4-3 Solving Quadratic Equations by Factoring

Example 1 Translate Sentences into Equations

Write a quadratic equation in standard form with $-\frac{1}{3}$ and 6 as its roots.

$(x-p)(x-q)=0$	Write the pattern.
$\left[x-\left(-\frac{1}{3}\right)\right](x-6)=0$	Replace p with $-\frac{1}{3}$ and q with 6.
$\left(x+\frac{1}{3}\right)(x-6)=0$	Simplify.
$x^2-\frac{17}{3}x-2=0$	Multiply.
$3x^2-17x-6=0$	Multiply each side by 3 so that b and c are integers.

Example 1 **Write a quadratic equation in standard form with the given root(s).**

1. $-8, 5$ $x^2+3x-40=0$
2. $\frac{3}{2}, \frac{1}{4}$ $8x^2-14x+3=0$
3. $-\frac{2}{3}, \frac{5}{2}$ $6x^2-11x-10=0$

$(1)(x+8)(x-5)=0$

$\left(x+\frac{2}{3}\right)\left(x-\frac{5}{2}\right)=0$

$(3x+2)(2x-5)=0$

$(4x$

$-15x$

$-11x$

KeyConcept Zero Product Property

Words	For any real numbers a and b, if $ab=0$, then either $a=0$, $b=0$, or both a and b equal zero.
Example	If $(x+3)(x-5)=0$, then $x+3=0$ or $x-5=0$.

Example 2 Factor the GCF

Solve $16x^2 + 8x = 0$.

$16x^2 + 8x = 0$	Original equation.
$8x(2x) + 8x\ (1) = 0$	Factor the GCF.
$8x(2x + 1) = 0$	Distributive Property
$8x = 0$ or $2x + 1 = 0$	Zero Product Property
$x = 0$ $\quad 2x = -1$	Solve both equations.
$x = -\frac{1}{2}$	

Example 3 Perfect Squares and Differences of Squares

Solve each equation.

a. $x^2 + 16x + 64 = 0$

$x^2 = (x)^2;\ 64 = (8)^2$	First and last terms are perfect squares.
$16x = 2(x)(8)$	Middle term equals $2ab$.

$x^2 + 16x + 64$ is a perfect square trinomial.

$x^2 + 16x + 64 = 0$	Original equation
$(x + 8)^2 = 0$	Factor using the pattern.
$x + 8 = 0$	Take the square root of each side.
$x = -8$	Solve.

b. $x^2 = 64$

$x^2 = 64$	Original equation
$x^2 - 64 = 0$	Subtract 64 from each side.
$x^2 - (8)^2 = 0$	Write in the form $a^2 - b^2$.
$(x + 8)(x - 8) = 0$	Factor the difference of squares.
$x + 8 = 0$ or $x - 8 = 0$	Zero Product Property
$x = -8$ $\quad x = 8$	Solve.

Examples 2–4 Factor each polynomial. **5. $(6x-1)(3x+4)$ 7. $(x-7)(x+3)$ 8. $(2x-5)(x+6)$**

4. $35x^2 - 15x$ **$5x(7x-3)$**

5. $18x^2 - 3x + 24x - 4$

6. $x^2 - 12x + 32$ **$(x-8)(x-4)$**

7. $x^2 - 4x - 21$

8. $2x^2 + 7x - 30$

9. $16x^2 - 16x + 3$ **$(4x-3)(4x-1)$**

Example 5 **Solve each equation.**

10. $x^2 - 36 = 0$ **−6, 6**

11. $12x^2 - 18x = 0$ **$0, \frac{3}{2}$**

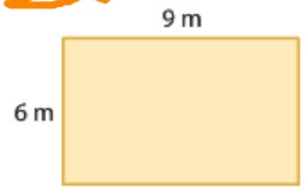
12. $12x^2 - 2x - 2 = 0$ **$-\frac{1}{3}, \frac{1}{2}$**

13. $x^2 - 9x = 0$ **0, 9**

14. $x^2 - 3x - 28 = 0$ **−4, 7**

15. $2x^2 - 24x = -72$ **6**

16. CCSS **SENSE-MAKING** Tamika wants to double the area of her garden by increasing the length and width by the same amount. What will be the dimensions of her garden then? **9 m by 12 m**



$6x(2x-3)=0$

2x

10 $x^2 - 36 = 0$

$+36 \quad +36$

$\sqrt{x^2} = \sqrt{36}$

$x = \pm 6$

$x^2 - 36 = 0$

$(x+6)(x-6)=0$

$x = \pm 6$

12. $12x^2 - 2x - 2 = 0$ $-\frac{1}{3}, \frac{1}{2}$

$\frac{12x^2}{2} - \frac{2x}{2} - \frac{2}{2} = \frac{0}{2}$

$6x^2 - 3x + 2x - 1 = 0$

$(3x + 1)$

	$6x^2$	$2x$
$2x$		
-1	$-3x$	-1

$3x + 1 = 0$

$-1 \quad -1$

$\frac{3x}{3} = \frac{-1}{3}$

$x = -\frac{1}{3}$

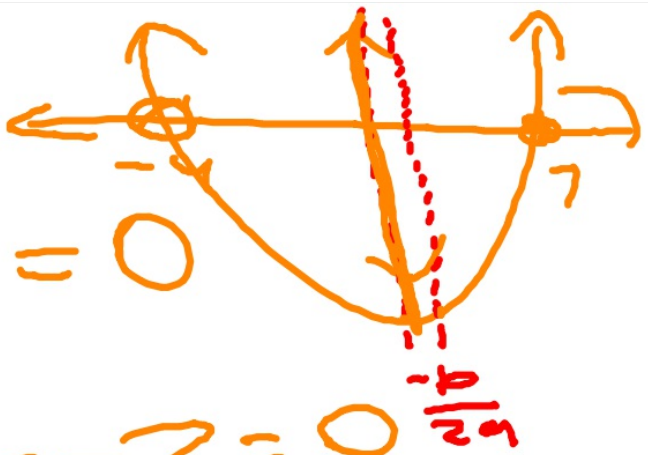
$2x - 1 = 0$

$+1 \quad +1$

$\frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$

14. $x^2 - 3x - 28 = 0$ $-4, 7$



$(x+4)(x-7)=0$

$x+4=0$ $\quad x-7=0$

$-4\ -4 \quad +7\ +7$

$x=-4 \quad x=7$

Examples 2–4 **Factor each polynomial.**

20. $40a^2 - 32a$ **$8a(5a - 4)$**
21. $51c^3 - 34c$ **$17c(3c^2 - 2)$**
22. $32xy + 40bx - 12ay - 15ab$
23. $3x^2 - 12$ **$3(x + 2)(x - 2)$**
24. $15y^2 - 240$
25. $48cg + 36cf - 4dg - 3df$
26. $x^2 + 13x + 40$
27. $x^2 - 9x - 22$
28. $3x^2 + 12x - 36$
29. $15x^2 + 7x - 2$
30. $4x^2 + 29x + 30$
31. $18x^2 + 15x - 12$
32. $8x^2z^2 - 4xz^2 - 12z^2$ **$4z^2(2x - 3)(x + 1)$**
33. $9x^2 - 25$ **$(3x + 5)(3x - 5)$**
34. $18x^2y^2 - 24xy^2 + 36y^2$ **$6y^2(3x^2 - 4x + 6)$**

Example 5 **Solve each equation.**

35. $15x^2 - 84x - 36 = 0$ **$-\frac{2}{5}, 6$**
36. $12x^2 + 13x - 14 = 0$ **$-\frac{7}{4}, \frac{2}{3}$**
37. $12x^2 - 108x = 0$ **0, 9**
38. $x^2 + 4x - 45 = 0$ **5, −9**
39. $x^2 - 5x - 24 = 0$ **8, −3**
40. $x^2 = 121$ **11, −11**
41. $x^2 + 13 = 17$ **2, −2**
42. $-3x^2 - 10x + 8 = 0$ **$-4, \frac{2}{3}$**
43. $-8x^2 + 46x - 30 = 0$ **5, $\frac{3}{4}$**

44. **GEOMETRY** The hypotenuse of a right triangle is 1 centimeter longer than one side and 4 centimeters longer than three times the other side. Find the dimensions of the triangle.
7 cm, 24 cm, 25 cm

45. **NUMBER THEORY** Find two consecutive even integers with a product of 624.
24 and 26 or −24 and −26

GEOMETRY **Find x and the dimensions of each rectangle.**

46. $A = 96$ ft^2; $x - 2$ ft; $x + 2$ ft
$x = 10$; 8 ft by 12 ft

47. $A = 432$ in.^{2}; $x - 2$ in.; $x + 4$ in.
$x = 20$; 24 in. by 18 in.

48. $A = 448$ ft^2; $3x - 4$ ft; $x + 2$ ft
$x = 12$; 14 ft by 32 ft