

4-2 Solving Quadratic Equations by Graphing

1 Solve Quadratic Equations Quadratic equations are quadratic functions that are set equal to a value. The **standard form** of a quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and a , b , and c are integers.

The solutions of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic equation is to find the **zeros** of the related quadratic function.

The zeros of the function are the x -intercepts of its graph.

Quadratic Function

$$f(x) = x^2 - x - 6$$

$$f(-2) = (-2)^2 - (-2) - 6 \text{ or } 0$$

$$f(3) = 3^2 - 3 - 6 \text{ or } 0$$

-2 and 3 are zeros of the function.

Quadratic Equation

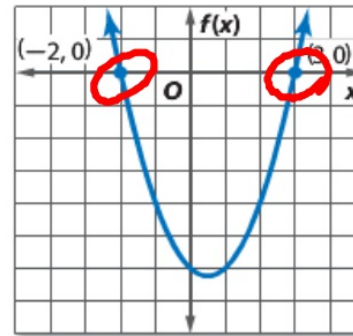
$$x^2 - x - 6 = 0$$

$$(-2)^2 - (-2) - 6 \text{ or } 0$$

$$3^2 - 3 - 6 \text{ or } 0$$

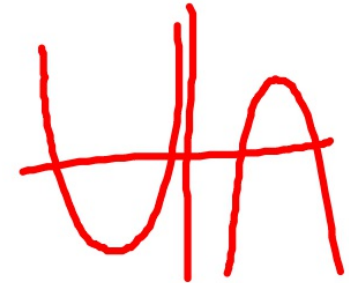
-2 and 3 are roots of the equation.

Graph of Function



The x -intercepts are **-2 and 3.**

↪
solution



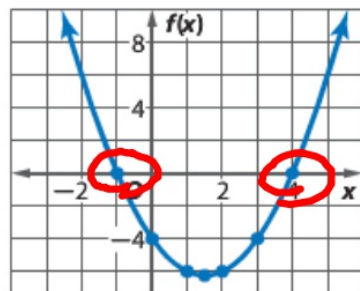
Example 1 Two Real Solutions

Solve $x^2 - 3x - 4 = 0$ by graphing.

Graph the related function, $f(x) = x^2 - 3x - 4$. The equation of the axis of symmetry is $x = -\frac{-3}{2(1)}$ or 1.5. Make a table using x -values around 1.5. Then graph each point.

x	-1	0	1	1.5	2	3	4
$f(x)$	0	-4	-6	-6.25	-6	-4	0

The zeros of the function are -1 and 4 . Therefore, the solutions of the equation are -1 and 4 or $\{x \mid x = -1, 4\}$.



Example 2 One Real Solution

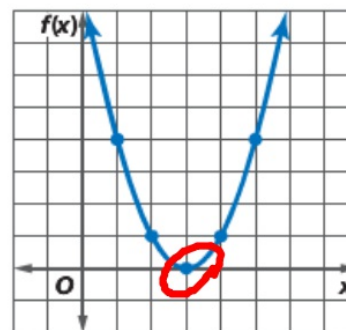
Solve $14 - x^2 = -6x + 23$ by graphing.

$$\begin{aligned} 14 - x^2 &= -6x + 23 && \text{Original equation} \\ 14 &= x^2 - 6x + 23 && \text{Add } x^2 \text{ to each side.} \\ 0 &= x^2 - 6x + 9 && \text{Subtract 14.} \end{aligned}$$

Graph the related function $f(x) = x^2 - 6x + 9$.

x	1	2	3	4	5
$f(x)$	4	1	0	1	4

The function has only one zero, 3 . Therefore, the solution is 3 or $\{x \mid x = 3\}$.



Example 3 No Real Solution

NUMBER THEORY Use a quadratic equation to find two real numbers with a sum of 15 and a product of 63.

Understand Let x represent one of the numbers. Then $15 - x$ is the other number.

Plan $x(15 - x) = 63$

$$15x - x^2 = 63$$

$$-x^2 + 15x - 63 = 0$$

The product of the numbers is 63.

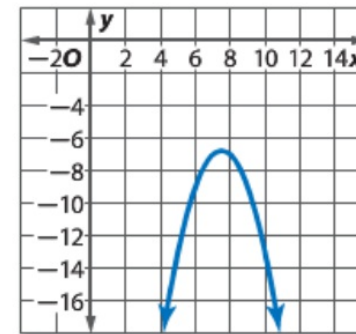
Distributive Property

Subtract 63.

Solve Graph the related function.

The graph has no x -intercepts. This means the original equation has no real solution. Thus, it is not possible for two real numbers to have a sum of 15 and a product of 63.

Check Try finding the product of several pairs of numbers with sums of 15. Is each product less than 63 as the graph suggests?

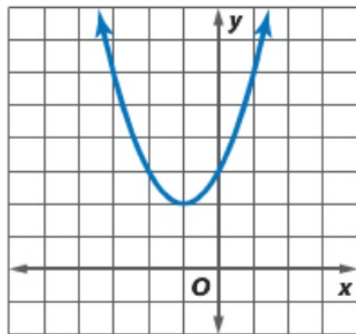


Example 1

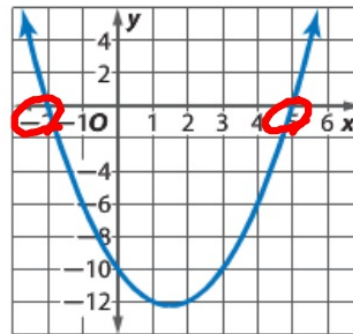
Use the related graph of each equation to determine its solutions.

1. no real solution

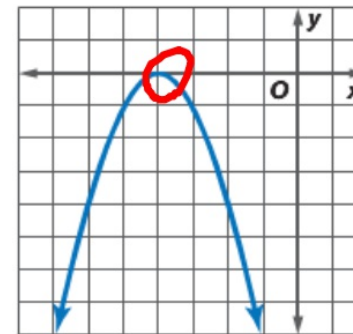
1. $x^2 + 2x + 3 = 0$



2. $x^2 - 3x - 10 = 0$ **-2, 5**



3. $-x^2 - 8x - 16 = 0$ **-4**

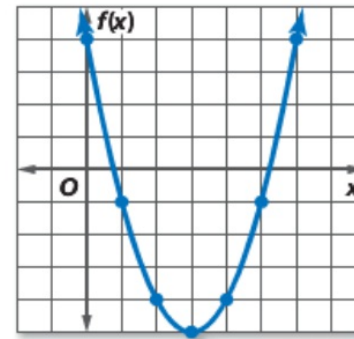



Example 4 Estimate Roots

Solve $x^2 - 6x + 4 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

x	0	1	2	3	4	5	6
$f(x)$	4	-1	-4	-5	-4	-1	4

The x -intercepts of the graph indicate that one solution is between 0 and 1, and the other solution is between 5 and 6.



Examples 2-5  **PRECISION** Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located. **4-11. See margin.**

4. $x^2 + 8x = 0$

$a = 1$
 $b = 8$

6. $4x - x^2 + 8 = 0$

8. $x^2 - 6x + 4 = -8$

10. $5x^2 + 10x - 4 = -6$

5. $x^2 - 3x - 18 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $-12 - 5x + 3x^2 = 0$

9. $9 - x^2 = 12$

11. $x^2 - 20 = 2 + x$

12. **NUMBER THEORY** Use a quadratic equation to find two real numbers with a sum of 2 and a product of -24 .

Additional Answers

4-11. See Ch. 4 Answer Appendix for graphs.

4. $0, -8$

5. $-3, 6$

6. between -2 and -1 , between 5 and 6

7. between -2 and -1 , 3

8. no real solution

9. no real solution

10. between -2 and -1 , between -1 and 0

11. between -5 and -4 , between 5 and 6

Handwritten work for problem 12:

$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-24)}}{2(1)} = \frac{8 \pm \sqrt{64 + 96}}{2} = \frac{8 \pm \sqrt{160}}{2}$

$\frac{8 \pm 4\sqrt{10}}{2} = 4 \pm 2\sqrt{10}$

$4 + 2\sqrt{10} \approx 10.32$

$4 - 2\sqrt{10} \approx -2.32$

Number line from -8 to 0 with tick marks at $-8, -7, -6, -5, -4, -3, -2, -1, 0$. The roots are approximately -2.32 and 10.32 .

Examples 2-5 **CCSS PRECISION** Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located. **4-11. See margin.**

4. $x^2 + 8x = 0$

6. $4x - x^2 + 8 = 0$

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5. $x^2 - 3x - 18 = 0$

7. $-12 - 5x + 3x^2 = 0$

9. $9 - x^2 = 12$

11. $x^2 - 20 = 2 + x$

12. **NUMBER THEORY** Use a quadratic equation to find two real numbers with a sum of 2 and a product of -24 . **6 and -4**

6. $-x^2 + 4x + 8 = 0$ $a = -1$

$x = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

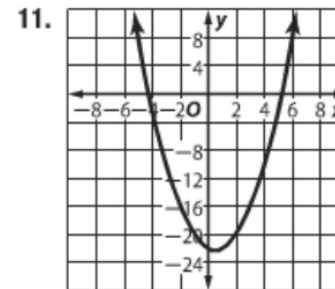
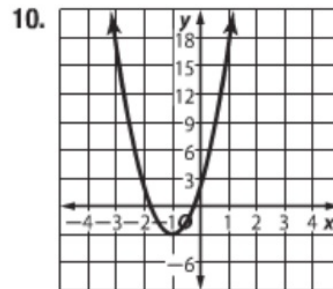
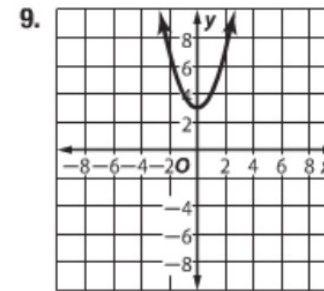
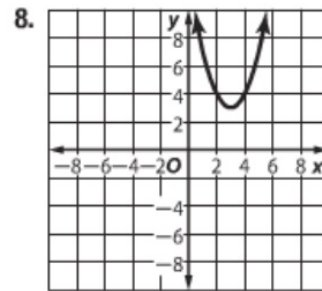
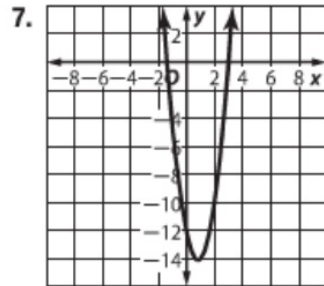
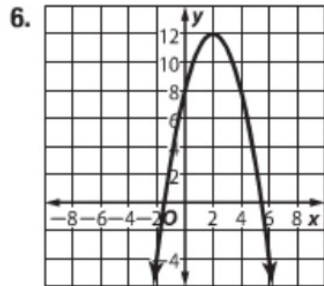
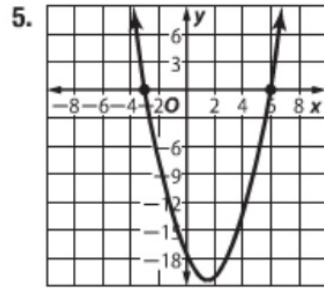
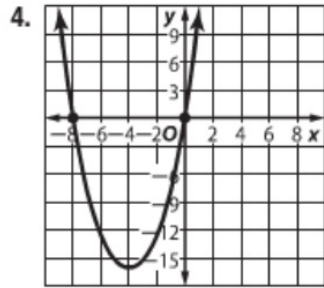
$x = 2$

$-1 - 4 + 8$
 $-5 + 8 = 3$
 $-4 - 8 + 8 = -4$

$4(7) - (8)^2 + 8$
 $32 - 64 + 8$

-2	-1	0	1	2	3	4	5	6
-4	3	8	11	12	11	8	3	-4

Lesson 4-2



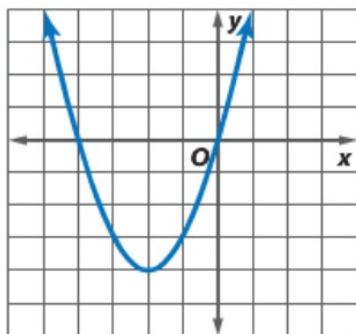
Example 6

13. **PHYSICS** How long will it take an object to fall from the roof of a building 400 feet above the ground? Use the formula $h(x) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet. **5 seconds**

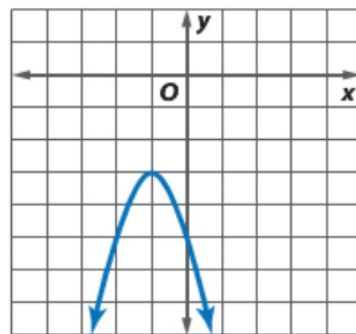
Example 1 Use the related graph of each equation to determine its solutions.

15. no real solution

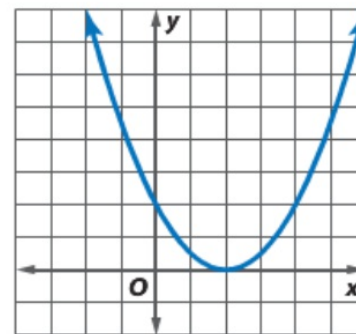
14. $x^2 + 4x = 0$ **-4, 0**



15. $-2x^2 - 4x - 5 = 0$

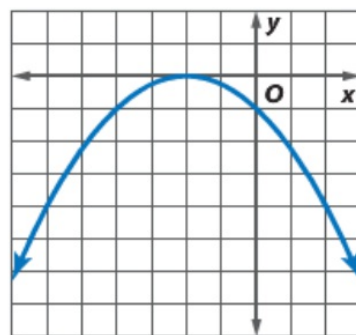


16. $0.5x^2 - 2x + 2 = 0$ **2**

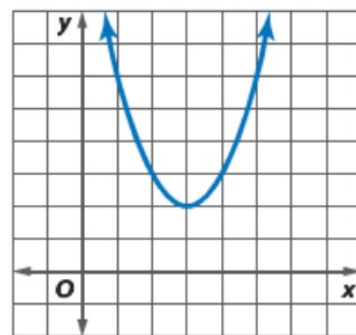


18. no real solution

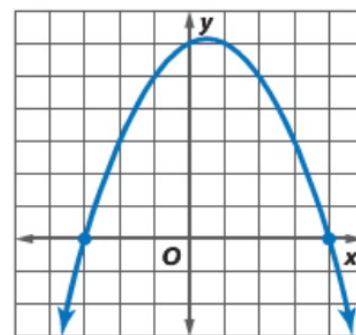
17. $-0.25x^2 - x - 1 = 0$ **-2**



18. $x^2 - 6x + 11 = 0$



19. $-0.5x^2 + 0.5x + 6 = 0$ **-3, 4**





Examples 2–4 Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located. **20–29. See Chapter 4 Answer Appendix for graphs.**

20. $x^2 = 5x$ **0, 5**

21. $-2x^2 - 4x = 0$ **-2, 0**

22. $x^2 - 5x - 14 = 0$ **-2, 7**

23. $-x^2 + 2x + 24 = 0$ **-4, 6**

24. $x^2 - 18x = -81$ **9**

25. $2x^2 - 8x = -32$ **no real solution**

26. $2x^2 - 3x - 15 = 4$ **between -3 and -2 and between 3 and 4**

27. $-3x^2 - 7 + 2x = -11$ **between -1 and 0 and between 1 and 2**

28. $-0.5x^2 + 3 = -5x - 2$

29. $-2x + 12 = x^2 + 16$ **no real solution**

28. between -1 and 0 and between 10 and 11

Example 5

Use the tables to determine the location of the zeros of each quadratic function.

30.

x	-7	-6	-5	-4	-3	-2	-1	0
f(x)	-8	-1	4	4	-1	-8	-22	-48

**between -6 and -5;
between -4 and -3**

31.

x	-2	-1	0	1	2	3	4	5
f(x)	32	14	2	-3	-3	2	14	32

**between 0 and 1;
between 2 and 3**

32.

x	-6	-3	0	3	6	9	12	15
f(x)	-6	-1	3	5	3	-1	-6	-14

**between -3 and 0;
between 6 and 9**

Example 6

NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

- 33** Their sum is -15 , and their product is -54 . **3 and -18**
- 34.** Their sum is 4 , and their product is -117 . **13 and -9**
- 35.** Their sum is 12 , and their product is -84 . **about -5 and 17**
- 36.** Their sum is -13 , and their product is 42 . **-6 and -7**
- 37.** Their sum is -8 and their product is -209 . **11 and -19**

