3-3 Rate of Change and Slope

KeyConcept Rate of Change

If x is the independent variable and y is the dependent variable, then

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}.$$

Real-World Example 1 Find Rate of Change

ENTERTAINMENT Use the table to find the rate of change. Then explain its meaning.

Number of Computer Games	Total Cost ($)
x	y
2	78
4	156
6	234

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} \quad \begin{matrix} \leftarrow \text{dollars} \\ \leftarrow \text{games} \end{matrix}$$

$$= \frac{\text{change in cost}}{\text{change in number of games}}$$

$$= \frac{156 - 78}{4 - 2}$$

$$= \frac{78}{2} \text{ or } \frac{39}{1}$$

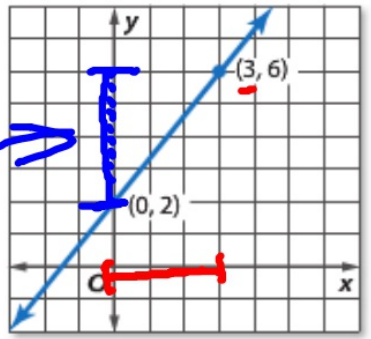
KeyConcept Rate of Change

If x is the independent variable and y is the dependent variable, then

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}.$$

Example 1 Find the rate of change represented in each table or graph.

1.



2.

x	y
3	−6
5	2
7	10
9	18
11	26

Example 2

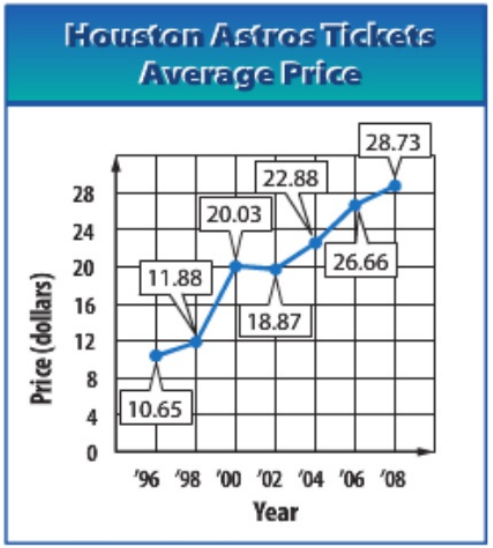
3. CCSS **SENSE-MAKING** Refer to the graph at the right.

a. Find the rate of change of prices from 2006 to 2008. Explain the meaning of the rate of change.

b. Without calculating, find a two-year period that had a greater rate of change than 2006–2008. Explain.

c. Between which years would you guess the new stadium was built? Explain your reasoning. **Sample answer: 1998–2000; Ticket prices show a sharp increase.**

$$\frac{\Delta y}{\Delta x}$$



Source: *Team Marketing Report*

x	y
1	−6
4	−8
7	−10
10	−12
13	−14

rate of change

$\frac{-8-(-6)}{4-1}$ or $-\frac{2}{3}$

$\frac{-10-(-8)}{7-4}$ or $-\frac{2}{3}$

$\frac{-12-(-10)}{10-7}$ or $-\frac{2}{3}$

$\frac{-14-(-12)}{13-10}$ or $-\frac{2}{3}$

The rate of change is constant.
Thus, the function is linear.

x	y
−3	10
−1	12
1	16
3	18
5	22

rate of change

$\frac{12-10}{-1-(-3)}$ or 1

$\frac{16-12}{1-(-1)}$ or 2

$\frac{18-16}{3-1}$ or 1

$\frac{22-18}{5-3}$ or 2

This rate of change is not constant.
Thus, the function is not linear.

$\frac{5-7}{12-8} = \frac{-2}{4} = -\frac{1}{2}$

Example 3 Determine whether each function is linear. Write *yes* or *no*. Explain.

4.

x	−7	−4	−1	2	5
y	5	4	3	2	1

5.

x	8	12	16	20	24
y	7	5	3	0	−2

No; the rate of change is not constant.

$\frac{0-3}{20-16} = -\frac{3}{4}$

Example 4 Positive, Negative and Zero Slope

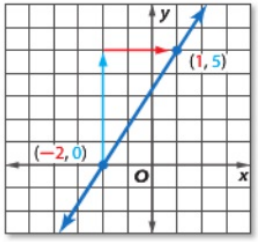
Find the slope of a line that passes through each pair of points.

a. (−2, 0) and (1, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{\text{rise}}{\text{run}}$$

$$= \frac{5 - 0}{1 - (-2)} \qquad (-2, 0) = (x_1, y_1) \text{ and } (1, 5) = (x_2, y_2)$$

$$= \frac{5}{3} \qquad \text{Simplify.}$$

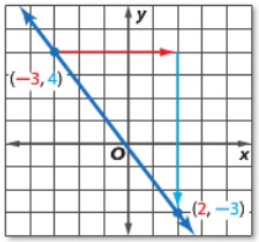


b. (−3, 4) and (2, −3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{\text{rise}}{\text{run}}$$

$$= \frac{-3 - 4}{2 - (-3)} \qquad (-3, 4) = (x_1, y_1) \text{ and } (2, -3) = (x_2, y_2)$$

$$= \frac{-7}{5} \text{ or } -\frac{7}{5} \qquad \text{Simplify.}$$

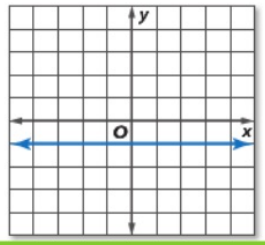


c. (−3, −1) and (2, −1)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{\text{rise}}{\text{run}}$$

$$= \frac{-1 - (-1)}{2 - (-3)} \qquad \text{Substitute.}$$

$$= \frac{0}{5} \text{ or } 0 \qquad \text{Simplify.}$$



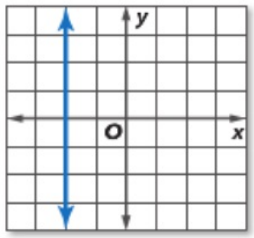
Example 5 Undefined Slope

Find the slope of the line that passes through (−2, 4) and (−2, −3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{\text{rise}}{\text{run}}$$

$$= \frac{-3 - 4}{-2 - (-2)} \qquad \text{Substitute.}$$

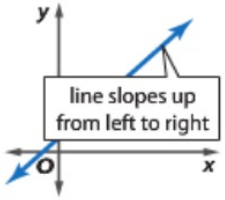
$$= \frac{-7}{0} \text{ or undefined} \qquad \text{Simplify.}$$



GuidedPractice

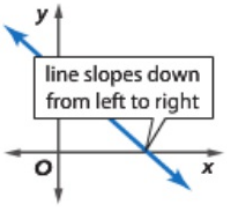
ConceptSummary Slope

positive slope



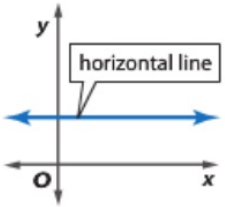
The function values are increasing over the entire domain.

negative slope



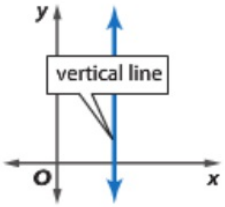
The function values are decreasing over the entire domain.

slope of 0



The function values are constant over the entire domain.

undefined slope



The relation is not a function.

Examples 4–5 Find the slope of the line that passes through each pair of points.

6. (5, 3), (6, 9)

7. (−4, 3), (−2, 1)

8. (6, −2), (8, 3)

9. (1, 10), (−8, 3)

10. (−3, 7), (−3, 4)

11. (5, 2), (−6, 2)

Example 6 Find Coordinates Given the Slope

Find the value of r so that the line through $(1, 4)$ and $(-5, r)$ has a slope of $\frac{1}{3}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope Formula

$$\frac{1}{3} = \frac{r - 4}{-5 - 1}$$ Let $(1, 4) = (x_1, y_1)$ and $(-5, r) = (x_2, y_2)$.

$$\frac{1}{3} = \frac{r - 4}{-6}$$ Subtract.

$$3(r - 4) = 1(-6)$$ Find the cross products.

$$3r - 12 = -6$$ Distributive Property

$$3r = 6$$ Add 12 to each side and simplify.

$$r = 2$$ Divide each side by 3 and simplify.

So, the line goes through $(-5, 2)$.

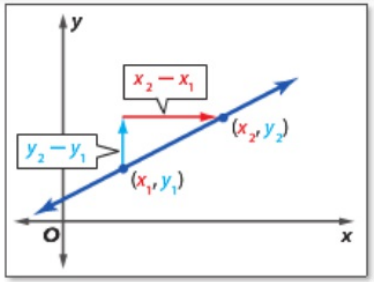
KeyConcept Slope

Words The slope of a nonvertical line is the ratio of the rise to the run.

Symbols The slope m of a nonvertical line through any two points, (x_1, y_1) and (x_2, y_2), can be found as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← change in y ← change in x

Graph



Example 6 **Find the value of r so the line that passes through each pair of points has the given slope.**

12. $(-4, r), (-8, 3), m = -5$ ☐

13. $(5, 2), (-7, r), m = \frac{5}{6}$ ☐

Practice and Problem Solving

Example 1 Find the rate of change represented in each table or graph.

14.

x	y
5	2
10	3
15	4
20	5

$\frac{1}{5}$

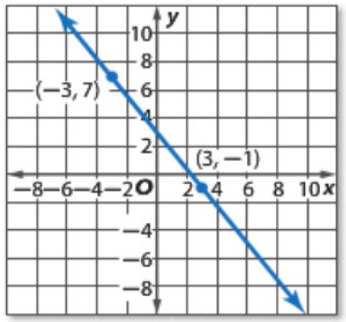
15

x	y
1	15
2	9
3	3
4	−3

−6

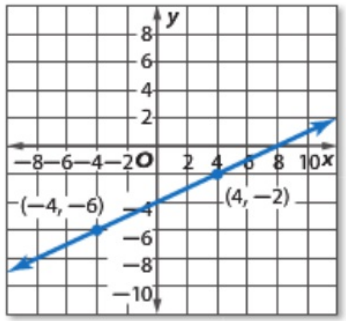
nple 1 Find the rate of change represented in each table or graph.

16.



$-\frac{4}{3}$

17.



$\frac{1}{2}$

Example 2

18. SPORTS What was the annual rate of change from 2004 to 2008 for women participating in collegiate lacrosse? Explain the meaning of the rate of change.

Year	Number of Women
2004	5545
2008	6830

18. 321.25; There was an average increase of 321.25 women per year participating in lacrosse.

19. RETAIL The average retail price in the spring of 2009 for a used car is shown in the table at the right.

Age (years)	Value ($)
2	17,378
3	16,157

a. Write a linear function to model the price of the car with respect to age.

b. Interpret the meaning of the slope of the line.

c. Assuming a constant rate of change predict the average retail price for a 7-year-old car. **$11,273**

19a. Sample answer: $p = -1221t + 19{,}820$

19b. The car value depreciates by $1221 each year.

Example 3 Determine whether each function is linear. Write *yes* or *no*. Explain.

20.

x	4	2	0	−2	−4
y	−1	1	3	5	7

21.

x	−7	−5	−3	−1	0
y	11	14	17	20	23

22.

x	−0.2	0	0.2	0.4	0.6
y	0.7	0.4	0.1	0.3	0.6

23.

x	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

20–23. See Chapter 3 Answer Appendix.

Examples 4–5 Find the slope of the line that passes through each pair of points.

24. (4, 3), (−1, 6) $-\frac{3}{5}$

25 (8, −2), (1, 1) $-\frac{3}{7}$

26. (2, 2), (−2, −2) 1

27. (6, −10), (6, 14)

28. (5, −4), (9, −4) 0

29. (11, 7), (−6, 2) $\frac{5}{17}$

30. (−3, 5), (3, 6) $\frac{1}{6}$

31. (−3, 2), (7, 2) 0

32. (8, 10), (−4, −6) $\frac{4}{3}$

33. (−8, 6), (−8, 4)

34. (−12, 15), (18, −13)

35. (−8, −15), (−2, 5) $\frac{10}{3}$

27. undefined
33. undefined
34. $-\frac{14}{15}$

Example 6 Find the value of r so the line that passes through each pair of points has the given slope.

36. (12, 10), (−2, r), $m = -4$ 66

37. (r, −5), (3, 13), $m = 8$ $\frac{3}{4}$

38. (3, 5), (−3, r), $m = \frac{3}{4}$ $\frac{1}{2}$

39. (−2, 8), (r, 4), $m = -\frac{1}{2}$ 6

20. Yes; the rate of change is constant.

21. No; the rate of change is not constant.

22. No; the rate of change is not constant.

23. Yes; the rate of change is constant.