4-1 Graphing Quadratic Functions

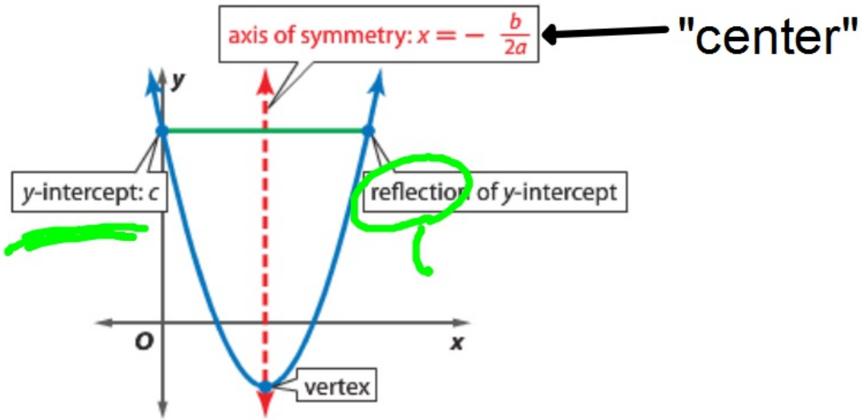
KeyConcept Graph of a Quadratic Function—Parabola

Words

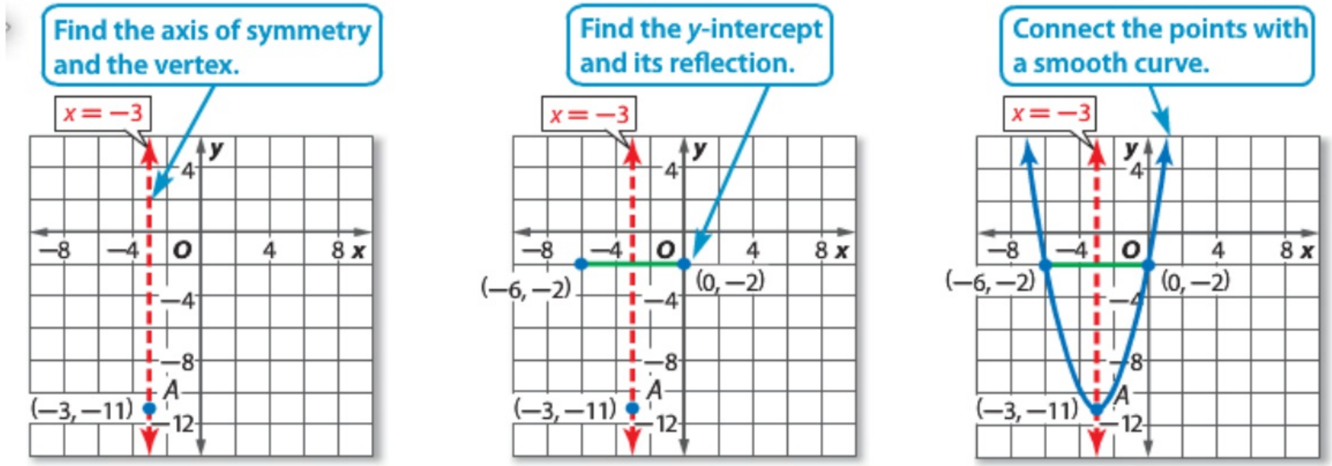
Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.

- The y-intercept is $a(0)^2 + b(0) + c$ or c.
- The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.

Model



Now you can use the axis of symmetry to help plot points and graph a parabola. For $y = x^2 + 6x - 2$ below, the axis of symmetry is $x = -\frac{b}{2a} = -\frac{6}{2(1)}$ or $x = -3$.



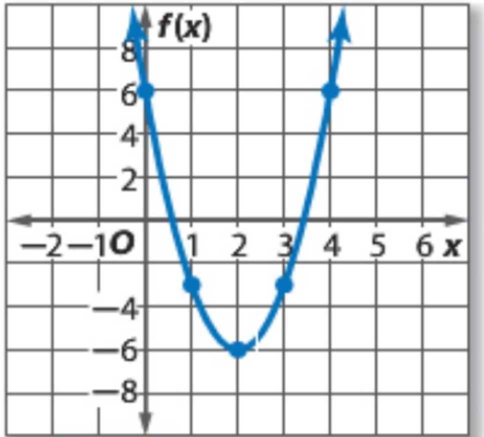
Example 1 Graph a Quadratic Function by Using a Table

Graph $f(x) = 3x^2 - 12x + 6$ by making a table of values.

$ax^2 + bx + c$

Choose integer values for x, and evaluate the function for each value. Graph the resulting coordinate pairs, and connect the points with a smooth curve.

x	$3x^2 - 12x + 6$	$f(x)$	$(x, f(x))$
0	$3(0)^2 - 12(0) + 6$	6	(0, 6)
1	$3(1)^2 - 12(1) + 6$	−3	(1, −3)
2	$3(2)^2 - 12(2) + 6$	−6	(2, −6)
3	$3(3)^2 - 12(3) + 6$	−3	(3, −3)
4	$3(4)^2 - 12(4) + 6$	6	(4, 6)



$$\frac{-b}{2a} = \frac{(-12)}{2(3)} = \frac{12}{6} = 2$$

GuidedPractice

Examples 1–2 Complete parts a–c for each quadratic function. **1–6. See Chapter 4 Answer Appendix.**

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = 3x^2$ $+0$
2. $f(x) = -6x^2$
3. $f(x) = x^2 - 4x$
4. $f(x) = -x^2 - 3x + 4$
5. $f(x) = 4x^2 - 6x - 3$
6. $f(x) = 2x^2 - 8x + 5$

$x = \frac{-b}{2a}$

$\frac{0}{2(3)} = 0$

$3(-2)^2$

(1)

x	y
−2	12
−1	3
0	0
1	3
2	12

$3(-1)^2$

(3) $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 4/2 = 2$

x	y
0	0
1	−3
2	−4
3	−3
4	0

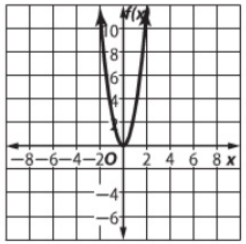
Lesson 4-1

1a. y-int $= 0$; axis of symmetry: $x = 0$; x-coordinate $= 0$

1b.

x	$f(x)$
−2	12
−1	3
0	0
1	3
2	12

1c.

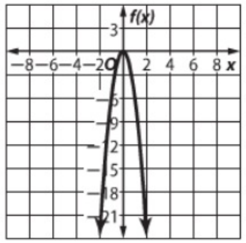


2a. y-int $= 0$; axis of symmetry: $x = 0$; x-coordinate $= 0$

2b.

x	$f(x)$
−2	−24
−1	−6
0	0
1	−6
2	−24

2c.

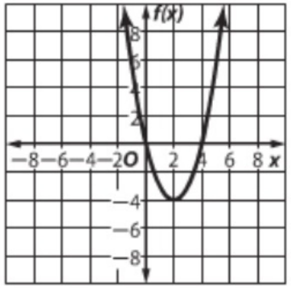


3a. *y*-int = 0; axis of symmetry: $x = 2$; *x*-coordinate = 2

3b.

x	$f(x)$
0	0
1	−3
2	−4
3	−3
4	0

3c.

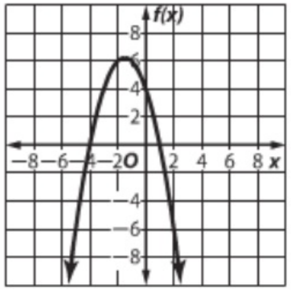


4a. *y*-int = 4; axis of symmetry: $x = -1.5$; *x*-coordinate = −1.5

4b.

x	$f(x)$
−3	4
−2	6
−1.5	6.25
−1	6
0	4

4c.

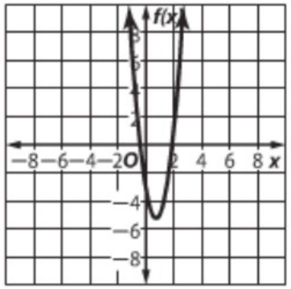


5a. *y*-int = −3; axis of symmetry: $x = 0.75$; *x*-coordinate = 0.75

5b.

x	$f(x)$
−1	7
0	−3
0.75	−5.25
1.5	−3
2.5	7

5c.

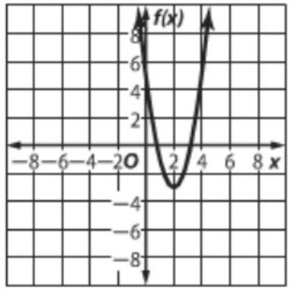


6a. *y*-int = 5; axis of symmetry: $x = 2$; *x*-coordinate = 2

6b.

x	$f(x)$
0	5
1	−1
2	−3
3	−1
4	5

6c.



2 Maximum and Minimum Values The y-coordinate of the vertex of a quadratic function is the **maximum value** or the **minimum value** of the function. These values represent the greatest or lowest possible value the function can reach.

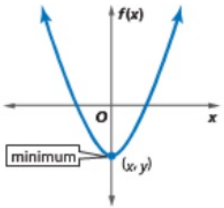
KeyConcept Maximum and Minimum Value

Words The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,

- opens up and has a minimum value when $a > 0$, and
- opens down and has a maximum value when $a < 0$.

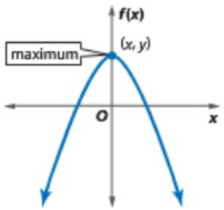
Model

a is positive.



The y-coordinate is the minimum value.

a is negative.



The y-coordinate is the maximum value.

Example 3 Maximum or Minimum Values

Consider $f(x) = -4x^2 + 12x + 18$.

a. Determine whether the function has a *maximum* or *minimum* value.

For this function, $a = -4$, so the graph opens down and the function has a maximum value.

b. State the maximum or minimum value of the function.

The maximum value of the function is the y-coordinate of the vertex.

The x-coordinate of the vertex is $-\frac{12}{2(-4)}$ or 1.5.

Find the y-coordinate of the vertex by evaluating the function for $x = 1.5$.

$f(x) = -4x^2 + 12x + 18$ — Original function

$= -4(1.5)^2 + 12(1.5) + 18$ — $x = 1.5$

$= -9 + 18 + 18$ or 27 — The maximum value of the function is 27.

c. State the domain and range of the function.

The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or $\{f(x) \mid f(x) \leq 27\}$.

Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

7. $f(x) = -x^2 + 6x - 1$

8. $f(x) = x^2 + 3x - 12$

9. $f(x) = 3x^2 + 8x + 5$

10. $f(x) = -4x^2 + 10x - 6$

9. min $= -\frac{1}{3}$; D = {all real numbers}, R $= \left\{ f(x) \,\middle|\, f(x) \geq -\frac{1}{3} \right\}$

7. max = 8; D = {all real numbers}, R $= \{f(x) | f(x) \leq 8\}$

8. min = −14.25; D = {all real numbers}, R $= \{f(x) | f(x) \geq -14.25\}$

10. max = 0.25; D = {all real numbers}, R $= \{f(x) | f(x) \leq 0.25\}$

(7)

opens down

$a = -1$ $b = 6$

maximum

$x = \frac{-(6)}{2(-1)} = \frac{-6}{-2} = 3$

$x = 3$

$f(3) = -(3)^2 + 6(3) - 1$

$-9 + 18 - 1 = 8$

Practice and Problem Solving

Extra Practice is on page R4.

Examples 1–2 Complete parts a–c for each quadratic function. **12–21. See Chapter 4 Answer Appendix.**

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

12. $f(x) = 4x^2$ **13.** $f(x) = -2x^2$

14. $f(x) = x^2 - 5$ **15.** $f(x) = x^2 + 3$

16. $f(x) = 4x^2 - 3$ **17.** $f(x) = -3x^2 + 5$

18. $f(x) = x^2 - 6x + 8$ **19.** $f(x) = x^2 - 3x - 10$

20. $f(x) = -x^2 + 4x - 6$ **21.** $f(x) = -2x^2 + 3x + 9$

Example 3 Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

22. $f(x) = 5x^2$ **23.** $f(x) = -x^2 - 12$

24. $f(x) = x^2 - 6x + 9$ **25.** $f(x) = -x^2 - 7x + 1$

26. $f(x) = 8x - 3x^2 + 2$ **27.** $f(x) = 5 - 4x - 2x^2$

28. $f(x) = 15 - 5x^2$ **29.** $f(x) = x^2 + 12x + 27$

30. $f(x) = -x^2 + 10x + 30$ **31.** $f(x) = 2x^2 - 16x - 42$

22. min = 0; D = {all real numbers}, R = $\{f(x) \mid f(x) \geq 0\}$

23. max = −12; D = {all real numbers}, R = $\{f(x) \mid f(x) \leq -12\}$

24. min = 0; D = {all real numbers}, R = $\{f(x) \mid f(x) \geq 0\}$

25. max = 13.25; D = {all real numbers}, R = $\{f(x) \mid f(x) \leq 13.25\}$

26. max = $\frac{22}{3}$; D = {all real numbers}, R = $\left\{f(x) \mid f(x) \leq \frac{22}{3}\right\}$

27. max = 7; D = {all real numbers}, R = $\{f(x) \mid f(x) \leq 7\}$

28. max = 15; D = {all real numbers}, R = $\{f(x) \mid f(x) \leq 15\}$

29. min = −9; D = {all real numbers}, R = $\{f(x) \mid f(x) \geq -9\}$

30. max = 55; D = {all real numbers}, R = $\{f(x) \mid f(x) \leq 55\}$

31. min = −74; D = {all real numbers}, R = $\{f(x) \mid f(x) \geq -74\}$

Example 4 **32.** CCSS **MODELING** A financial analyst determined that the cost, in thousands of dollars, of producing bicycle frames is $C = 0.000025f^2 - 0.04f + 40$, where f is the number of frames produced.

a. Find the number of frames that minimizes cost. **800**

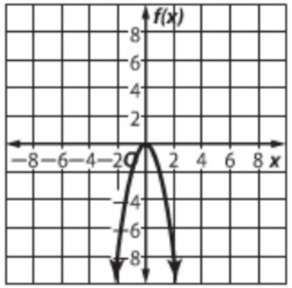
b. What is the total cost fo

13a. y-int $= 0$; axis of symmetry: $x = 0$; x-coordinate $= 0$

13b.

x	$f(x)$
−2	−8
−1	−2
0	0
1	−2
2	−8

13c.

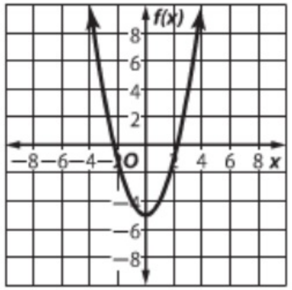


14a. y-int $= -5$; axis of symmetry: $x = 0$; x-coordinate $= 0$

14b.

x	$f(x)$
−2	−1
−1	−4
0	−5
1	−4
2	−1

14c.

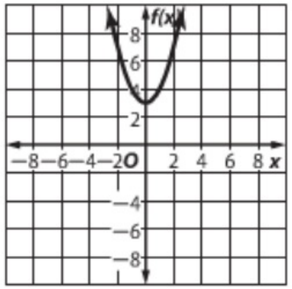


15a. y-int $= 3$; axis of symmetry: $x = 0$; x-coordinate $= 0$

15b.

x	$f(x)$
−2	7
−1	4
0	3
1	4
2	7

15c.

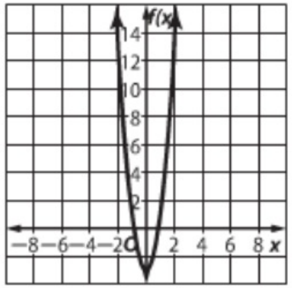


16a. y-int $= -3$; axis of symmetry: $x = 0$; x-coordinate $= 0$

16b.

x	$f(x)$
−2	13
−1	1
0	−3
1	1
2	13

16c.

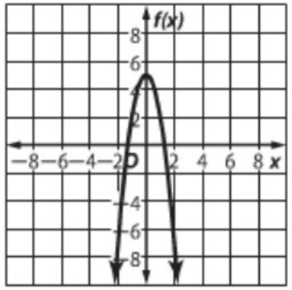


17a. y-int $= 5$; axis of symmetry: $x = 0$; x-coordinate $= 0$

17b.

x	$f(x)$
−2	−7
−1	2
0	5
1	2
2	−7

17c.

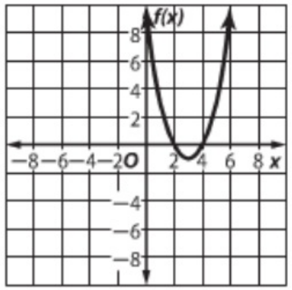


18a. y-int $= 8$; axis of symmetry: $x = 3$; x-coordinate $= 3$

18b.

x	$f(x)$
1	3
2	0
3	−1
4	0
5	3

18c.

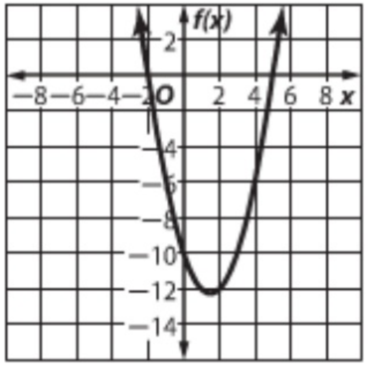


19a. y-int = −10; axis of symmetry: $x = 1.5$; x-coordinate = 1.5

19b.

x	$f(x)$
0	−10
1	−12
1.5	−12.25
2	−12
3	−10

19c.

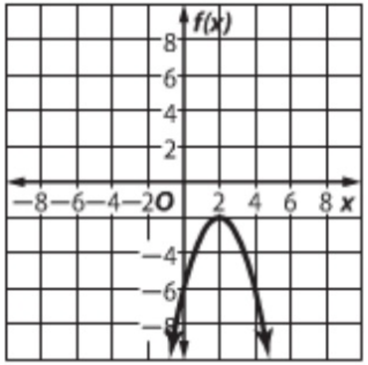


20a. y-int = −6; axis of symmetry: $x = 2$; x-coordinate = 2

20b.

x	$f(x)$
0	−6
1	−3
2	−2
3	−3
4	−6

20c.



21a. y-int = 9; axis of symmetry: $x = 0.75$; x-coordinate = 0.75

21b.

x	$f(x)$
−1	4
0	9
0.75	10.125
1.5	9
2.5	4

21c.