

LESSON 4-4 Parallel and Perpendicular Lines

EXAMPLE 1 Parallel Line Through a Given Point

Write the slope-intercept form of an equation for the line that passes through $(4, -2)$ and is parallel to the graph of $y = \frac{1}{2}x - 7$.

The line parallel to $y = \frac{1}{2}x - 7$ has the same slope, $\frac{1}{2}$.

Replace m with $\frac{1}{2}$, and (x_1, y_1) with $(4, -2)$ in the point-slope form.

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EXAMPLE 1

Parallel Line Through a Given Point

$$y = \frac{1}{2}x - 4$$

Write the equation in slope-intercept form.

EXAMPLE 1



Check Your Progress

Write the slope-intercept form of an equation for the line that passes through $(2, 3)$ and is parallel to the graph of $y = \frac{1}{2}x - 1$.

A. $y = -2x + 3$

B. $y = \frac{1}{2}x + 3$

C. $y = \frac{1}{2}x + 2$

D. $y = -2x - 1$

EXAMPLE 1



Check Your Progress

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A. $y = -2x + 3$

B. $y = \frac{1}{2}x + 3$

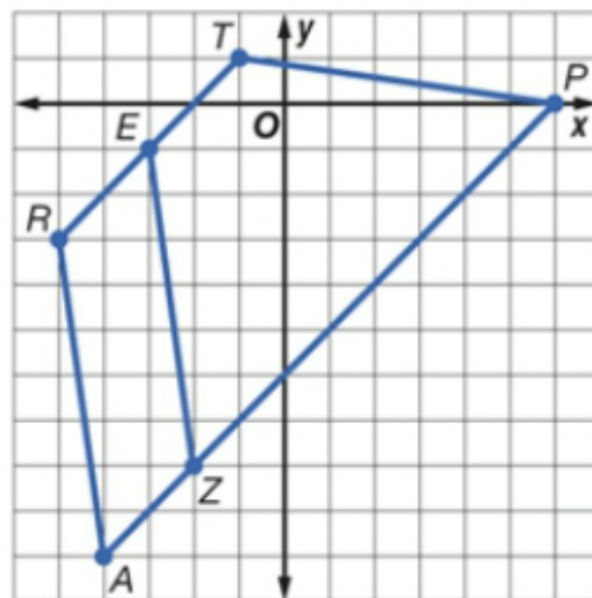
C. $y = \frac{1}{2}x + 2$

D. $y = -2x - 1$

Real-World Example 2

Slopes of Perpendicular Lines

A. GEOMETRY The height of a trapezoid is the length of a segment that is perpendicular to both bases. In trapezoid $ARTP$, \overline{RT} and \overline{AP} are bases. Can \overline{EZ} be used to measure the height of the trapezoid? Explain.



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Real-World Example 2 Slopes of Perpendicular Lines

Find the slope of each segment.

$$\text{Slope of } \overline{RT}: m = \frac{1 - (-3)}{-1 - (-5)} \text{ or } 1$$

$$\text{Slope of } \overline{AP}: m = \frac{0 - (-10)}{6 - (-4)} \text{ or } 1$$

$$\text{Slope of } \overline{EZ}: m = \frac{-8 - (-1)}{-2 - (-3)} \text{ or } -7$$

 Real-World Example 2 Slopes of Perpendicular Lines

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$$\text{Slope of } \overline{EZ}: m = \frac{-8 - (-1)}{-2 - (-3)} \text{ or } -7$$

Answer: The slope of \overline{RT} and \overline{AP} is 1 and the slope of \overline{EZ} is -7 . Since $1(-7) \neq -1$, \overline{EZ} is not perpendicular to \overline{RT} and \overline{AP} . So, it cannot be used to measure the height of $ARTP$.

Real-World Example 2

Slopes of Perpendicular Lines

B. GEOMETRY The height of a trapezoid is the length of a segment that is perpendicular to both bases. In trapezoid $ARTP$, \overline{RT} and \overline{AP} are bases. Are the bases parallel?

$$\text{Slope of } \overline{RT}: m = \frac{1 - (-3)}{-1 - (-5)} \text{ or } 1$$

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Real-World Example 2

Slopes of Perpendicular Lines

B. GEOMETRY The height of a trapezoid is the length of a segment that is perpendicular to both bases. In trapezoid $ARTP$, \overline{RT} and \overline{AP} are bases. Are the bases parallel?

$$\text{Slope of } \overline{RT}: m = \frac{1 - (-3)}{-1 - (-5)} \text{ or } 1$$

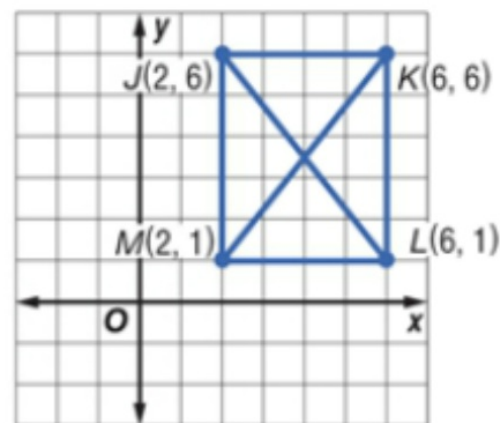
$$\text{Slope of } \overline{AP}: m = \frac{0 - (-10)}{6 - (-4)} \text{ or } 1$$

Answer: Yes, both \overline{RT} and \overline{AP} have a slope of 1.

 Real-World Example 2

Check Your Progress

The graph shows the diagonals of a rectangle. Determine whether \overline{JL} is perpendicular to \overline{KM} .



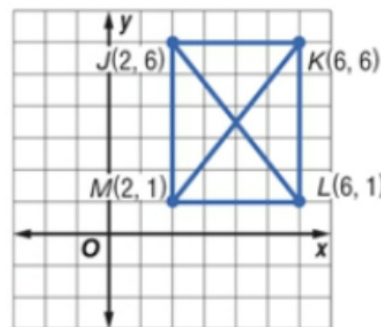
- A. \overline{JL} is not perpendicular to \overline{KM} .
- B. \overline{JL} is perpendicular to \overline{KM} .
- C. cannot be determined

LESSON 4-4 Parallel and Perpendicular Lines

Real-World Example 2

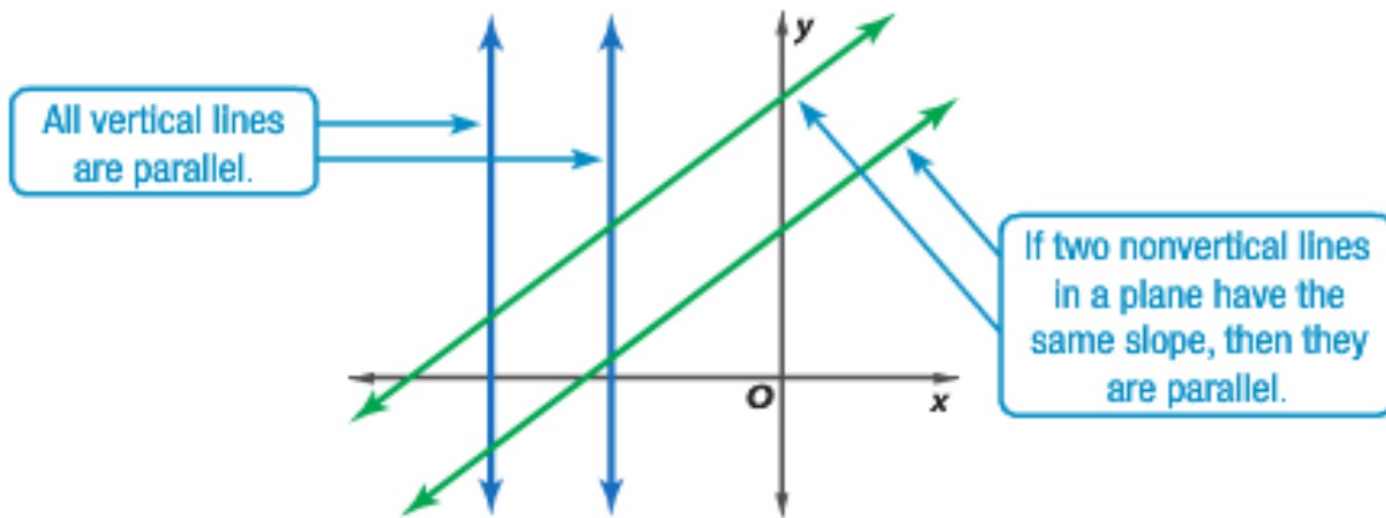
Check Your Progress

The graph shows the diagonals of a rectangle. Determine whether \overline{JL} is perpendicular to \overline{KM} .

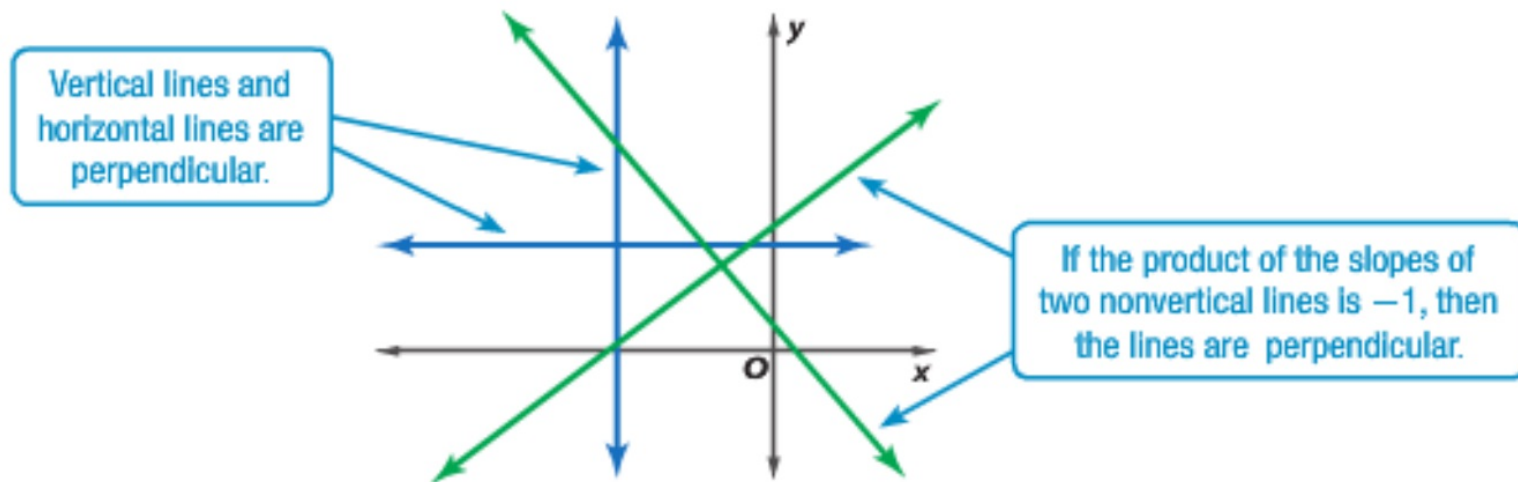


- A. \overline{JL} is not perpendicular to \overline{KM} .
- B. \overline{JL} is perpendicular to \overline{KM} .
- C. cannot be determined

Parallel Lines:



Perpendicular Lines:



4-4 Parallel and Perpendicular Lines

EXAMPLE 3

Parallel or Perpendicular Lines

Determine whether the graphs of $3x + y = 12$, $y = \frac{1}{3}x + 2$, and $2x - 6y = -5$ are *parallel* or *perpendicular*. Explain.

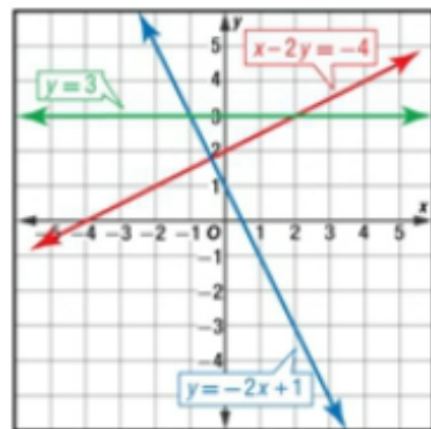
LESSON 4-4 Parallel and Perpendicular Lines

EXAMPLE 3

Check Your Progress

Determine whether the graphs of $y = -2x + 1$, $x - 2y = -4$, and $y = 3$ are *parallel* or *perpendicular*.

- A. $y = -2x + 1$ and $x - 2y = -4$ are perpendicular. None of the lines are parallel.
- B. $y = -2x + 1$ and $y = 3$ are perpendicular. None of the lines are parallel.
- C. $y = -2x + 1$ and $x - 2y = -4$ are parallel. None of the lines are perpendicular.
- D. None of the lines are parallel or perpendicular.



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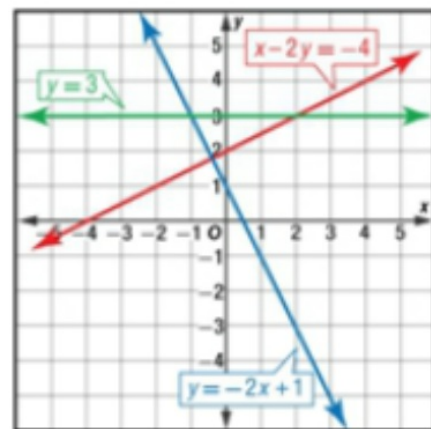
EXAMPLE 3



Check Your Progress

Determine whether the graphs of $y = -2x + 1$, $x - 2y = -4$, and $y = 3$ are *parallel* or *perpendicular*.

- A.** $y = -2x + 1$ and $x - 2y = -4$ are perpendicular. None of the lines are parallel.
- B.** $y = -2x + 1$ and $y = 3$ are perpendicular. None of the lines are parallel.
- C.** $y = -2x + 1$ and $x - 2y = -4$ are parallel. None of the lines are perpendicular.
- D.** None of the lines are parallel or perpendicular.



Check Your Understanding

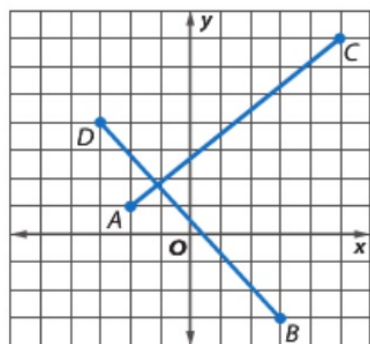
 = Step-by-Step Solutions begin on page R13.



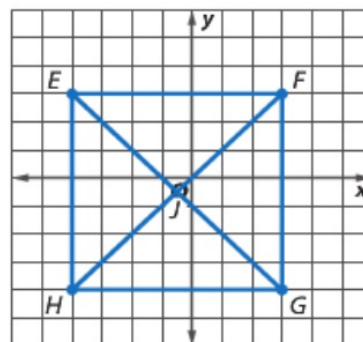
Example 1 Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.

1. $(-1, 2)$, $y = \frac{1}{2}x - 3$ $y = \frac{1}{2}x + 2\frac{1}{2}$ 2. $(0, 4)$, $y = -4x + 5$ $y = -4x + 4$

Example 2 3. **GARDENS** A garden is in the shape of a quadrilateral with vertices $A(-2, 1)$, $B(3, -3)$, $C(5, 7)$, and $D(-3, 4)$. Two paths represented by \overline{AC} and \overline{BD} cut across the garden. Are the paths perpendicular? Explain.



4. **CCSS PRECISION** A square is a quadrilateral that has opposite sides parallel, consecutive sides that are perpendicular, and diagonals that are perpendicular. Determine whether the quadrilateral is a square. Explain. **See margin.**



Example 3 Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

5. $y = -2x$, $2y = x$, $4y = 2x + 4$ 6. $y = \frac{1}{2}x$, $3y = x$, $y = -\frac{1}{2}x$

6. **None are parallel or perpendicular; none of the slopes are equal or opposite reciprocals.**

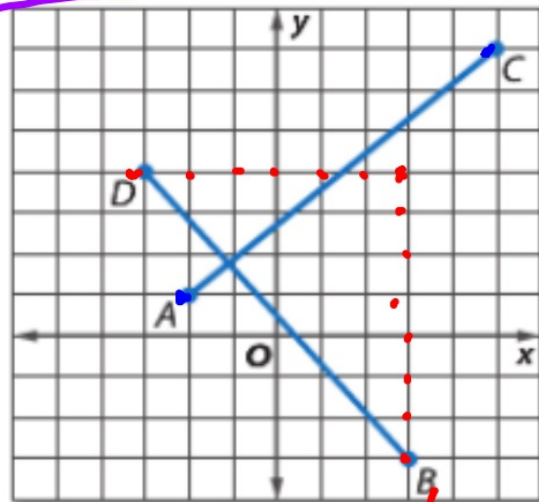
Additional Answer

4. Since \overline{EH} and \overline{FG} are parallel to the y -axis, they are parallel. Since \overline{EF} and \overline{HG} are parallel to the x -axis they are parallel and \overline{EH} is perpendicular to \overline{EF} and \overline{HG} . Likewise, \overline{FG} is perpendicular to \overline{EF} and \overline{HG} . The slope of \overline{EG} is -1 and the slope of \overline{FH} is 1 . Since the slopes are opposite reciprocals, $\overline{EG} \perp \overline{FH}$. The quadrilateral is a square.

Example 2

3. **GARDENS** A garden is in the shape of a quadrilateral with vertices $A(-2, 1)$, $B(3, -3)$, $C(5, 7)$, and $D(-3, 4)$. Two paths represented by \overline{AC} and \overline{BD} cut across the garden. Are the paths perpendicular? Explain.

yes



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 7}{-2 - 5}$$

$$= \frac{-6}{-7}$$

$$m = \frac{1}{6}$$

$$m = -\frac{7}{6}$$

6

Example 1

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.

1. $(-1, 2)$, $y = \frac{1}{2}x - 3$ $y = \frac{1}{2}x + 2\frac{1}{2}$

2. $(0, 4)$, $y = -4x + 5$ $y = -4x + 4$

$$y = \frac{1}{2}x - 3$$

parallel \rightarrow $m = \frac{1}{2}$

$$y = \frac{1}{2}x + 2\frac{1}{2}$$

$$y = mx + b$$

$$(2) = \left(\frac{1}{2}\right)(-1) + b$$

$$2 = -\frac{1}{2} + b$$

$$b = 2\frac{1}{2}$$

5/2

Example 3

Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

5. $y = -2x$, $2y = x$, $4y = 2x + 4$

6. $y = \frac{1}{2}x$, $3y = x$, $y = -\frac{1}{2}x$

6. None are parallel or perpendicular; none of the slopes are equal or opposite reciprocals.

with the slope of \overline{EG} , slopes are opposite reciprocals. $\overline{EG} \perp \overline{FH}$. The quadrilateral is a square.

perpendicular lines have negative reciprocals for their slopes...



Example 3

Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

5 $y = -2x$, $2y = x$, $4y = 2x + 4$

6. $y = \frac{1}{2}x$, $3y = x$, $y = -\frac{1}{2}x$

... slopes are opposite
 $\overline{EG} \perp \overline{FH}$. The quadrilateral is a square.

6. None are parallel or perpendicular; none of the slopes are equal or opposite reciprocals.

① $y = -2x$
 $m = -\frac{2}{1}$

② $\frac{2y}{2} = \frac{x}{2}$
 $y = \frac{1}{2}x$
 $m = \frac{1}{2}$

③ $\frac{4y}{4} = \frac{2x+4}{4}$
 $y = \frac{1}{2}x + 1$
 $m = \frac{1}{2}$

① is ~~parallel~~ perpendicular to ② and ③

② and ③ are parallel

2. $(0, 4), y = -4x + 5$ $y = -4x + 4$

$$m = -4$$

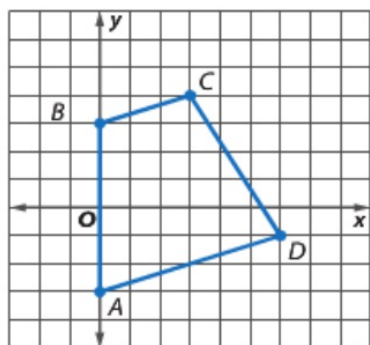
$$(4) = -4(0) + b$$

$$4 = b$$

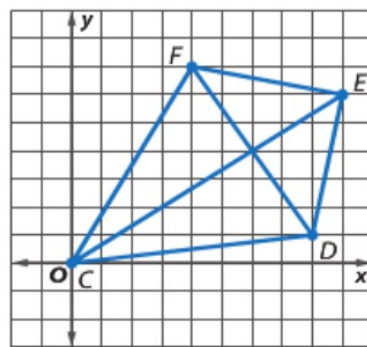
Example 1 Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.

11. $(3, -2)$, $y = x + 4$ 12. $(4, -3)$, $y = 3x - 5$ 13. $(0, 2)$, $y = -5x + 8$
 14. $(-4, 2)$, $y = -\frac{1}{2}x + 6$ 15. $(-2, 3)$, $y = -\frac{3}{4}x + 4$ 16. $(9, 12)$, $y = 13x - 4$

Example 2 17. **GEOMETRY** A trapezoid is a quadrilateral that has exactly one pair of parallel opposite sides. Is $ABCD$ a trapezoid? Explain your reasoning. **See margin.**



18. **GEOMETRY** $CDEF$ is a kite. Are the diagonals of the kite perpendicular? Explain your reasoning.



11. $y = x - 5$
 12. $y = 3x - 15$
 13. $y = -5x + 2$
 14. $y = -\frac{1}{2}x$
 15. $y = -\frac{3}{4}x + 1\frac{1}{2}$
 16. $y = 13x - 105$
 18. The slope of \overline{CE} is $\frac{2}{3}$ and the slope of \overline{DF} is $-\frac{3}{2}$. The diagonals are perpendicular because the slopes are opposite reciprocals.

19. Determine whether the graphs of $y = -6x + 4$ and $y = \frac{1}{6}x$ are perpendicular. Explain. **Yes; the slopes are -6 and $\frac{1}{6}$.**

20. **MAPS** On a map, Elmwood Drive passes through $R(4, -11)$ and $S(0, -9)$, and Taylor Road passes through $J(6, -2)$ and $K(4, -5)$. If they are straight lines, are the two streets perpendicular? Explain.

No; the slopes are $-\frac{1}{2}$ and $\frac{3}{2}$.

Additional Answers

17. Yes; the line containing \overline{AD} and the line containing \overline{BC} have the same slope, $\frac{1}{3}$. Therefore, one pair of sides is parallel. The slope of \overline{AB} is undefined and the slope of \overline{CD} is $\frac{1}{3}$.

Example 3**PERSEVERANCE** Determine whether the graphs of the following equations are parallel or perpendicular. Explain.

21. $2x - 8y = -24$, $4x + y = -2$, $x - 4y = 4$

22. $3x - 9y = 9$, $3y = x + 12$, $2x - 6y = 12$

21. $2x - 8y = -24$ and $4x + y = -2$ are perpendicular; $2x - 8y = -24$ and $x - 4y = 4$ are parallel.

22. All of the lines are parallel.

Example 4Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation. **23–28. See margin.**

23. $(-3, -2)$, $y = -2x + 4$

24. $(-5, 2)$, $y = \frac{1}{2}x - 3$

25. $(-4, 5)$, $y = \frac{1}{3}x + 6$

26. $(2, 6)$, $y = -\frac{1}{4}x + 3$

27. $(3, 8)$, $y = 5x - 3$

28. $(4, -2)$, $y = 3x + 5$

23. $y = \frac{1}{2}x - \frac{1}{2}$

24. $y = -2x - 8$

25. $y = -3x - 7$

26. $y = 4x - 2$

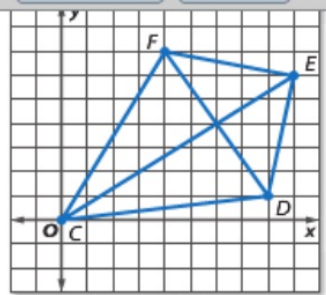
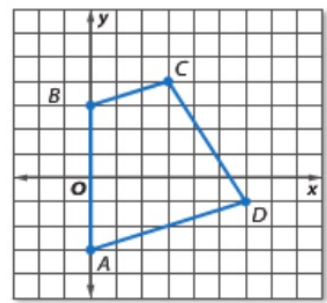
27. $y = -\frac{1}{5}x + 8\frac{3}{5}$

28. $y = -\frac{1}{3}x - \frac{2}{3}$

29. $y = 2x + 16$

30. $y = -\frac{3}{2}x + \frac{27}{2}$


31. $y = -\frac{1}{5}x - \frac{3}{25}$



15. $y = -\frac{1}{4}x + 1\frac{1}{2}$
 16. $y = 13x - 105$
 18. The slope of \overline{CE} is $\frac{2}{3}$ and the slope of \overline{DF} is $-\frac{3}{2}$. The diagonals are perpendicular because the slopes are opposite reciprocals.

19. Determine whether the graphs of $y = -6x + 4$ and $y = \frac{1}{6}x$ are perpendicular. Explain. **Yes; the slopes are -6 and $\frac{1}{6}$.**
20. **MAPS** On a map, Elmwood Drive passes through $R(4, -11)$ and $S(0, -9)$, and Taylor Road passes through $J(6, -2)$ and $K(4, -5)$. If they are straight lines, are the two streets perpendicular? Explain.

No; the slopes are $-\frac{1}{2}$ and $\frac{3}{2}$.

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21. $2x - 8y = -24$ and $4x + y = -2$ are perpendicular; $2x - 8y = -24$ and $x - 4y = 4$ are parallel.
 22. All of the lines are parallel.

Example 4 Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation. **23–28. See margin.**

23. $(-3, -2)$, $y = -2x + 4$ 24. $(-5, 2)$, $y = \frac{1}{2}x - 3$ 25. $(-4, 5)$, $y = \frac{1}{3}x + 6$
 26. $(2, 6)$, $y = -\frac{1}{4}x + ?$ 27. $(2, 8)$, $y = -5x$ 28. $(4, -2)$, $y = 3x + 5$

R Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.