3-1 Graphing Linear Equations

1 Linear Equations and Intercepts A **linear equation** is an equation that forms a line when it is graphed. Linear equations are often written in the form $Ax + By = C$. This is called the **standard form** of a linear equation. In this equation, C is called a **constant**, or a number. Ax and By are variable terms.

KeyConcept Standard Form of a Linear Equation

Words	The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero, and A, B, and C are integers with a greatest common factor of 1.
Examples	In $3x + 2y = 5$, $A = 3$, $B = 2$, and $C = 5$. In $x = -7$, $A = 1$, $B = 0$, and $C = -7$.

Example 1 Identify Linear Equations

Determine whether each equation is a linear equation. Write the equation in standard form.

a. $y = 4 - 3x$

Rewrite the equation so that it appears in standard form.

$y = 4 - 3x$	Original equation
$y + 3x = 4 - 3x + 3x$	Add $3x$ to each side.
$3x + y = 4$	Simplify.

The equation is now in standard form where $A = 3$, $B = 1$, and $C = 4$. This is a linear equation.

b. $6x - xy = 4$

Since the term xy has two variables, the equation cannot be written in the form $Ax + By = C$. Therefore, this is not a linear equation.

$3\left(\frac{2}{3}x - \frac{1}{3}y\right) = 3\left(\frac{2}{3}\right)$

Example 1 **Determine whether each equation is a linear equation. Write *yes* or *no*. If yes, write the equation in standard form.**

1. $x = y - 5$ — yes; $x - y = -5$
2. $-2x - 3 = y$ — yes; $2x + y = -3$
3. $-4y + 6 = 2$ — yes; $y = 1$
4. $\frac{2}{3}x - \frac{1}{3}y = 2$ — yes; $2x - y = 6$

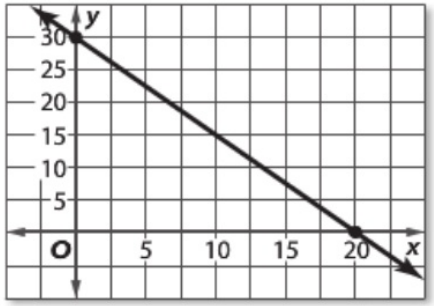
$+3 \quad -y \quad +3$

$0x + 1y = 1$

$-(-2x - y) = (3)$

Standardized Test Example 2 Find Intercepts from a Graph

Find the x- and y-intercepts of the line graphed at the right.



A x-intercept is 0; y-intercept is 30.

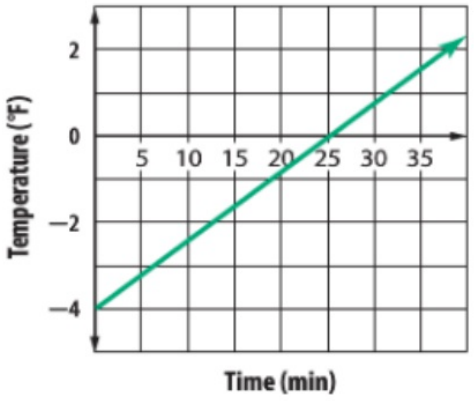
B x-intercept is 20; y-intercept is 30.

C x-intercept is 20; y-intercept is 0.

D x-intercept is 30; y-intercept is 20.

Examples 2–3 **Find the x- and y-intercepts of the graph of each linear function. Describe what the intercepts mean.**

5. **Increasing Temperature**



5. 25, −4; The x-intercept 25 means that after 25 minutes, the temperature is 0°F. The y-intercept −4 means that at time 0, the temperature is −4°F.

Real-World Example 3 Find Intercepts from a Table

SWIMMING POOL A swimming pool is being drained at a rate of 720 gallons per hour. The table shows the function relating the volume of water in a pool and the time in hours that the pool has been draining.

Draining a Pool	
Time (h)	Volume (gal)
x	y
0	10,080
2	8640
6	5760
10	2880
12	1440
14	0

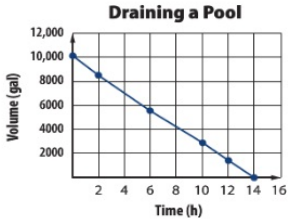
a. Find the x- and y-intercepts of the graph of the function.

x-intercept $= 14$ 14 is the value of x when $y = 0$.
y-intercept $= 10{,}080$ 10,080 is the value of y when $x = 0$.

b. Describe what the intercepts mean in this situation.

The x-intercept 14 means that after 14 hours, the water has a volume of 0 gallons, or the pool is completely drained.

The y-intercept 10,080 means that the pool contained 10,080 gallons of water at time 0, or before it started to drain. This is shown in the graph.



Examples 2–3 Find the x- and y-intercepts of the graph of each linear function. Describe what the intercepts mean.

6.

Position of Scuba Diver	
Time (s)	Depth (m)
x	y
0	−24
3	−18
6	−12
9	−6
12	0

6. 12, −24; The x-intercept 12 means that after 12 seconds, the scuba diver is at a depth of 0 meters, or at the surface. The y-intercept −24 means that at time 0, the scuba diver is at a depth of −24 meters, or 24 meters below sea level.

Example 4 Graph by Using Intercepts

Graph $2x + 4y = 16$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$.

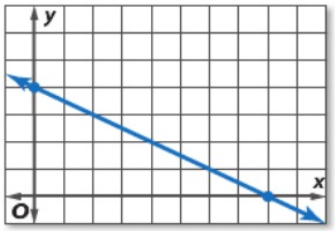
$2x + 4y = 16$	Original equation
$2x + 4(0) = 16$	Replace y with 0.
$2x = 16$	Simplify.
$x = 8$	Divide each side by 2.

To find the y-intercept, let $x = 0$.

$2x + 4y = 16$	Original equation
$2(0) + 4y = 16$	Replace x with 0.
$4y = 16$	Simplify.
$y = 4$	Divide each side by 4.

The y-intercept is 4. This means the graph intersects the y-axis at (0, 4).

Plot these two points and then draw a line through them.



(−4, 0)

Example 4 **Graph each equation by using the x- and y-intercepts.**

7. $y = 4 + x$ (0, 4)

7–8. See Ch. 3 Answer Appendix.

(y=0) x-int. $0 = 4 + x$, $-4 \quad -4$, $x = -4$

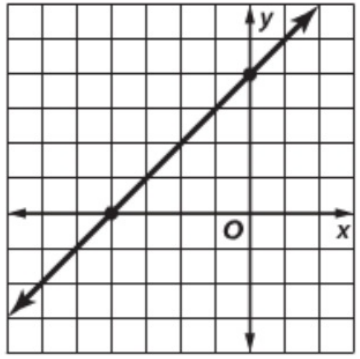
y-int (x=0) $y = 4 + 0$, $y = 4$

8. $2x - 5y = 1$

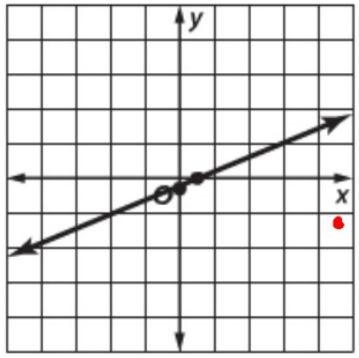
$2x - 5(0) = 1$, $2x = 1$, $x = \frac{1}{2} = .5$

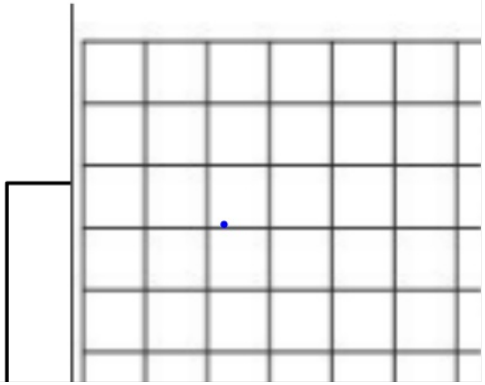
$2(0) - 5y = 1$, $-5y = 1$, $y = -\frac{1}{5} = -.2$

7.



8.



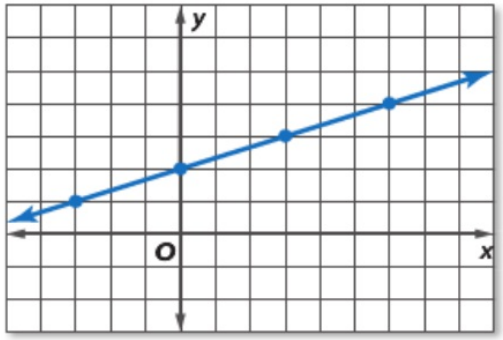


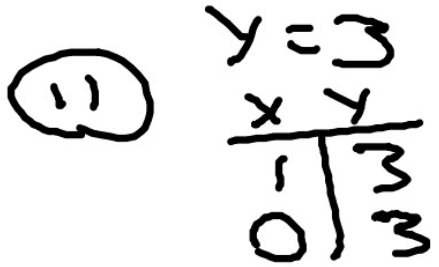
Example 5 Graph by Making a Table

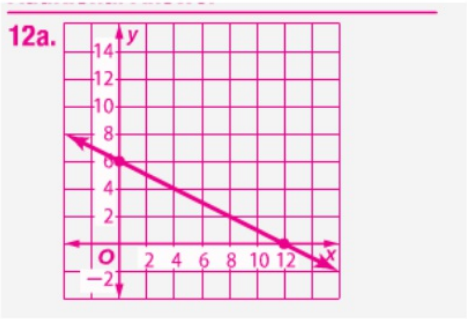
Graph $y = \frac{1}{3}x + 2$.

The domain is all real numbers. Select values from the domain and make a table. When the x-coefficient is a fraction, select a number from the domain that is a multiple of the denominator. Create ordered pairs and graph them.

x	$\frac{1}{3}x + 2$	y	(x, y)
-3	$\frac{1}{3}(-3) + 2$	1	$(-3, 1)$
0	$\frac{1}{3}(0) + 2$	2	$(0, 2)$
3	$\frac{1}{3}(3) + 2$	3	$(3, 3)$
6	$\frac{1}{3}(6) + 2$	4	$(6, 4)$







Example 5 **Graph each equation by making a table.** **9–11. See Ch. 3 Answer Appendix.**

9. $x + 2y = 4$ **10.** $-3 + 2y = -5$ **11.** $y = 3$

12. CCSS **REASONING** The equation $5x + 10y = 60$ represents the number of children x and adults y who can attend the rodeo for \$60.

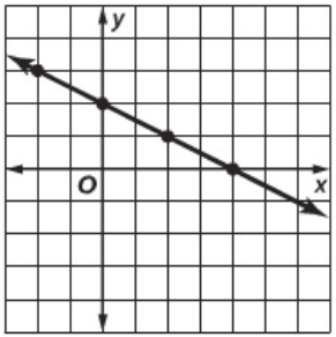
a. Use the x- and y-intercepts to graph the equation. **See margin.**

b. Describe what these values mean.

The *x*-intercept means that 12 children and 0 adults can attend for \$60.
The *y*-intercept means that 0 children and 6 adults can attend for \$60.

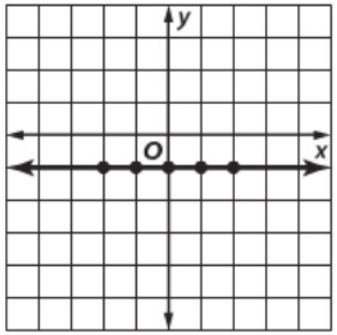
9.

x	$y = 2 - \frac{x}{2}$	y	(x, y)
-4	$y = 2 - \frac{(-4)}{2}$	4	$(-4, 4)$
-2	$y = 2 - \frac{(-2)}{2}$	3	$(-2, 3)$
0	$y = 2 - \frac{0}{2}$	2	$(0, 2)$
2	$y = 2 - \frac{2}{2}$	1	$(2, 1)$
4	$y = 2 - \frac{4}{2}$	0	$(4, 0)$



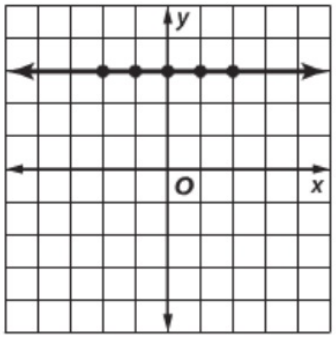
10.

x	$-3 + 2y = -5$	y	(x, y)
-2	$-3 + 2y = -5$	-1	$(-2, -1)$
-1	$-3 + 2y = -5$	-1	$(-1, -1)$
0	$-3 + 2y = -5$	-1	$(0, -1)$
1	$-3 + 2y = -5$	-1	$(1, -1)$
2	$-3 + 2y = -5$	-1	$(2, -1)$



11.

x	$y = 3$	y	(x, y)
-2	$y = 3$	3	$(-2, 3)$
-1	$y = 3$	3	$(-1, 3)$
0	$y = 3$	3	$(0, 3)$
1	$y = 3$	3	$(1, 3)$
2	$y = 3$	3	$(2, 3)$



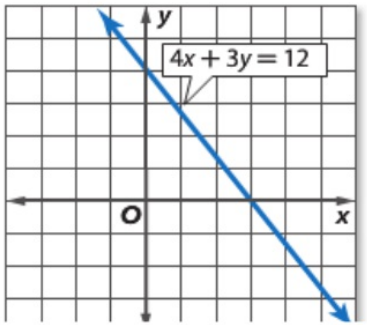
Practice and Problem Solving

Extra Practice is on page R3.

Example 1 Determine whether each equation is a linear equation. Write *yes* or *no*. If yes, write the equation in standard form.

13. $5x + y^2 = 25$ no
14. $8 + y = 4x$ yes; $4x - y = 8$
15. $9xy - 6x = 7$ no
16. $4y^2 + 9 = -4$ no
17. $12x = 7y - 10y$ yes; $4x + y = 0$
18. $y = 4x + x$ yes; $5x - y = 0$

Example 2 Find the x- and y-intercepts of the graph of each linear function.

19. 

3, 4

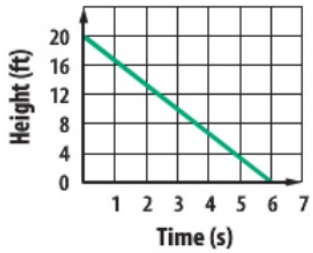
20.

x	y
−3	−1
−2	0
−1	1
0	2
1	3

−2, 2

Example 3 Find the x- and y-intercepts of each linear function. Describe what the intercepts mean.

21. **Descent of Eagle**



6, 20; The x-intercept represents the number of seconds that it takes the eagle to land. The y-intercept represents the initial height of the eagle.

22.

Eva's Distance from Home	
Time (min)	Distance (mi)
x	y
0	4
2	3
4	2
6	1
8	0

8, 4; The x-intercept 8 means that it took Eva 8 minutes to get home. The y-intercept 4 means that Eva was initially 4 miles from home.

Example 4 Graph each equation by using the x- and y-intercepts. **23–28. See margin.**

23. $y = 4 + 2x$ **24.** $5 - y = -3x$ **25.** $x = 5y + 5$

26. $x + y = 4$ **27.** $x - y = -3$ **28.** $y = 8 - 6x$

Example 5 Graph each equation by making a table. **29–34. See Chapter 3 Answer Appendix.**

29. $x = -2$ **30.** $y = -4$ **31.** $y = -8x$

32. $3x = y$ **33.** $y - 8 = -x$ **34.** $x = 10 - y$

35 **TV RATINGS** The number of people who watch a singing competition can be given by $p = 0.15v$, where p represents the number of people in millions who saw the show and v is the number of potential viewers in millions.

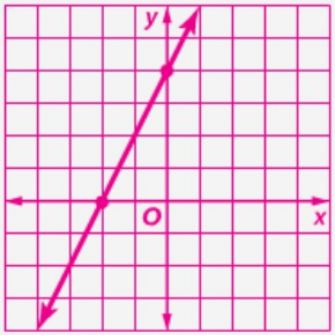
a. Make a table of values for the points (v, p). **35a–b. See Chapter 3 Answer Appendix.**

b. Graph the equation.

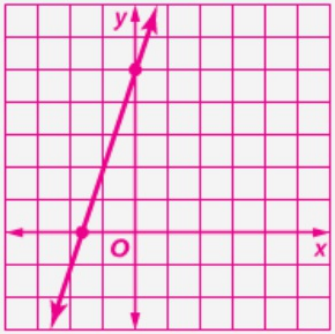
c. Use the graph to estimate the number of people who saw the show if there are 14 million potential viewers. **≈2.1 million**

d. Explain why it would not make sense for v to be a negative number. **There cannot be fewer than 0 viewers.**

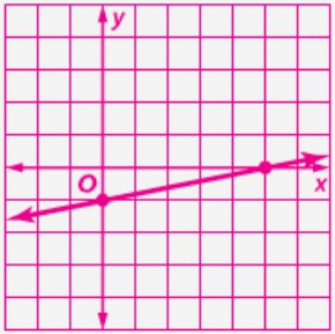
23.



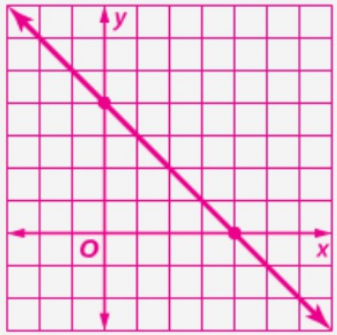
24.



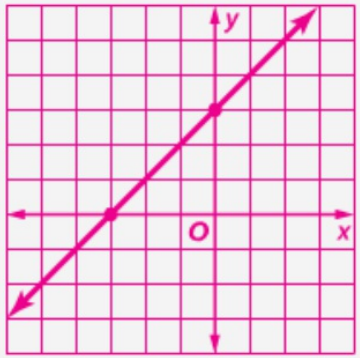
25.



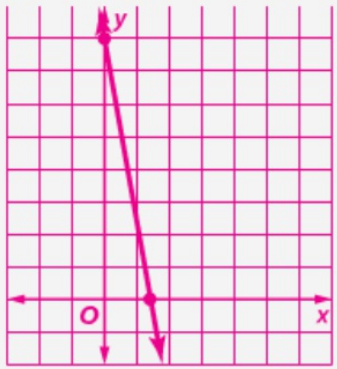
26.



27.

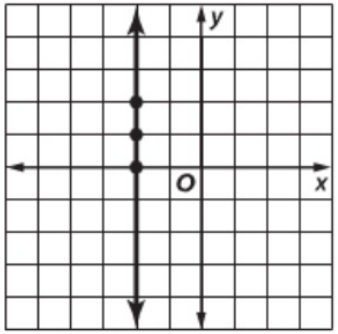


28.



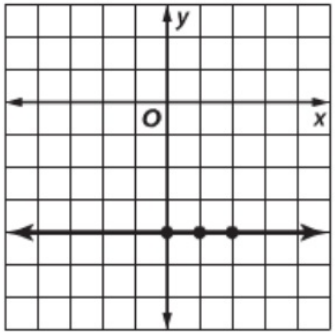
29.

x	y
−2	0
−2	1
−2	2



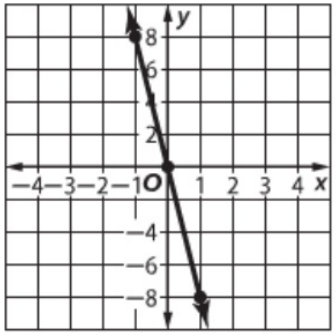
30.

x	y
0	−4
1	−4
2	−4



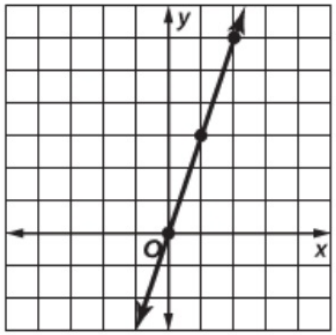
31.

x	y
−1	8
0	0
1	−8



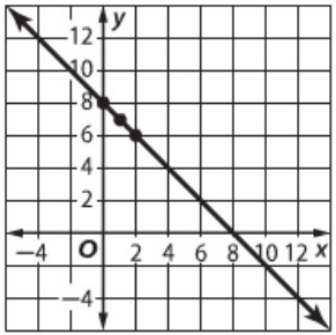
32.

x	y
0	0
1	3
2	6



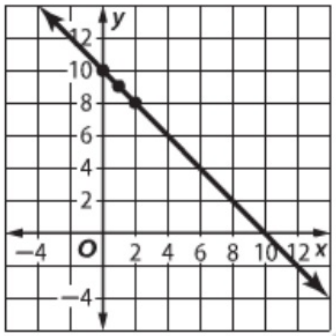
33.

x	y
0	8
1	7
2	6



34.

x	y
0	10
1	9
2	8



35a.

v	p = 0.15v	p	(v, p)
0	p = 0.15(0)	0	(0, 0)
2	p = 0.15(2)	0.3	(2, 0.3)
4	p = 0.15(4)	0.6	(4, 0.6)
6	p = 0.15(6)	0.9	(6, 0.9)
8	p = 0.15(8)	1.2	(8, 1.2)
10	p = 0.15(10)	1.5	(10, 1.5)

35b.