3-6 Proportional and Nonproportional Relationships

KeyConcept Proportional Relationship

Words
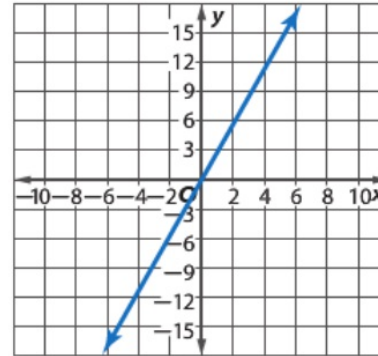
A relationship is proportional if its equation is of the form $y = kx$, $k \neq 0$. The graph passes through $(0, 0)$.

Example

$y = 3x$

x	0	1	2	3	4
y	0	3	6	9	12

The ratio of the value of x to the value of y is constant when $x \neq 0$.

BONUS PAY Marcos is a personal trainer at a gym. In addition to his salary, he receives a bonus for each client he sees.

Number of Clients	1	2	3	4	5
Bonus Pay ($)	45	90	135	180	225

a. Graph the data. What can you deduce from the pattern about the relationship between the number of clients and the bonus pay?

The graph demonstrates a linear relationship between the number of clients and the bonus pay.

The graph also passes through the point (0, 0) because when Marcos sees 0 clients, he does not receive any bonus money. Therefore, the relationship is proportional.
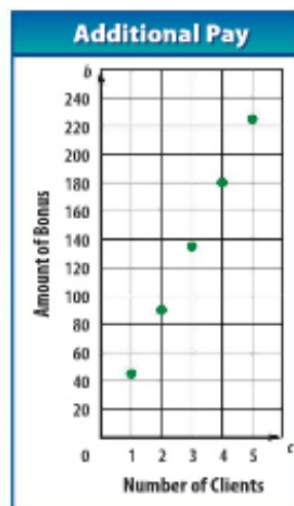
b. Write an equation to describe this relationship.

Look for a pattern that can be described in an equation.

+1   +1   +1   +1

Number of Clients	1	2	3	4	5
Bonus Pay ($)	45	90	135	180	225

+45 +45 +45 +45

The difference between the values for the number of clients c is 1. The difference in the values for the bonus pay b is 45. This suggests that the k-value is $\frac{45}{1}$ or 45. So the equation is $b = 45c$. You can check this equation by substituting values for c into the equation.

CHECK   If $c = 1$, then $b = 45(1)$ or 45. ✓
    If $c = 5$, then $b = 45(5)$ or 225. ✓

c. Use this equation to predict the amount of Marcos's bonus if he sees 8 clients.

$b = 45c$ Original equation
  $= 45(8)$ or 360 $c = 8$

Marcos will receive a bonus of $360 if he sees 8 clients.

Additional Pay

Amount of Bonus / Number of Clients

Example 1

1. **GEOMETRY** The table shows the perimeter of a square with sides of a given length.

Side Length (in.)	1	2	3	4	5
Perimeter (in.)	4	8	12	16	20

a. Graph the data.

b. Write an equation to describe the relationship. $y = 4x$

c. What conclusion can you make regarding the relationship between the side and the perimeter? **The perimeter is 4 times the length of the side.**

Additional Answers

1a.
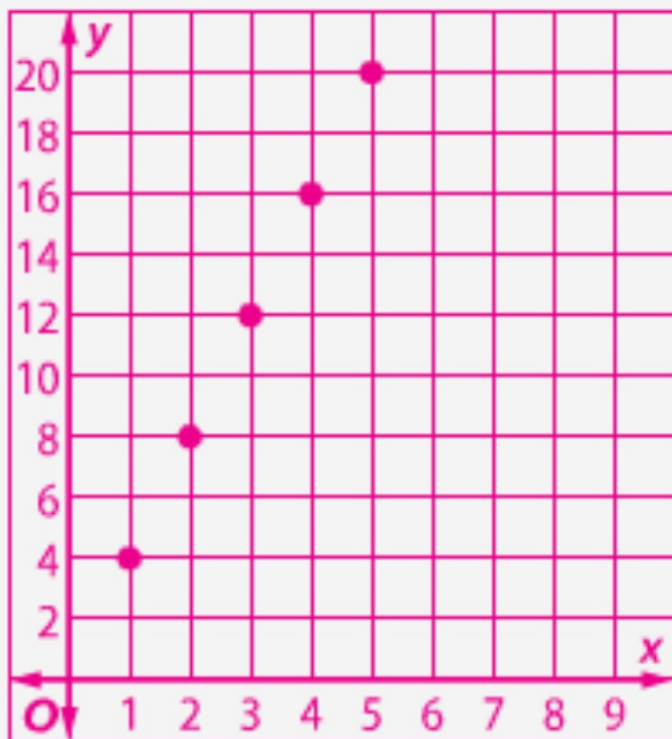
Example 2 Nonproportional Relationships

Write an equation in function notation for the graph.

Understand You are asked to write an equation of the relation that is graphed in function notation.

Plan Find the difference between the x-values and the difference between the y-values.

Solve Select points from the graph and place them in a table.
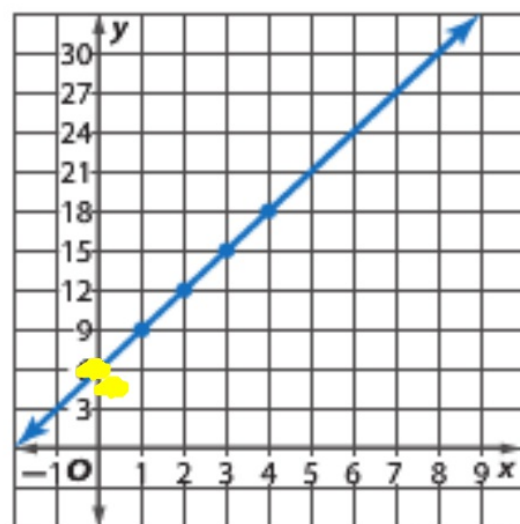
$$+1 \quad +1 \quad +1$$

x	1	2	3	4
y	9	12	15	18

$$+3 \quad +3 \quad +3$$

Notice that

$$\frac{1}{9} \neq \frac{2}{12} \neq \frac{3}{15} \neq \frac{4}{18}.$$

The difference between the x-values is 1, while the difference between the y-values is 3. This suggests that $y = 3x$ or $f(x) = 3x$.
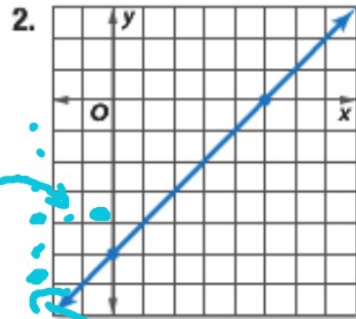
If $x = 1$, then $y = 3(1)$ or 3. But the y-value for $x = 1$ is 9. Let's try some other values and see if we can detect a pattern.
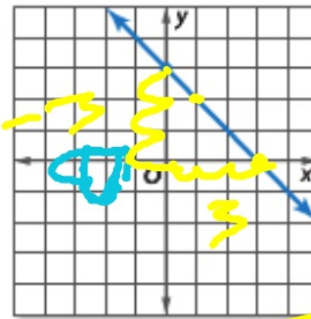
x	1	2	3	4

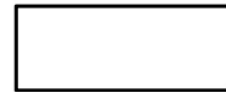**Example 2** Write an equation in function notation for each relation.

$y = mx + b$

2. $f(x) = x - 5$

3. $f(x) = -x + 3$

$\frac{-1}{1} = -1$

$-\frac{3}{3}$

$b = 3$

$m = \frac{1}{1} = b$

-5

Example 1

4. **CCSS** **STRUCTURE** The table shows the pages of comic books read.

Books Read	1	2	3	4	5
Pages Read	35	70	105	140	175

a. Graph the data. **See margin.**

b. Write an equation to describe the relationship. $y = 35x$

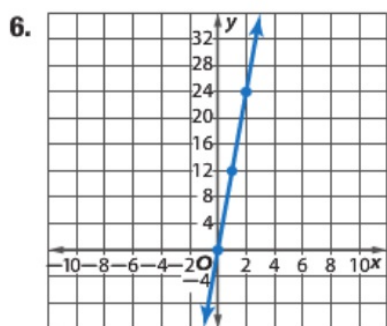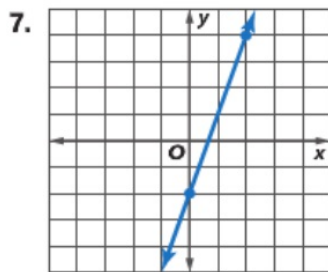**c.** Find the number of pages read if 8 comic books were read. **280**
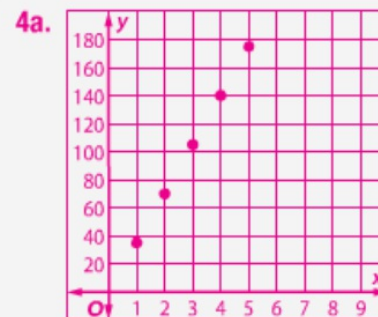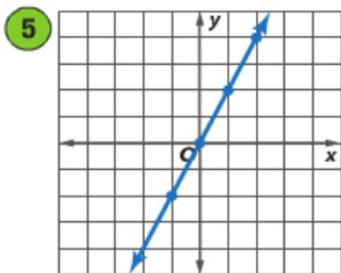
4a.

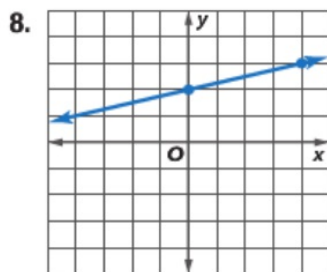**Example 2**   **Write an equation in function notation for each relation.**

5

$f(x) = 2x$

6.

$f(x) = 12x$

7.

$f(x) = 3x - 2$

8.

$f(x) = \frac{1}{4}x + 2$

9. $0, 3, 6, \ldots$

10. $-4, 0, 4, \ldots$ $f(n) = 4n - 8$; nonproportional; the graph of the line does not pass through (0, 0).

11. BOWLING Marielle is bowling with her friends. The table shows prices for renting a pair of shoes and bowling. Write an equation to represent the total price y if Marielle buys x games.
$y = 2.25x + 2.50$

Games Bowled	Total Price ($)
2	7.00
4	11.50
6	16.00
8	20.50

12. SNOWFALL The total snowfall each hour of a winter snowstorm is shown in the table below.

9. $f(n) = 3n - 3$; nonproportional; the function does not describe a direct variation.

Hour	1	2	3	4
Inches of Snowfall	1.65	3.30	4.95	6.60

a. Write an equation to fit the data in the table. $a_n = 1.65n$ The relationship is proportional.

b. Describe the relationship between the hour and inches of snowfall.

13 FUNDRAISER The Cougar Pep Squad wants to sell T-shirts in the bookstore for the spring dance. The cost in dollars to order T-shirts in their school colors is represented by the equation $C = 2t + 3$.

a. Make a table of values that represents this relationship.   See Chapter 3 Answer Appendix.

b. Rewrite the equation in function notation. $C(t) = 2t + 3$

c. Graph the function.   See Chapter 3 Answer Appendix.

d. Describe the relationship between the number of T-shirts and the cost.
This relationship is nonproportional.

Lesson 3-6

13a. Sample answer:

Number of T-Shirts Ordered	5	10	15	20	25
Cost ($)	13	23	33	43	53

13c.