3.3 Rules for Differentiation

Copy this down;

Basic Properties and Formulas

If $f(x)$ and $g(x)$ are differentiable functions (the derivative exists), c and n are any real numbers,

1. $(cf)' = cf'(x)$
2. $(f \pm g)' = f'(x) \pm g'(x)$
3. $(fg)' = f'g + fg'$ – **Product Rule**
4. $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ – **Quotient Rule**
5. $\frac{d}{dx}(c) = 0$
6. $\frac{d}{dx}(x^n) = nx^{n-1}$ – **Power Rule**
7. $\boxed{\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)}$
   This is the **Chain Rule**

We'll deal with this next week...

If you scan through the lesson, you'll notice that there's a proof for each of the rules...

The trick for today; you need to be able to change the problem so that you can use the differentiation rules easier!

$x^1 \rightarrow x^0 = 1$

power rule

$$\frac{d}{dx}\left[x^n\right] = n \cdot x^{n-1}$$

In Exercises 1–6, write the expression as a sum of powers of x.

1. $(x^2 - 2)(x^{-1} + 1)$

$x + x^2 - 2x^{-1} - 2$

$\frac{d}{dx}$

$= 1 + 2x + 2x^{-2}$

2. $\left(\frac{x}{x^2+1}\right)^{-1}$ $\quad x + x^{-1}$

4. $\frac{3x^4 - 2x^3 + 4}{2x^2}$

$= \frac{3}{2}x^2 - x + \frac{1}{2x^2}$

$= \frac{3}{2}x^2 - x + \frac{1}{2}x^{-2}$

Ex: $\frac{d}{dx}\left[x^3\right]$

$= 3x^2$

5. $(x^{-1} + 2)(x^{-2} + 1)$

6. $\frac{x^{-1} + x^{-2}}{x^{-3}}$

RULE 5 The Product Rule $uv' + vu'$

The product of two differentiable functions u and v is differentiable, and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

$(x+3)^{1.5}$

13. Let $y = (x+1)(x^2+1)$. Find dy/dx **(a)** by applying the Product Rule, and **(b)** by multiplying the factors first and then differentiating. (a) $3x^2 + 2x + 1$ (b) $3x^2 + 2x + 1$

a)

$u = x+1 \quad v = x^2+1$

$u' = 1 \quad v' = 2x$

$(x+1)(2x) + (x^2+1)(1)$

$2x^2 + 2x + x^2 + 1$

$= 3x^2 + 2x + 1$

b)

b) $x^3 + x + x^2 + 1$

$3x^2 + 1 + 2x$

$3x^2 + 2x + 1$

RULE 2 Power Rule for Positive Integer Powers of x

If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

Sometimes there's more than one way to derive the function!

...other times, there's only one way to derive...

In Exercises 15–22, find dy/dx. Support your answer graphically.

17. $y = \dfrac{2x+5}{3x-2}$ $\quad -\dfrac{19}{(3x-2)^2}$

18. $y = \dfrac{x^2+5x-1}{x^2}$ $\quad -\dfrac{5}{x^2} + \dfrac{2}{x^3}$

18 $\quad y = 1 + \dfrac{5}{x} - \dfrac{1}{x^2}$

$= 1 + 5x^{-1} - x^{-2}$

then, derive

4. $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$ – **Quotient Rule**

17. $y = \dfrac{2x+5}{3x-2}$ $-\dfrac{19}{(3x-2)^2}$ ✓

f

g

$f = 2x+5$	$g = 3x-2$
$f' = 2$	$g' = 3$

$$= \frac{2(3x-2) - (2x+5)(3)}{(3x-2)^2}$$

$$= \frac{\cancel{6x} - 4 \cancel{-6x} - 15}{(3x-2)^2} = \frac{-19}{(3x-2)^2}$$

$y' = 0$

EXAMPLE 2 Finding Horizontal Tangents

Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where?

SOLUTION

The horizontal tangents, if any, occur where the slope dy/dx is zero. To find these points, we

(a) calculate dy/dx:

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2) = 4x^3 - 4x.$$

(b) solve the equation $dy/dx = 0$ for x:

$$4x^3 - 4x = 0$$
$$4x(x^2 - 1) = 0$$
$$x = 0, 1, -1.$$

The curve has horizontal tangents at $x = 0, 1,$ and -1. The corresponding points on the curve (found from the equation $y = x^4 - 2x^2 + 2$) are $(0, 2)$, $(1, 1)$, and $(-1, 1)$. You might wish to graph the curve to see where the horizontal tangents go.

Now try Exercise 7.

1) derive the function
2) set equal to zero, solve for x.

In Exercises 7–12, find the horizontal tangents of the curve.

7. $y = x^3 - 2x^2 + x + 1$

8. $y = x^3 - 4x^2 + x + 2$

7

23. Suppose u and v are functions of x that are differentiable at $x = 0$, and that $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$, $v'(0) = 2$. Find the values of the following derivatives at $x = 0$.

(a) $\frac{d}{dx}(uv)$ 13

(b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ -7

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ $\frac{7}{25}$

(d) $\frac{d}{dx}(7v - 2u)$ 20

1. $(c\,f)' = c\,f'(x)$
2. $(f \pm g)' = f'(x) \pm g'(x)$
3. $(f\,g)' = f'g + f\,g'$ – **Product Rule**
4. $\left(\frac{f}{g}\right)' = \frac{f'g - f\,g'}{g^2}$ – **Quotient Rule**

"hi" "ho"

$$\frac{\text{ho d hi} - \text{hi d ho}}{\text{ho ho}}$$

b $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$

$$= \frac{(-1)(-3) - (5)(2)}{(-1)^2}$$

$$\frac{3 - 10}{1} =$$

c $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{uv' - vu'}{u^2}$

$$\frac{(5)(2) - (-1)(-3)}{5^2}$$

$$\frac{10 - 3}{25} = \frac{7}{25}$$

In Exercises 29–32, find dy/dx.

29. $y = 4x^{-2} - 8x + 1$ $\quad -8x^{-3} - 8$

30. $y = \dfrac{x^{-4}}{4} - \dfrac{x^{-3}}{3} + \dfrac{x^{-2}}{2} - x^{-1} + 3$ $\quad -x^{-5} + x^{-4} - x^{-3} + x^{-2}$

31. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$ $\quad \dfrac{1}{\sqrt{x}(\sqrt{x} + 1)^2}$

32. $y = 2\sqrt{x} - \dfrac{1}{\sqrt{x}}$ $\quad \dfrac{1}{\sqrt{x}} + \dfrac{1}{2x^{3/2}}$

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$\frac{1}{2} - 1$

31

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$$

$$f = x^{\frac{1}{2}} - 1 \qquad g = x^{\frac{1}{2}} + 1$$

$$f' = \frac{1}{2}x^{-\frac{1}{2}} \qquad g' = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f' = \frac{(x^{\frac{1}{2}} + 1)(\frac{1}{2}x^{-\frac{1}{2}}) - (x^{\frac{1}{2}} - 1)(\frac{1}{2}x^{-\frac{1}{2}})}{(x^{\frac{1}{2}} + 1)^2}$$

$$= \frac{\cancel{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - \cancel{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}}{(x^{\frac{1}{2}} + 1)^2}$$

$$= \frac{x^{-\frac{1}{2}}}{(x^{\frac{1}{2}} + 1)^2} = \frac{1}{x^{\frac{1}{2}}(x^{\frac{1}{2}} + 1)^2}$$

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21. $y = \dfrac{x^2}{1 - x^3}$ $\quad \dfrac{x^4 + 2x}{(1 - x^3)^2}$

22. $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$ $\quad \dfrac{12 - 6x^2}{(x^2 - 3x + 2)^2}$

In Exercises 37–42, support your answer graphically.

37. Find an equation of the line perpendicular to the tangent to the

21) $y = \dfrac{x^2}{1-x^3}$

$f = x^2 \qquad g = 1 - x^3$

$f' = 2x \qquad g' = -3x^2$

$$= \frac{(1-x^3)(2x) - (x^2)(-3x^2)}{(1-x^3)^2}$$

$$= \frac{2x - 2x^4 + 3x^4}{(1-x^3)^2} = \frac{x^4 + 2x}{(1-x^3)^2}$$

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EXAMPLE 8 Finding Higher Order Derivatives

Find the first four derivatives of $y = x^3 - 5x^2 + 2$.

SOLUTION

The first four derivatives are:

First derivative: $y' = 3x^2 - 10x$;

Second derivative: $y'' = 6x - 10$;

Third derivative: $y''' = 6$;

Fourth derivative: $y^{(4)} = 0$.

This function has derivatives of all orders, the fourth and higher order derivatives all being zero. *Now try Exercise 33.*

In Exercises 33–36, find the first four derivatives of the function.

33. $y = x^4 + x^3 - 2x^2 + x - 5$ **34.** $y = x^2 + x + 3$

EXAMPLE 7 Using the Power Rule

Find an equation for the line tangent to the curve

$$y = \frac{x^2 + 3}{2x}$$

at the point (1, 2). Support your answer graphically.

SOLUTION

We could find the derivative by the Quotient Rule, but it is easier to first simplify the function as a sum of two powers of x.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{x^2}{2x} + \frac{3}{2x}\right) \\ &= \frac{d}{dx}\left(\frac{1}{2}x + \frac{3}{2}x^{-1}\right) \\ &= \frac{1}{2} - \frac{3}{2}x^{-2}\end{aligned}$$

The slope at $x = 1$ is

$$\left.\frac{dy}{dx}\right|_{x=1} = \left[\frac{1}{2} - \frac{3}{2}x^{-2}\right]_{x=1} = \frac{1}{2} - \frac{3}{2} = -1.$$

step 1: find the value of the slope

The line through $(1, 2)$ with slope $m = -1$ is

$$y - 2 = (-1)(x - 1)$$
$$y = -x + 1 + 2$$
$$y = -x + 3.$$

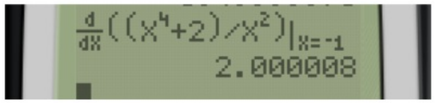
We graph $y = (x^2 + 3)/2x$ and $y = -x + 3$ (Figure 3.20), observing that the line appears to be tangent to the curve at $(1, 2)$. Thus, we have graphical support that our computations are correct. ***Now try Exercise 27.***

step 2: find the equation of the line that goes through the given point.

In Exercises 27 and 28, find an equation for the line tangent to the curve at the given point.

27. $y = \dfrac{x^3 + 1}{2x}$, $x = 1$ $\quad y = \dfrac{1}{2}x + \dfrac{1}{2}$

28. $y = \dfrac{x^4 + 2}{x^2}$, $x = -1$ $\quad y = 2x + 5$



HW: p.124 #1-42

In Exercises 37–42, support your answer graphically.

37. Find an equation of the line perpendicular to the tangent to the curve $y = x^3 - 3x + 1$ at the point $(2, 3)$. $y = -\frac{1}{9}x + \frac{29}{9}$

...find the derivative, THEN find the negative reciprocal...

let $x=2$...

$(2,3)$

$y' = 3x^2 - 3$

$y' = 3(2)^2 - 3 = 12 - 3 = 9$

$m = -\frac{1}{9}$

$y - 3 = -\frac{1}{9}(x-2)$

$y - 3 = -\frac{1}{9}x + \frac{2}{9} \quad + \frac{3}{1}$

$+3$

$y = -\frac{1}{9}x + \frac{29}{9}$