are dots at (1, 1) and (2, 0), so the linear function is defined for $\{x|1 \le x \le 2\}$.

The right portion of the graph is the constant function $f(x) = 3$. There is a circle at (2, 3), so the constant function is defined for $\{x|x > 2\}$.

Write the piecewise-defined function.

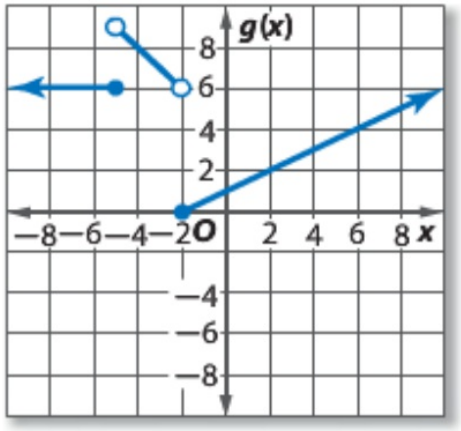
$$f(x) = \begin{cases} 2x + 3 \text{ if } x < 1 \\ -x + 2 \text{ if } 1 \le x \le 2 \\ 3 \text{ if } x > 2 \end{cases}$$

3. $g(x) = \begin{cases} x + 4 \text{ if } x < -2 \\ -3 \text{ if } -2 \le x \le 3 \\ -2x + 12 \text{ if } x > 3 \end{cases}$

4. $g(x) = \begin{cases} 6 \text{ if } x \le -5 \\ -x + 4 \text{ if } -5 < x < -2 \\ \frac{1}{2}x + 1 \text{ if } x \ge -2 \end{cases}$

Example 2 **Write the piecewise-defined function shown in each graph.** 3, 4. See Chap

3.



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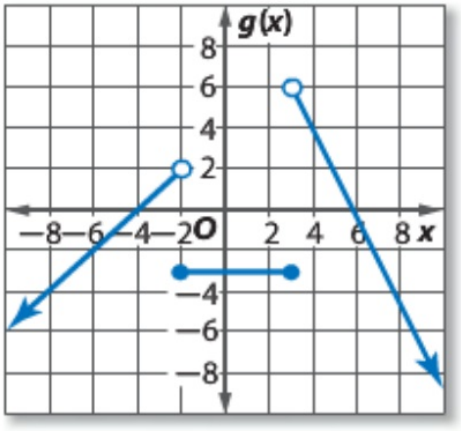
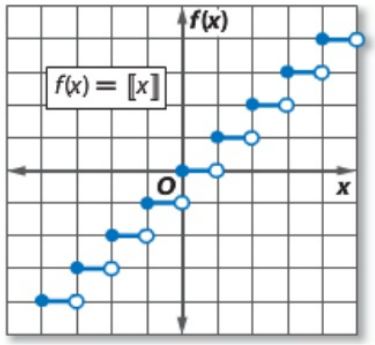


2 Step Functions and Absolute Value Functions

Unlike a piecewise-defined function, a **piecewise-linear function** contains a single expression. A common piecewise-linear function is the step function. The graph of a **step function** consists of line segments.

The **greatest integer function**, written $f(x) = [\![x]\!]$, is one kind of step function. The symbol $[\![x]\!]$ means *the greatest integer less than or equal to* x. For example, $[\![3.25]\!] = 3$ and $[\![-4.6]\!] = -5$.



KeyConcept Parent Function of Absolute Value Functions

Parent function: $f(x) = |x|$, defined as

$$f(x) = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

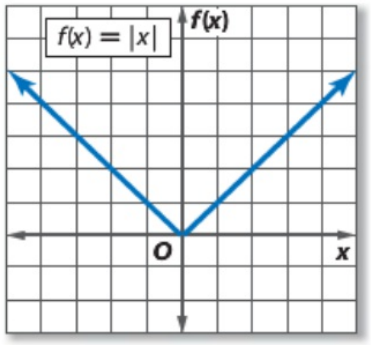
Type of graph: V-shaped

Domain: all real numbers

Range: all nonnegative real numbers

Intercepts: $x = 0$, $f(x) = 0$

Not defined: $f(x) < 0$



Example 4 Absolute Value Functions

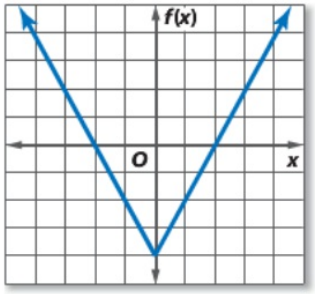
Graph $f(x) = |2x| - 4$. Identify the domain and range.

Create a table of values.

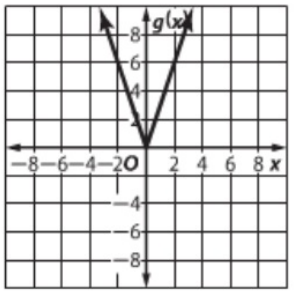
x	$\|2x\| - 4$
−3	2
−2	0
−1	−2
0	−4
1	−2
2	0
3	2

Graph the points and connect them.

The domain is the set of all real numbers. The range is $\{f(x) | f(x) \geq -4\}$.

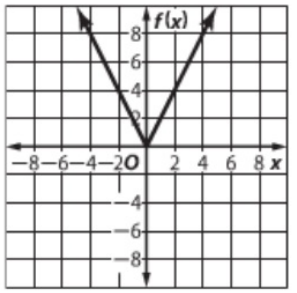


8.



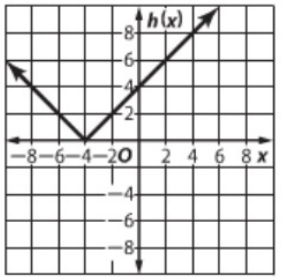
D = {all real numbers};
R = $\{g(x) | g(x) \geq 0\}$

9.



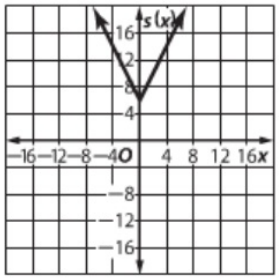
D = {all real numbers};
R = $\{f(x) | f(x) \geq 0\}$

10.



D = {all real numbers};
R = $\{h(x) | h(x) \geq 0\}$

11.



D = {all real numbers};
R = $\{s(x) | s(x) \geq 6\}$

Example 4 **Graph each function. Identify the domain and range.**

8. $g(x) = |-3x|$

9. $f(x) = 2|x|$

10. $h(x) = |x + 4|$

11. $s(x) = |-2x| + 6$

Example 1 Graph each function. Identify the domain and range. **12–15. See margin.**

12. $f(x) = \begin{cases} -3x \text{ if } x \le -4 \\ x \text{ if } 0 < x \le 3 \\ 8 \text{ if } x > 3 \end{cases}$

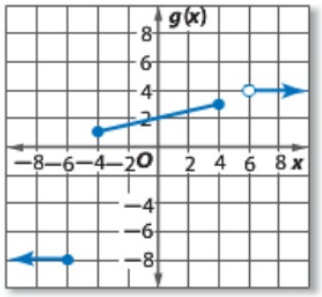
13. $f(x) = \begin{cases} 2x \text{ if } x \le -6 \\ 5 \text{ if } -6 < x \le 2 \\ -2x + 1 \text{ if } x > 4 \end{cases}$

14. $g(x) = \begin{cases} 2x + 2 \text{ if } x < -6 \\ x \text{ if } -6 \le x \le 2 \\ -3 \text{ if } x > 2 \end{cases}$

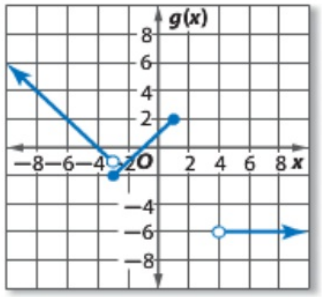
15. $g(x) = \begin{cases} -2 \text{ if } x < -4 \\ x - 3 \text{ if } -1 \le x \le 5 \\ 2x - 15 \text{ if } x > 7 \end{cases}$

Example 2 Write the piecewise-defined function shown in each graph. **16–19. See Chapter 2 Answer Appendix.**

16.



17



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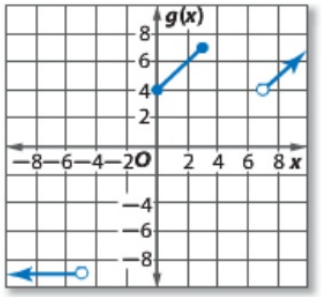

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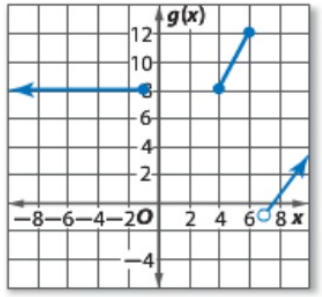
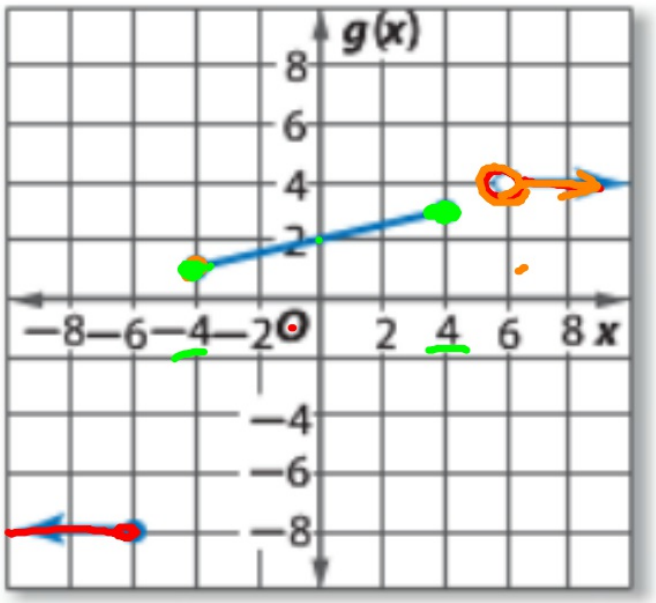


16.

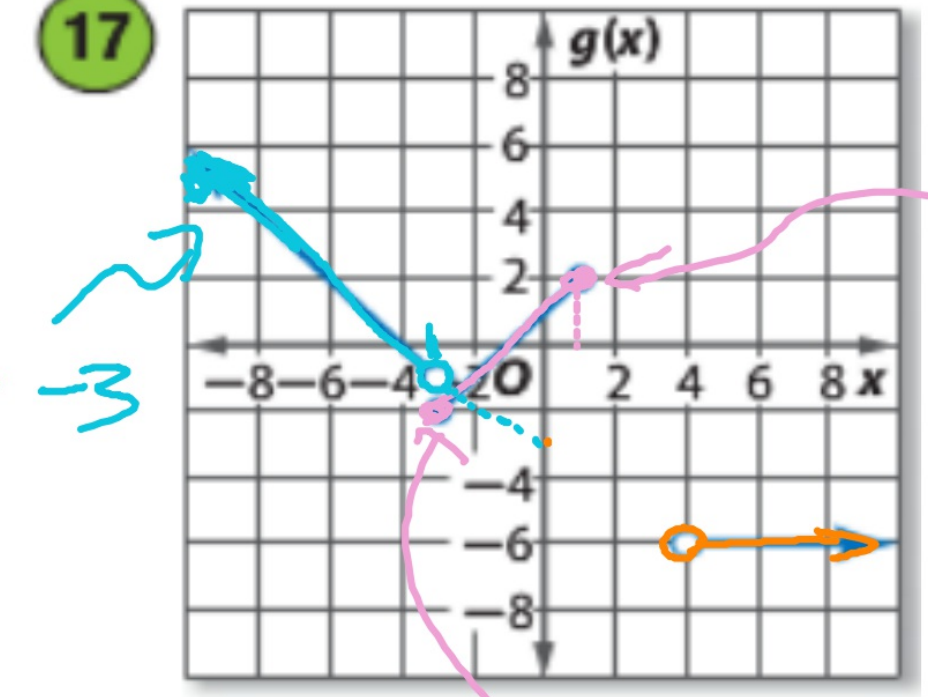


16

$$g(x) = \begin{cases} -8, & x \le -6 \\ \frac{1}{4}x + 2, & -4 \le x \le 4 \\ 4, & x > 6 \end{cases}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{2}{8} = \frac{1}{4}$$

17



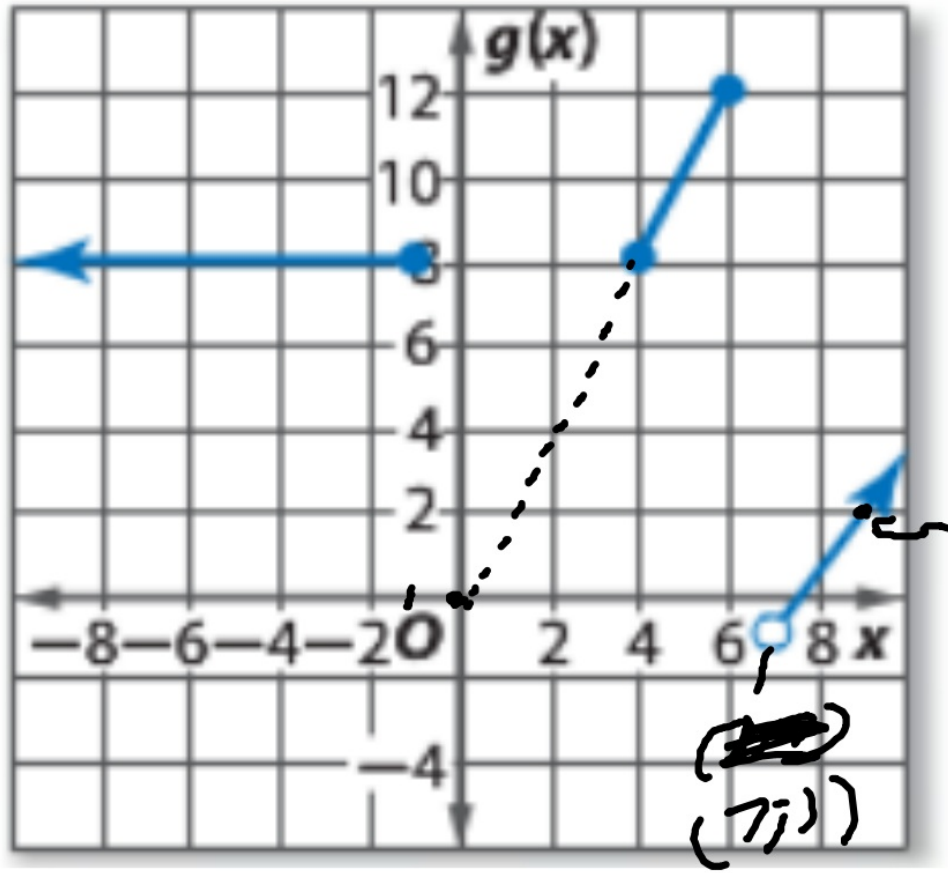
$-x-3$

$(1,2)$

$\begin{cases} -x-3, & x<-3 \\ \frac{3}{4}x+1, & -3\le x\le 1 \\ -6, & x>4 \end{cases}$

$(-3,-1)$

$\frac{\Delta y}{\Delta x}=\frac{3}{4}$



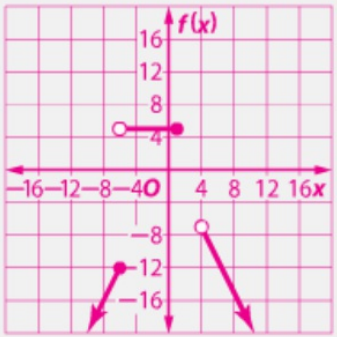
$\frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$

$g(x) = \begin{cases} 8, & x \le -1 \\ 2x, & 4 \le x \le 6 \\ \text{(9,2)} \; \text{⌊} & x > 7 \end{cases}$

19. $g(x) = \begin{cases} 8 \text{ if } x \le -1 \\ 2x \text{ if } 4 \le x \le 6 \\ 2x - 15 \text{ if } x > 7 \end{cases}$

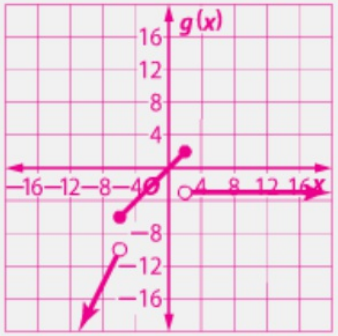
Additional Answers

13.



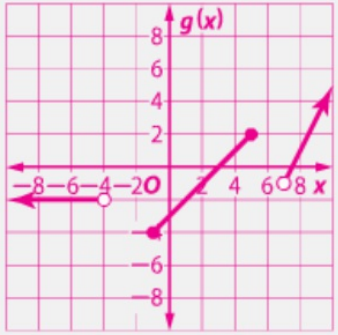
D = {x | x ≤ 2 or x > 4};
R = {f(x) | f(x) < −7, or f(x) = 5}

14.



D = {all real numbers};
R = {g(x) | g(x) < −10 or
−6 ≤ g(x) ≤ 2}

15.



D = {x | x < −4, −1 ≤ x ≤ 5, or
x > 7};
R = {g(x) | g(x) ≥ −4}

16. $g(x) = \begin{cases} -8 \text{ if } x \le -6 \\ 0.25x + 2 \text{ if } -4 \le x \le 4 \\ 4 \text{ if } x > 6 \end{cases}$

17. $g(x) = \begin{cases} -x - 4 \text{ if } x < -3 \\ x + 1 \text{ if } -3 \le x \le 1 \\ -6 \text{ if } x > 4 \end{cases}$

18. $g(x) = \begin{cases} -9 \text{ if } x < -5 \\ x + 4 \text{ if } 0 \le x \le 3 \\ x - 3 \text{ if } x > 7 \end{cases}$

19. $g(x) = \begin{cases} 8 \text{ if } x \le -1 \\ 2x \text{ if } 4 \le x \le 6 \\ 2x - 15 \text{ if } x > 7 \end{cases}$

Example 3 **Graph each function. Identify the domain and range.** **20–23. See Chapter 2 Answer Appendix.**

20. $f(x) = [\![x]\!] - 6$

21. $h(x) = [\![3x]\!] - 8$

22. $f(x) = [\![3x + 2]\!]$

23. $g(x) = 2[\![0.5x + 4]\!]$

Example 4 **Graph each function. Identify the domain and range.** **24–29. See Chapter 2 Answer Appendix.**

24. $f(x) = |x - 5|$

25. $g(x) = |x + 2|$

26. $h(x) = |2x| - 8$

27. $k(x) = |-3x| + 3$

28. $f(x) = 2|x - 4| + 6$

29. $h(x) = -3|0.5x + 1| - 2$

30. GIVING Patrick is donating money and volunteering his time to an organization that restores homes for the needy. His employer will match his monetary donations up to $100.

a. Identify the type of function that models the total amount of money received by the charity when Patrick donates x dollars. **piecewise**

b. Write and graph a function for the situation. **See Chapter 2 Answer Appendix.**

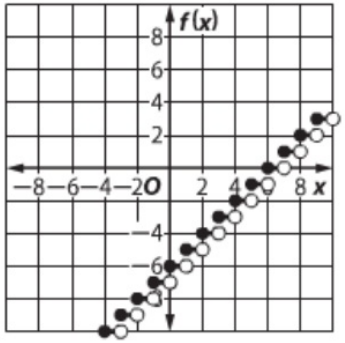
31. CCSS **SENSE-MAKING** A car's speedometer reads 60 miles an hour.

a. Write an absolute value function for the difference between the car's actual speed a and the reading on the speedometer. **$f(a) = |a - 60|$**

b. What is an appropriate domain for the function? Explain your reasoning. **$\{a|a \geq 0\}$**

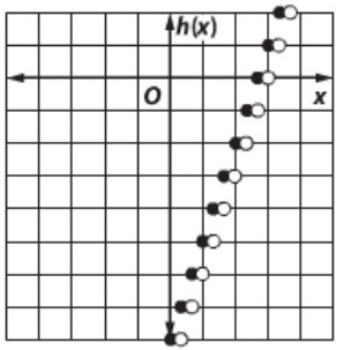
c. Use the domain to graph the function. **See Chapter 2 Answer Appendix.**

20.



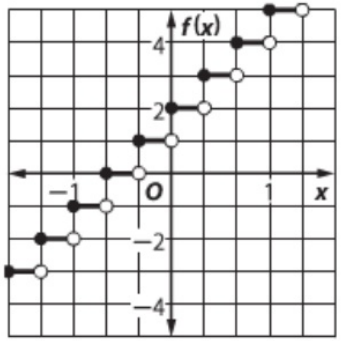
D = {all real numbers};
R = {all integers}

21.



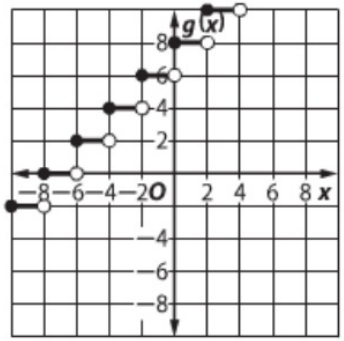
D = {all real numbers};
R = {all integers}

22.



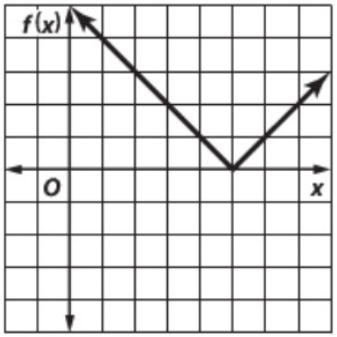
D = {all real numbers};
R = {all integers}

23.



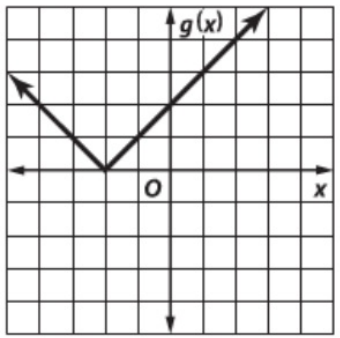
D = {all real numbers};
R = {all even integers}

24.



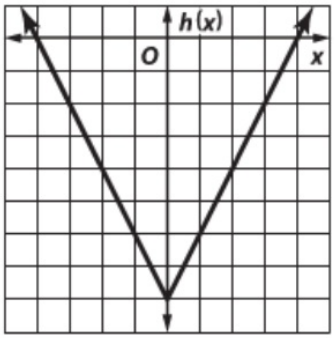
D = {all real numbers};
R = $\{f(x) \mid f(x) \geq 0\}$

25.



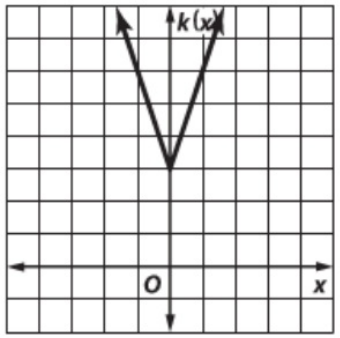
D = {all real numbers};
R = $\{g(x) \mid g(x) \geq 0\}$

26.



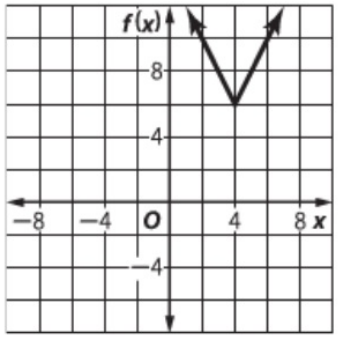
D = {all real numbers};
R = $\{h(x) \mid h(x) \geq -8\}$

27.



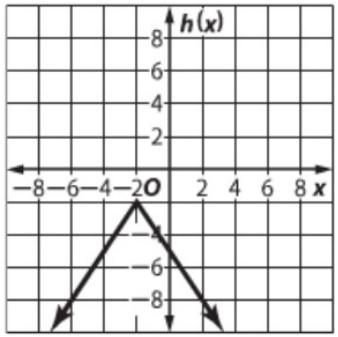
D = {all real numbers};
R = $\{k(x) \mid k(x) \geq 3\}$

28.



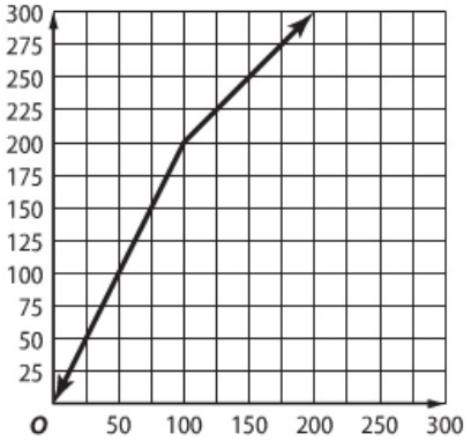
D = {all real numbers};
R = $\{f(x) \mid f(x) \geq 6\}$

29.



D = {all real numbers};
R = $\{h(x) \mid h(x) \leq -2\}$

30b. $f(x) = \begin{cases} 2x \text{ if } 0 < x \leq 100 \\ x + 100 \text{ if } x > 100 \end{cases}$



31c.