Check Your Understanding

Example 1

1. CCSS **MODELING** Use the table that shows the city and highway gas mileage of five different types of vehicles.

Vehicle	SUV	Mini-van	Sedan	Compact	APV
City	23	21	21	42	61
Highway	25	24	32	49	70

Source: Auto Hoppers

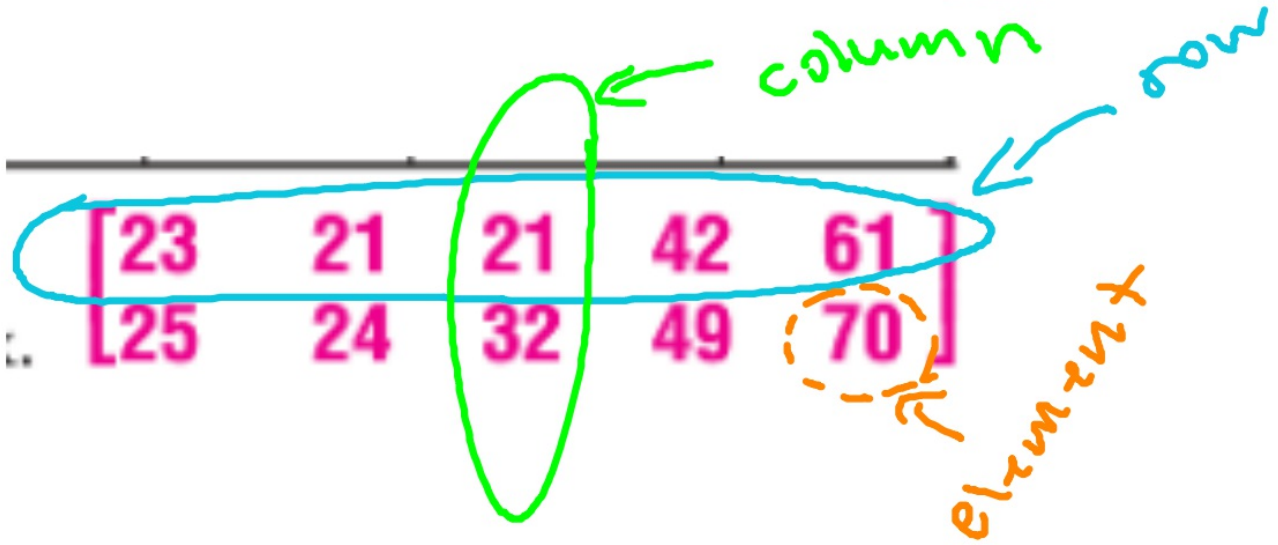
a. Organize the gas mileages in a matrix. $\begin{bmatrix} 23 & 21 & 21 & 42 & 61 \\ 25 & 24 & 32 & 49 & 70 \end{bmatrix}$

b. Add the elements of each row and interpret the results.

c. Add the elements of each column and interpret the results.

1b. Sample answer: City: The sum is 168. However, this value is irrelevant since it is the sum of 5 different types of data. Highway: The sum is 200. However, this value is irrelevant since it is the sum of 5 different types of data.

1c. Sample answer: The sums are 48, 45, 53, 91, and 131. These values are irrelevant since they are the sums of 2 different types of data.



Example 2 Add and Subtract Matrices

Find each of the following for $A = \begin{bmatrix} 16 & 2 \\ -9 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix}$, and $C = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$.

a. $A + B$

$$A + B = \begin{bmatrix} 16 & 2 \\ -9 & 8 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix}$$ Substitution

$$= \begin{bmatrix} 16 + (-4) & 2 + (-1) \\ -9 + (-3) & 8 + (-7) \end{bmatrix}$$ Add corresponding elements.

$$= \begin{bmatrix} 12 & 1 \\ -12 & 1 \end{bmatrix}$$ Simplify.

b. $B - C$

$$B - C = \begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix} - \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$ Substitution

Since the dimensions of B and C are different, you cannot subtract the matrices.

c. $B - A$

$$B - A = \begin{bmatrix} -4 & -1 \\ -3 & -7 \end{bmatrix} - \begin{bmatrix} 16 & 2 \\ -9 & 8 \end{bmatrix}$$ Substitution

$$= \begin{bmatrix} -4 - 16 & -1 - 2 \\ -3 - (-9) & -7 - 8 \end{bmatrix}$$ Subtract corresponding elements.

$$= \begin{bmatrix} -20 & -3 \\ 6 & -15 \end{bmatrix}$$ Simplify.

KeyConcept Adding and Subtracting Matrices

Words To add or subtract two matrices with the same dimensions, add or subtract their corresponding elements.

Symbols

$$A + B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} = A + B$$

$$A - B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix} = A - B$$

Example $\begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -9 & 10 \end{bmatrix} = \begin{bmatrix} 3+2 & -5+0 \\ 1+(-9) & 7+10 \end{bmatrix}$

Example 2 Perform the indicated operations. If the matrix does not exist, write *impossible*.

2–5. See margin.

2. $\begin{bmatrix} -8 & 2 & 6 \end{bmatrix} + \begin{bmatrix} 11 & -7 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 9 & -8 & 4 \end{bmatrix} + \begin{bmatrix} 12 & 2 \end{bmatrix}$

4. $\begin{bmatrix} 7 & -12 \\ 15 & 4 \end{bmatrix} - \begin{bmatrix} 9 & 6 \\ 4 & -9 \end{bmatrix}$

5. $\begin{bmatrix} 5 & 13 & -6 \\ 3 & -17 & 2 \end{bmatrix} - \begin{bmatrix} -2 & -18 & 8 \\ 2 & -11 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -5 & 7 \end{bmatrix}$

3. impossible

4. $\begin{bmatrix} -2 & -18 \\ 11 & 13 \end{bmatrix}$ **5.** $\begin{bmatrix} 7 & 31 & -14 \\ 1 & -6 & 2 \end{bmatrix}$

ReadingMath

Scalar Think of a scalar as a coefficient for a variable, but instead it is for a matrix.

KeyConcept Multiplying by a Scalar

Words To multiply a matrix by a scalar k, multiply each element by k.

$k \cdot A = kA$

Symbols $k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

Example $-3\begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix} = \begin{bmatrix} -3(4) & -3(1) \\ -3(7) & -3(-2) \end{bmatrix}$

6. $\begin{bmatrix} 18 & 12 & 0 \\ -6 & 42 & -24 \\ -12 & -18 & 21 \end{bmatrix}$

7. $\begin{bmatrix} -90 & 54 & -12 & -18 \\ -36 & 66 & -84 & 12 \\ -24 & 48 & 60 & -162 \end{bmatrix}$

Example 3 Multiply a Matrix by a Scalar

If $R = \begin{bmatrix} -12 & 8 & 6 \\ -16 & 4 & 19 \end{bmatrix}$, find $5R$.

$5R = 5\begin{bmatrix} -12 & 8 & 6 \\ -16 & 4 & 19 \end{bmatrix}$ Substitution

$= \begin{bmatrix} 5(-12) & 5(8) & 5(6) \\ 5(-16) & 5(4) & 5(19) \end{bmatrix}$ Distribute the scalar.

$= \begin{bmatrix} -60 & 40 & 30 \\ -80 & 20 & 95 \end{bmatrix}$ Multiply.

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erform the indicated operations. If the matrix does not exist, write *impossible*.

6, 7. See margin.

6. $3\begin{bmatrix} 6 & 4 & 0 \\ -2 & 14 & -8 \\ -4 & -6 & 7 \end{bmatrix}$

7. $-6\begin{bmatrix} 15 & -9 & 2 & 3 \\ 6 & -11 & 14 & -2 \\ 4 & -8 & -10 & 27 \end{bmatrix}$

Example 4 Multi-Step Operations

If $A = \begin{bmatrix} -9 & 12 \\ 2 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -8 \\ 2 & -3 \end{bmatrix}$, find $-4B - 3A$.

$$-4B - 3A = -4\begin{bmatrix} -4 & -8 \\ 2 & -3 \end{bmatrix} - 3\begin{bmatrix} -9 & 12 \\ 2 & -6 \end{bmatrix}$$ Substitution

$$= \begin{bmatrix} -4(-4) & -4(-8) \\ -4(2) & -4(-3) \end{bmatrix} - \begin{bmatrix} 3(-9) & 3(12) \\ 3(2) & 3(-6) \end{bmatrix}$$ Distribute the scalars in each matrix.

$$= \begin{bmatrix} 16 & 32 \\ -8 & 12 \end{bmatrix} - \begin{bmatrix} -27 & 36 \\ 6 & -18 \end{bmatrix}$$ Multiply.

$$= \begin{bmatrix} 16 - (-27) & 32 - 36 \\ -8 - 6 & 12 - (-18) \end{bmatrix}$$ Subtract corresponding elements.

$$= \begin{bmatrix} 43 & -4 \\ -14 & 30 \end{bmatrix}$$ Simplify.

Example 4 Use matrices A, B, C, and D to find the following.

$A = \begin{bmatrix} 6 & -4 \\ 3 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 8 & -1 \\ -2 & 7 \end{bmatrix}$ $C = \begin{bmatrix} -4 & -6 \\ 12 & -7 \end{bmatrix}$ $D = \begin{bmatrix} 9 & 6 & 0 \\ -2 & 8 & 0 \end{bmatrix}$

8–11. See margin.

8. $4B - 2A$

9. $-8C + 3A$

10. $-5B - 2D$

11. $-4C - 5B$

12a. Test 1: $\begin{bmatrix} 85 \\ 75 \\ 96 \end{bmatrix}$ **Test 2:** $\begin{bmatrix} 72 \\ 74 \\ 83 \end{bmatrix}$

12b. $\begin{bmatrix} 157 \\ 149 \\ 179 \end{bmatrix}$

(8) $4B - 2A$

$$4\begin{bmatrix} 8 & -1 \\ -2 & 7 \end{bmatrix} - 2\begin{bmatrix} 6 & -4 \\ 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 32 & -4 \\ -8 & 28 \end{bmatrix} - \begin{bmatrix} 12 & 8 \\ 6 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -12 \\ -14 & 38 \end{bmatrix}$$

Example 5

12c. $\begin{bmatrix} 13 \\ 1 \\ 13 \end{bmatrix}$

12. **GRADES** Geraldo, Olivia, and Nikki have had two tests in their math class. The table shows the test grades for each student.

a. Write a matrix for the information from each test.

b. Find the sum of the scores from the two tests expressed as a matrix.

c. Express the difference in scores from test 1 to test 2 as a matrix.

Student	Test 1	Test 2
Geraldo	85	72
Olivia	75	74
Nikki	96	83

a. $\begin{bmatrix} 85 & 72 \\ 75 & 74 \\ 96 & 83 \end{bmatrix}$

b $85+72$..

13b. Sample answer: Brand C; it was given the highest rating possible for cost and comfort, and a high rating for looks, and it will last a fairly long time.

Practice and Problem Solving

Extra Practice is on page R3.

Example 1

13. **SHOES** A consumer service company rated several pairs of shoes by cost, level of comfort, look, and longevity using a scale of 1–5, with 1 being low and 5 being high.

13a. See margin.

a. Write a 4×4 matrix to organize this information.

b. Which shoe would you buy based on this information, and why?

c. Would finding the sum of the rows or columns provide any useful information? Explain your reasoning.

Brand	Cost	Comfort	Look	Longevity
A	3	2	2	1
B	4	3	2	3
C	5	5	4	4
D	1	5	5	2

Example 2

Perform the indicated operations. If the matrix does not exist, write *impossible*.

14. $\begin{bmatrix} 12 & -5 \\ -8 & -3 \end{bmatrix} + \begin{bmatrix} -6 & 11 \\ -7 & 2 \end{bmatrix}$

15 $\begin{bmatrix} 9 & 5 \\ -2 & 16 \end{bmatrix} + \begin{bmatrix} -6 & -3 & 7 \\ 12 & 2 & -4 \end{bmatrix}$

14. $\begin{bmatrix} 6 & 6 \\ -15 & -1 \end{bmatrix}$

impossible

13c. Sample answer: Yes; finding the sum of the rows and then calculating the average will provide an easy way to compare the data.

13a. $\begin{bmatrix} 3 & 2 & 2 & 1 \\ 4 & 3 & 2 & 3 \\ 5 & 5 & 4 & 4 \\ 1 & 5 & 5 & 2 \end{bmatrix}$

Examples 3–4 **16. BUSINESS** The drink menu from a fast-food restaurant is shown at the right. The store owner has decided that all of the prices must be increased by 10%. **a–d. See margin.**

Drink	Small	Medium	Large
Soda	$0.95	$1.00	$1.05
Iced tea	$0.75	$0.80	$0.85
Lemonade	$0.75	$0.80	$0.85
Coffee	$1.00	$1.10	$1.20

a. Write matrix C to represent the current prices.

b. What scalar can be used to determine a matrix N to represent the new prices?

c. Find N.

d. What is $N - C$? What does this represent in this situation?

17–20. **See margin.**

Use matrices A, B, C, and D to find the following.

$$A = \begin{bmatrix} 0 & -7 \\ 8 & 12 \end{bmatrix} \quad B = \begin{bmatrix} 11 & 4 \\ -3 & -17 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 2 & -2 \\ 1 & -9 & 15 \end{bmatrix} \quad D = \begin{bmatrix} -2 & -8 & 0 \\ 4 & 13 & 1 \end{bmatrix}$$

17. $-3B + 2A$

18. $9C - 4D$

19. $2C + 11A$

20. $7A - 2B$

Example 5

21. CCSS **MODELING** Library A has 10,000 novels, 5000 biographies, and 5000 children's books. Library B has 15,000 novels, 10,000 biographies, and 2500 children's books. Library C has 4000 novels, 700 biographies, and 800 children's books. **a–d. See margin.**

a. Express each library's number of books as a matrix. Label the matrices A, B, and C.

b. Find the total number of each type of book in all 3 libraries. Express as a matrix.

c. How many more books of each type does Library A have than Library C?

d. Find $A + B$. Does the matrix have meaning in this situation? Explain.

Additional Answers

16a. $\begin{bmatrix} \$0.95 & \$1.00 & \$1.05 \\ \$0.75 & \$0.80 & \$0.85 \\ \$0.75 & \$0.80 & \$0.85 \\ \$1.00 & \$1.10 & \$1.20 \end{bmatrix}$

16b. 1.1

16c. $\begin{bmatrix} \$1.05 & \$1.10 & \$1.16 \\ \$0.83 & \$0.88 & \$0.94 \\ \$0.83 & \$0.88 & \$0.94 \\ \$1.10 & \$1.21 & \$1.32 \end{bmatrix}$

16d. $\begin{bmatrix} \$0.10 & \$0.10 & \$0.11 \\ \$0.08 & \$0.08 & \$0.09 \\ \$0.08 & \$0.08 & \$0.09 \\ \$0.10 & \$0.11 & \$0.12 \end{bmatrix}$

Sample answer: this matrix represents the price increases for each item.

21a. Library A: $\begin{bmatrix} 10{,}000 \\ 5000 \\ 5000 \end{bmatrix}$; Library B: $\begin{bmatrix} 15{,}000 \\ 10{,}000 \\ 2500 \end{bmatrix}$; Library C: $\begin{bmatrix} 4000 \\ 700 \\ 800 \end{bmatrix}$

21b. $\begin{bmatrix} 29{,}000 \\ 15{,}700 \\ 8300 \end{bmatrix}$ **21c.** $\begin{bmatrix} 6000 \\ 4300 \\ 4200 \end{bmatrix}$

21d. $\begin{bmatrix} 25{,}000 \\ 15{,}000 \end{bmatrix}$; The sum represents the combined size of

17. $\begin{bmatrix} -33 & -26 \\ 25 & 75 \end{bmatrix}$

18. $\begin{bmatrix} 80 & 50 & -18 \\ -7 & -133 & 131 \end{bmatrix}$

19. impossible

20. $\begin{bmatrix} -22 & -57 \\ 62 & 118 \end{bmatrix}$

21a. Library A: $\begin{bmatrix} 10{,}000 \\ 5000 \end{bmatrix}$;