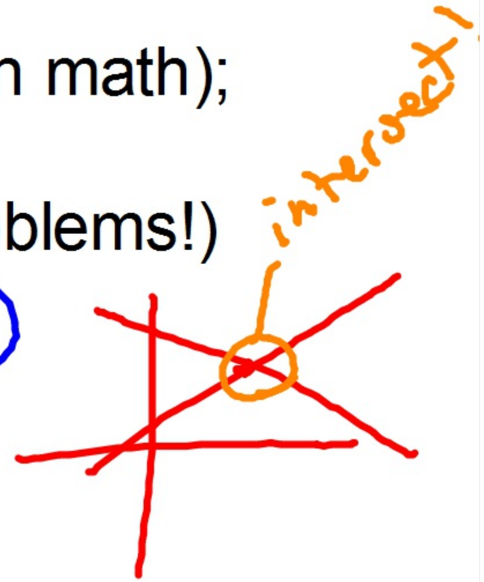
3-1 Solving Systems of Equations

Three ways to solve systems (or anything in math);

1) Numerically (skipping, don't do ex. 1 problems!)

$$2x + y = 5$$
$$x - y = 1$$

$(2, 1)$
$x \quad y$



2) Graphically (Use graph paper for Example 2!)

3)Algebraically (using substitution and elimination in example 4 and 5!)

Example 2 Solve by Graphing

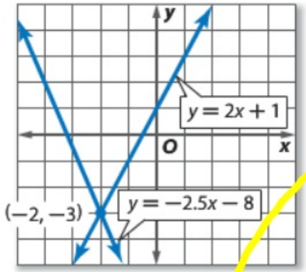
Solve the system of equations by graphing.

$$2x - y = -1 \qquad 2y + 5x = -16$$

Write each equation in slope-intercept form.

$2x - y = -1 \quad \rightarrow \quad y = 2x + 1$

$2y + 5x = -16 \quad \rightarrow \quad y = -2.5x - 8$



The graphs of the lines appear to intersect at (−2, −3).

CHECK Substitute the coordinates into each original equation.

$2x - y = -1$	$2y + 5x = -16$	Original equations
$2(-2) - (-3) \stackrel{?}{=} -1$	$2(-3) + 5(-2) \stackrel{?}{=} -16$	$x = -2$ and $y = -3$
$-1 = -1$ ✓	$-16 = -16$ ✓	Simplify.

The solution of the system is (−2, −3).

$y = mx + b$

5 $y = \frac{1}{2}x + 4$

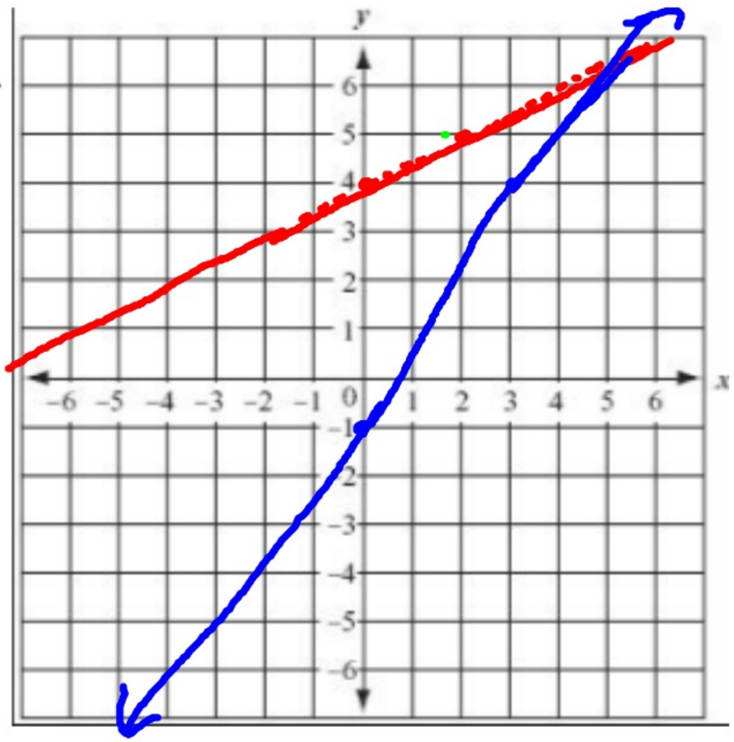
$y = \frac{4}{3}x - 1$

Example 2 **Solve each system of equations by graphing.**

3. $y = -3x + 6$
$2y = 10x - 36$ **(3, −3)**

5. $y = 0.5x + 4$
$3y = 4x - 3$ **(6, 7)**

7 $4x + 5y = -41$
$3y - 5x = 5$ **(−4, −5)**

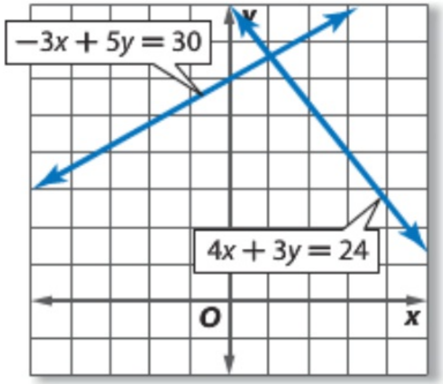


Systems of equations can be classified by the number of solutions. A system is **consistent** if it has at least one solution and **inconsistent** if it has no solutions. If it has exactly one solution, it is **independent**, and if it has an infinite number of solutions, it is **dependent**.

Example 3 Classify Systems

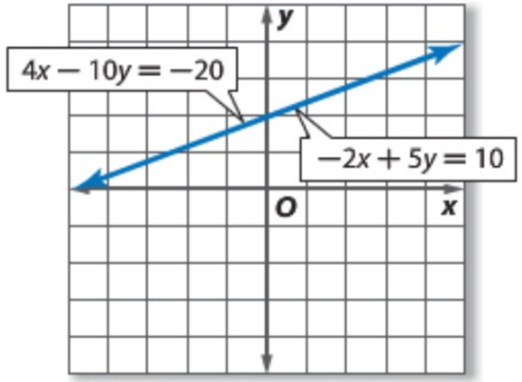
Graph each system of equations and describe them as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

a. $4x + 3y = 24$
$-3x + 5y = 30$



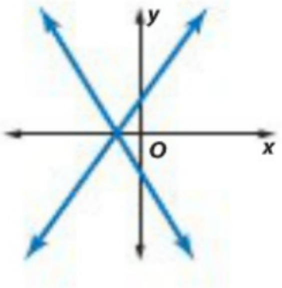
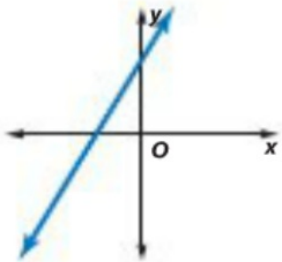
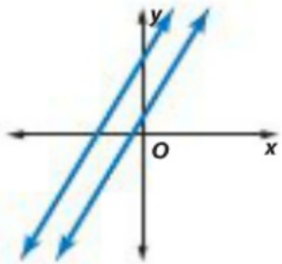
The graphs of the lines intersect at one point, so there is one solution. The system is consistent and independent.

b. $-2x + 5y = 10$
$4x - 10y = -20$



Because the equations are equivalent, their graphs are the same line. The system is consistent and dependent.

ConceptSummary Characteristics of Linear Systems

Consistent and Independent	Consistent and Dependent	Inconsistent
		
intersecting lines; one solution	same line; infinitely many solutions	parallel lines; no solution

Example 3 **Graph each system of equations and describe it as** ***consistent and independent, consistent and dependent,*** **or** ***inconsistent.*** **10–12. See margin.**

10. $y + 4x = 12$
$3y = 8 - 12x$

11. $-2x - 3y = 9$
$4x + 6y = -18$

12. $9x - 2y = 11$
$5x + 4y = 13$

Real-World Example 4 Use the Substitution Method

BUSINESS **Alejandro has a computer support business. He estimates that the cost to run his business can be represented by $y = 48x + 500$, where x is the number of customers. He also estimates that his income can be represented by $y = 65x - 145$. How many customers will Alejandro need in order to break even? What will his profit be if he has 60 customers?**

$y = 65x - 145$	Income equation
$48x + 500 = 65x - 145$	Substitute $48x + 500$ for y.
$500 = 17x - 145$	Subtract $48x$ from each side.
$645 = 17x$	Add 145 to each side.
$37.9 \approx x$	Divide each side by 17.

Alejandro needs 38 customers to break even. If he has 60 customers, his income will be 65(60) − 145 or \$3755, and his costs will be 48(60) + 500 or \$3380, so his profit will be 3755 − 3380 or \$375.

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$2x - 10 = -4x + 8$

$+4x \quad +10 \qquad +4x \quad +10$

$6x = 18$

$x = 3$

Example 4 **Solve each system of equations by using substitution.**

13. $x + 5y = 3$
$3x - 2y = -8$ **(−2, 1)**

14. $y = 2x - 10$
$y = -4x + 8$ **(3, −4)**

8
8

15. $2a + 8b = -8$
$3a - 5b = 22$ **(4, −2)**

16. $a - 3b = -22$
$4a + 2b = -4$ **(−4, 6)**

17. $6x - 7y = 23$
$8x + 4y = 44$ **(5, 1)**

18. $9c - 3d = -33$
$6c + 5d = -8$ **(−3, 2)**

$y = 2(3) - 10$

$y = 6 - 10$

$y = -4$

Example 5 Solve by Using Elimination

Use the elimination method to solve the system of equations.

Examples 5–6 Solve each system of equations by using elimination.

19. $-6w - 8z = -44$
$3w + 6z = 36$ **(−2, 7)**

20. $4x - 3y = 29$
$4x + 3y = 35$ **(8, 1)**

21. $3a + 5b = -27$
$4a + 10b = -46$ **(−4, −3)**

22. $8a - 3b = -11$
$5a + 2b = -3$ **(−1, 1)**

23. $5a + 15b = -24$
$-2a - 6b = 28$ **no solution**

24. $6x - 4y = 30$
$12x + 5y = -18$ **(1, −6)**

$$\begin{array}{rl} (+)\ -8x - 3y = 25 & \text{Equation 2} \times (-1) \\ \hline -3x \quad = 6 & \text{Add the equations.} \\ x = -2 & \text{Divide each side by } -3. \end{array}$$

Step 3 Substitute -2 for x into either original equation.

$$\begin{array}{rl} 8x + 3y = -25 & \text{Equation 2} \\ 8(-2) + 3y = -25 & x = -2 \\ -16 + 3y = -25 & \text{Multiply.} \\ 3y = -9 & \text{Add 16 to each side.} \\ y = -3 & \text{Divide each side by 3.} \end{array}$$

The solution is $(-2, -3)$.

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$-6w - 8z = -44 \Rightarrow$
$3w + 6z = 36 \times 2$

$-6w - 8z = -44$
$6w + 12z = 72$ 72

$4z = 28$
$z = 7$

22
$8a - 3b = -11 \times 2$
$5a + 2b = -3 \times 3$

$16a - 6b = -22$
$15a + 6b = -9$

$31a = -31$
$a = -1$

23. $5a + 15b = -24$
$-2a - 6b = 28$ no solution

$5a + 15b = -24 \quad \times 2$
$-2a - 6b = 28 \quad \times 5$

$\cancel{10a} + \cancel{30b} = -48$
$\cancel{-10a} - \cancel{30b} = 140$

$0 = 92$

false

$\begin{array}{r} 4 \\ 28 \\ \times\ 5 \\ \hline 0 \end{array}$

Standardized Test Example 6 No Solution and Infinite Solutions

Solve the system of equations.

$5x + 3y = 52$

$15x + 9y = 54$

A (3, 1) **B** (8, 4) **C** no solution **D** infinite solutions

Read the Test Item

You are given a system of two linear equations and are asked to find the solution.

Solve the Test Item

Neither variable has a common coefficient. The coefficients of the y-variables are 3 and 9 and their least common multiple is 9, so multiply each equation by the value that will make the y-coefficient 9.

$$5x + 3y = 52 \xrightarrow{\text{Multiply by 4.}} 15x + 9y = 156$$

$$\begin{array}{lr} 15x + 9y = 54 & (-)\ 15x + 9y = 54 \\ \hline & 0 = 102 \end{array}$$

Subtract the equations.

Because $0 = 102$ is not true, this system has no solution.

The correct answer is C.

23. $5a + 15b = -24$
$-2a - 6b = 28$ no solution

43. $9y + 3x = 18$
$-3y - x = -6$ infinite solutions

HW: p. 141-142 #1-24, 43,
50-58 even (use any method!)