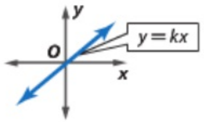
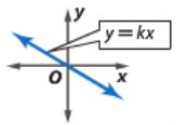
3-4 Direct Variation

ConceptSummary **Direct Variation Graphs**

- Direct variation equations are of the form $y = kx$, where $k \neq 0$. "y varies directly as x"
- The graph of $y = kx$ always passes through the origin.
- The slope is positive if $k > 0$.



- The slope is negative if $k < 0$.



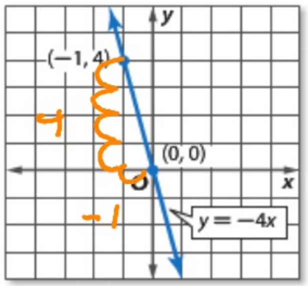
$$\frac{y}{x} = \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \underset{k}{.5}$$

As "x" increases, "y" increases ← "direct"

Example 1 Slope and Constant of Variation

Name the constant of variation for each equation. Then find the slope of th line that passes through each pair of points.

a.



The constant of variation is −4.

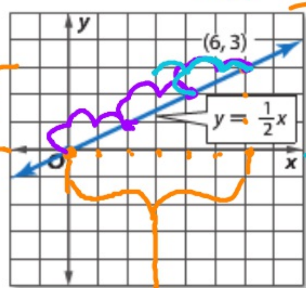
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$

$$= \frac{4 - 0}{-1 - 0} \quad (x_1, y_1) = (0, 0),\ (x_2, y_2) = (-1, 4)$$

$$= -4 \quad \text{The slope is } -4.$$

b.

$\Delta y = 3$



$\Delta x = 6$

$\frac{\text{rise}}{\text{run}}$

The constant of variation is $\frac{1}{2}$.

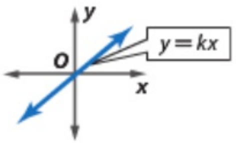
$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$

$$= \frac{3 - 0}{6 - 0} \quad (x_1, y_1) = (0, 0),\ (x_2, y_2) = (6, 3)$$

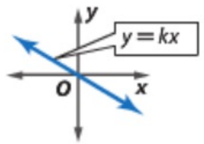
$$= \frac{1}{2} \quad \text{The slope is } \frac{1}{2}.$$

ConceptSummary Direct Variation Graphs

- Direct variation equations are of the form $y = kx$, where $k \neq 0$.
- The graph of $y = kx$ always passes through the origin.
- The slope is positive if $k > 0$.



- The slope is negative if $k < 0$.

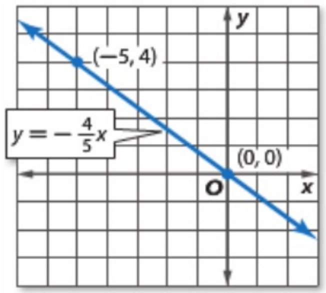


Check Your Understanding

= Step-by-Step Solutions begin on pag

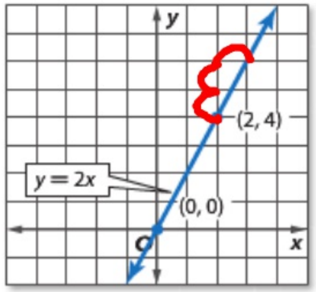
Example 1 **Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.**

1.



$-\frac{4}{5}; -\frac{4}{5}$

2.



2; 2

$\frac{2}{1}$

Example 2 **Graph a Direct Variation**

Graph $y = -6x$.

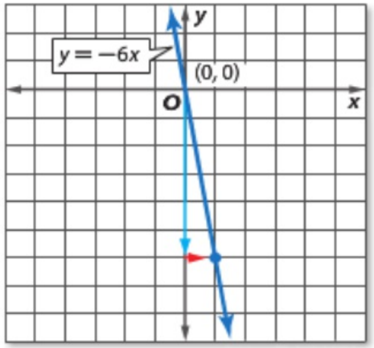
Step 1 Write the slope as a ratio.

$-6 = \frac{-6}{1}$ $\quad \frac{\text{rise}}{\text{run}}$

Step 2 Graph (0, 0).

Step 3 From the point (0, 0), move down 6 units and right 1 unit. Draw a dot.

Step 4 Draw a line containing the points.

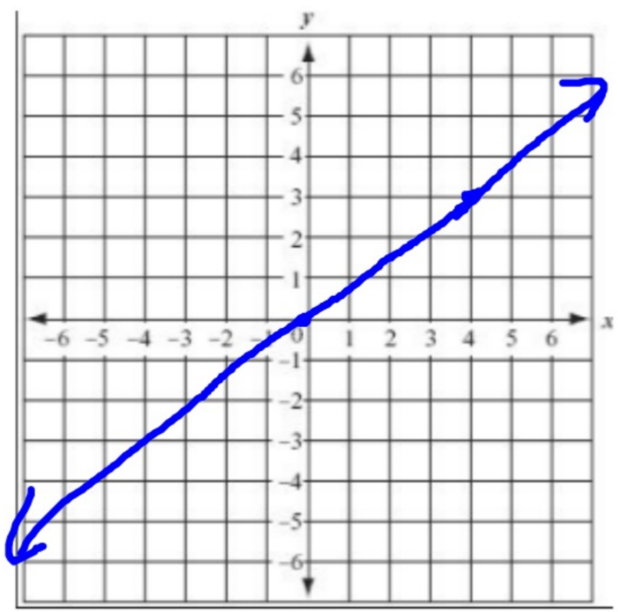


$\frac{\text{rise}}{\text{run}}$

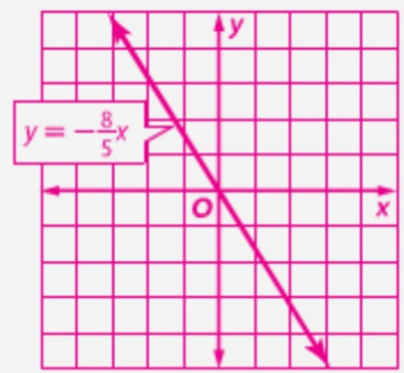
Example 2 Graph each equation. **3–6. See margin.**

3. $y = -x$ **4.** $y = \frac{3}{4}x$ **5.** $y = -8x$ **6.** $y = -\frac{8}{5}$

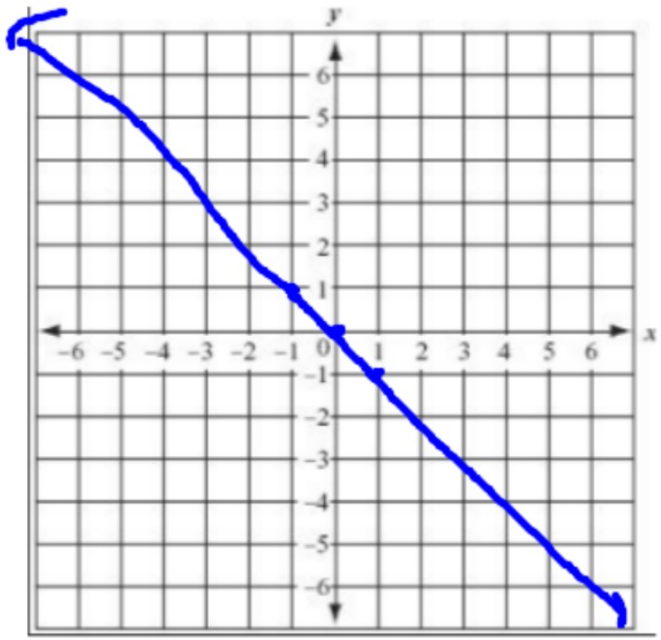
4



6.



3



Example 3 Write and Solve a Direct Variation Equation

Suppose y varies directly as x, and $y = 72$ when $x = 8$.

a. Write a direct variation equation that relates x and y.

$y = kx$	Direct variation formula
$72 = k(8)$	Replace y with 72 and x with 8.
$9 = k$	Divide each side by 8.

Therefore, the direct variation equation is $y = 9x$.

b. Use the direct variation equation to find x when $y = 63$.

$y = 9x$	Direct variation formula
$63 = 9x$	Replace y with 63.
$7 = x$	Divide each side by 9.

Therefore, $x = 7$ when $y = 63$.

Step 1: Find k!

$y = kx$

$(15) = k(12)$

$k = \frac{15}{12} = \frac{5}{4}$

Example 3 **Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.**

7. If $y = 15$ when $x = 12$, find y when $x = 32$. $y = \frac{5}{4}x$; 40

8. If $y = -11$ when $x = 6$, find x when $y = 44$. $y = -\frac{11}{6}x$; -24

Step 2: use "k" to find y!

$y = kx$

$y = \left(\frac{5}{4}\right)(32)$

$y = 40$

$y = kx$

$-11 = k(6)$

$k = \frac{-11}{6}$

$\left(-\frac{6}{11}\right) 44 = \left(\frac{-11}{6}\right) x \left(-\frac{6}{11}\right)$

Real-World Example 4 Estimate Using Direct Variation

TRAVEL **The distance a jet travels varies directly as the number of hours it flies. A jet traveled 3420 miles in 6 hours.**

a. Write a direct variation equation for the distance d flown in time t.

Words	Distance	equals	rate	times	time.
Variable	Let r = rate.				
Equation	3420	=	r	×	6

Solve for the rate.

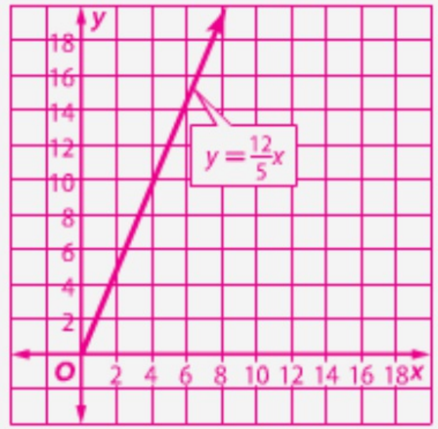
$3420 = r(6)$ Original equation

$\frac{3420}{6} = \frac{r(6)}{6}$ Divide each side by 6.

$570 = r$ Simplify.

Therefore, the direct variation equation is $d = 570t$. The airliner flew at a rate of 570 miles per hour.

9a.



y

y

Example 4 **9.** **CCSS REASONING** You find that the number of messages you receive on your message board varies directly as the number of messages you post. When you post 5 messages, you receive 12 messages in return.

x x

a. Write a direct variation equation relating your posts to the messages received. Then graph the equation. $y = \frac{12}{5}x$; **see margin for graph.**

b. Find the number of messages you need to post to receive 96 messages. **40**

$y = kx$

$12 = 5k$

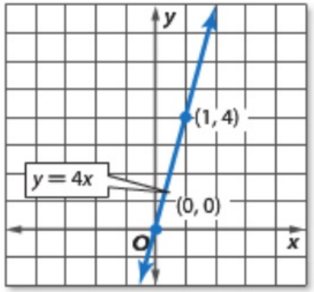
$\frac{12}{5} = \frac{5k}{5}$

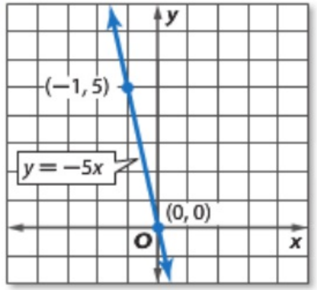
$k = \frac{12}{5}$.

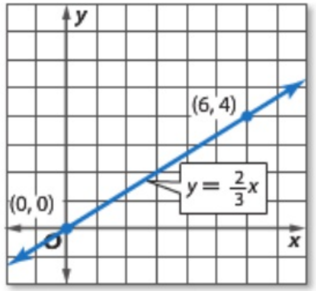
Practice and Problem Solving

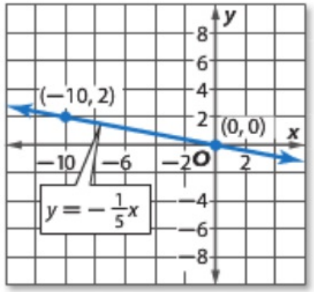
Extra Practice is on page R3.

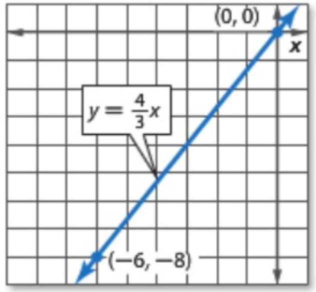
Example 1 Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points. **10–15. See margin.**

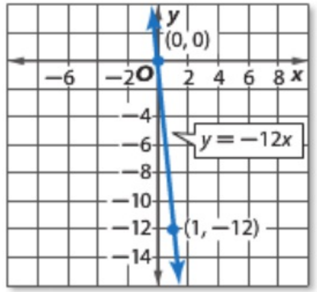
10.


11.


12.


13.


14.


15.


10. 4; 4 **11.** −5; −5
12. $\frac{2}{3}$; $\frac{2}{3}$ **13.** $-\frac{1}{5}$; $-\frac{1}{5}$
14. $\frac{4}{3}$; $\frac{4}{3}$ **15.** −12; −12

mple 2 **Graph each equation.** **16–23. See Chapter 3 Answer Appendix.**

16. $y = 10x$ **17.** $y = -7x$ **18.** $y = x$ **19.** $y = \frac{7}{6}x$

20. $y = \frac{1}{6}x$ **21.** $y = \frac{2}{9}x$ **22.** $y = \frac{6}{5}x$ **23.** $y = -\frac{5}{4}x$

Example 3 **Suppose y varies directly as x. Write a direct variation equation that relates x and y. Then solve.**

24. If $y = 6$ when $x = 10$, find x when $y = 18$. **$y = \frac{3}{5}x$; 30**

25 If $y = 22$ when $x = 8$, find y when $x = -16$. **$y = \frac{11}{4}x$; -44**

26. If $y = 4\frac{1}{4}$ when $x = \frac{3}{4}$, find y when $x = 4\frac{1}{2}$. **$y = 5\frac{2}{3}x$; $25\frac{1}{2}$**

27. If $y = 12$ when $x = \frac{6}{7}$, find x when $y = 16$. **$y = 14x$; $1\frac{1}{7}$**

Example 4 **28.** **SPORTS** The distance a golf ball travels at an altitude of 7000 feet varies directly with the distance the ball travels at sea level, as shown.

Hitting a Golf Ball		
Altitude (ft)	0 (sea level)	7000
Distance (yd)	200	210

a. Write and graph an equation that relates the distance a golf ball travels at an altitude of 7000 feet y with the distance at sea level x. **$y = 1.05x$; See Chapter 3 Answer Appendix for graph.**

b. What would be a person's average driving distance at 7000 feet if his average driving distance at sea level is 180 yards? **189 yd**

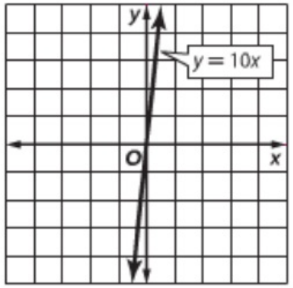
29. **FINANCIAL LITERACY** Depreciation is the decline in a car's value over the course of time. The table below shows the values of a car with an average depreciation.

Age of Car (years)	1	2	3	4	5
Value (dollars)	12,000	10,200	8400	6600	4800

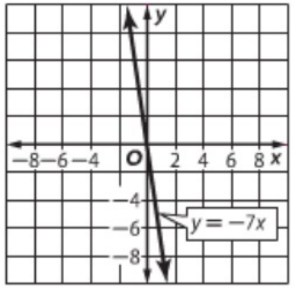
a. Write an equation that relates the age x of the car to the value y that it lost after each year. **$y = 1800x$**

b. Find the age of the car if the value is \$300. **7 yr 6 mo**

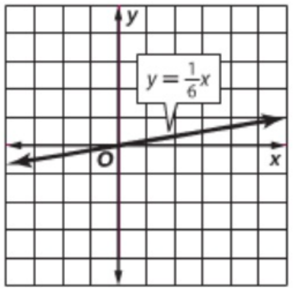
16.



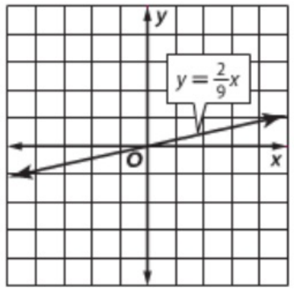
17.



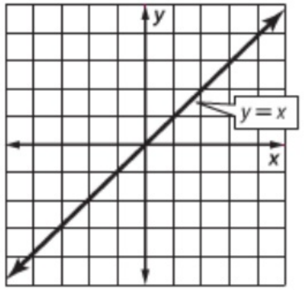
20.



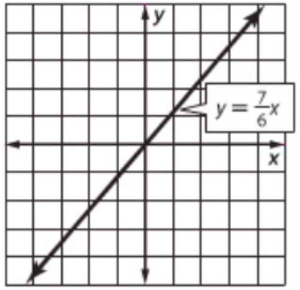
21.



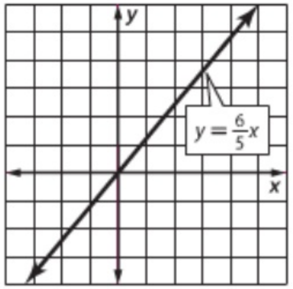
18.



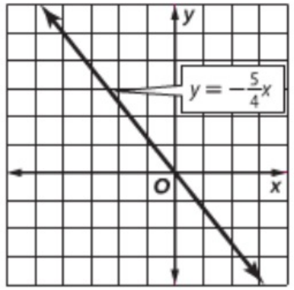
19.



22.



23.



28a.

Golf Ball Distance at High Altitude