4-5 Completing the Square

Example 1 Equation with Rational Roots

Solve $x^2 + 6x + 9 = 36$ by using the Square Root Property.

$x^2 + 6x + 9 = 36$	Original equation
$(x + 3)^2 = 36$	Factor the perfect square trinomial.
$x + 3 = \pm\sqrt{36}$	Square Root Property
$x + 3 = \pm 6$	$\sqrt{36} = 6$
$x = -3 \pm 6$	Subtract 3 from each side.
$x = -3 + 6$ or $x = -3 - 6$	Write as two equations.
$= 3$ $\quad\quad$ $= -9$	Simplify.

The solution set is $\{-9, 3\}$ or $\{x|x = -9, 3\}$.

Example 2 Equation with Irrational Roots

Solve $x^2 - 10x + 25 = 27$ by using the Square Root Property.

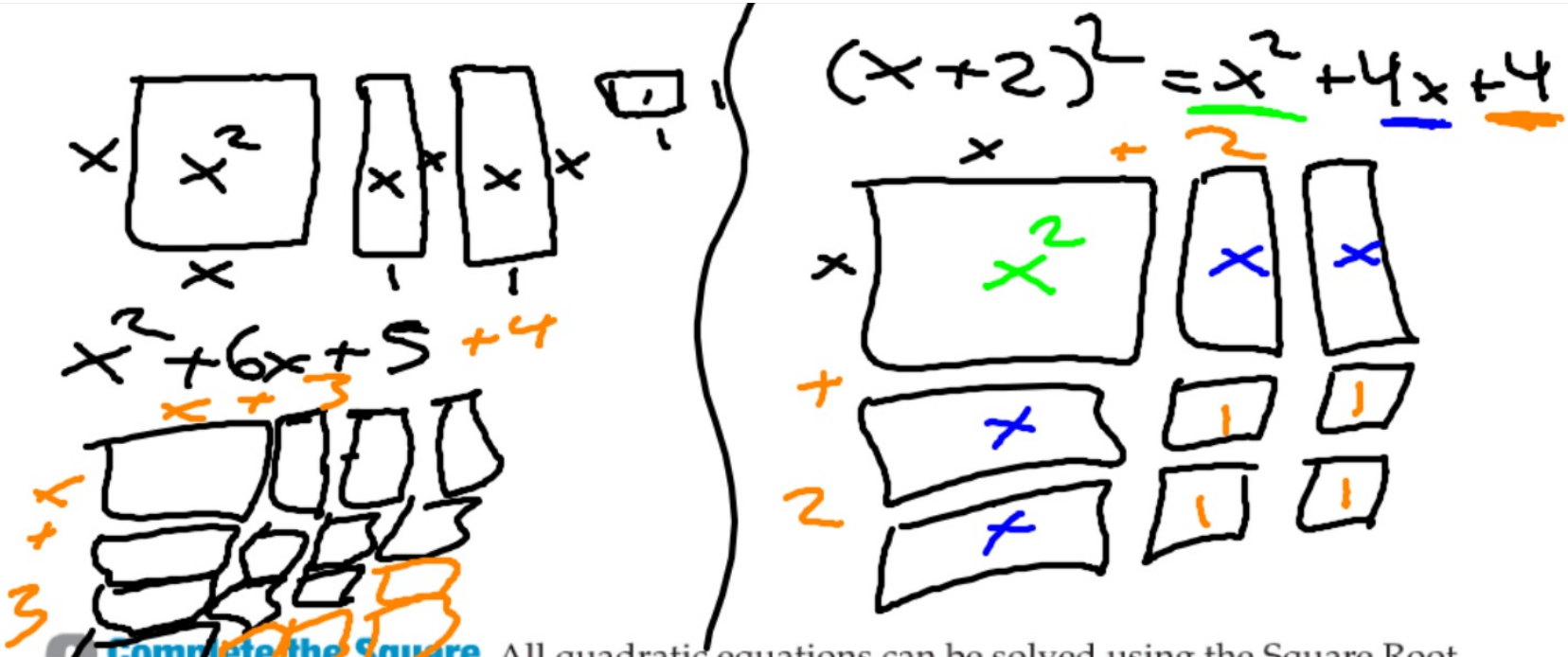
$x^2 - 10x + 25 = 27$		Original equation
$(x - 5)^2 = 27$		Factor the perfect square trinomial.
$x - 5 = \pm\sqrt{27}$		Square Root Property
$x = 5 \pm 3\sqrt{3}$		Add 5 to each side; $\sqrt{27} = 3\sqrt{3}$.
$x = 5 + 3\sqrt{3}$	or $x = 5 - 3\sqrt{3}$	Write as two equations.
≈ 10.2	≈ -0.2	Use a calculator.

The exact solutions of this equation are $5 + 3\sqrt{3}$ and $5 - 3\sqrt{3}$. The approximate solutions are -0.2 and 10.2. Check these results by finding and graphing the related quadratic function.

Examples 1–2 **Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary.**

1. $x^2 + 12x + 36 = 6$ {−8.45, −3.55}
2. $x^2 - 8x + 16 = 13$ {0.39, 7.61}
3. $x^2 + 18x + 81 = 15$ {−12.87, −5.13}
4. $9x^2 + 30x + 25 = 11$ {−2.77, −0.56}

5. **LASER LIGHT SHOW** The area A in square feet of a projected laser light show is given by $A = 0.16d^2$, where d is the distance from the laser to the screen in feet. At what distance will the projected laser light show have an area of 100 square feet? 25 ft



2 Complete the Square All quadratic equations can be solved using the Square Root Property by manipulating the equation until one side is a perfect square. This method is called **completing the square**.

$x^2+6x+9 = (x+3)(x+3)$.

Consider $x^2 + 16x = 9$. Remember to perform each operation on each side of the

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Consider $x^2 + 16x = 9$. Remember to perform each operation on each side of the equation.

$x^2 + 16x + \blacksquare = 9$	What value is needed for the perfect square?
$x^2 + 16x + 64 = 9 + 64$	$\left(\frac{16}{2}\right)^2 = 64$; add 64 to each side.
$x^2 + 16x + 64 = 73$	Simplify.
$(x + 8)^2 = 73$	We can now use the Square Root Property.

Example 3 Complete the Square

Find the value of c that makes $x^2 + 16x + c$ a perfect square. Then write the trinomial as a perfect square.

Step 1 Find one half of 16. $\frac{16}{2} = 8$

Step 2 Square the result in Step 1. $8^2 = 64$

Step 3 Add the result of Step 2 to $x^2 + 16x$. $x^2 + 16x + 64$

The trinomial $x^2 + 16x + 64$ can be written as $(x + 8)^2$.

$\left(\frac{-5}{2}\right)^2$

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

6. $x^2 - 10x + c$ **25; $(x - 5)^2$**

7. $x^2 - 5x + c$ **6.25; $(x - 2.5)^2$**

$\frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2$

Example 4 Solve an Equation by Completing the Square

Solve $x^2 + 10x - 11 = 0$ by completing the square.

$x^2 + 10x - 11 = 0$	Notice that $x^2 + 10x - 11$ is not a perfect square.
$x^2 + 10x = 11$	Rewrite so the left side is of the form $x^2 + bx$.
$x^2 + 10x + 25 = 11 + 25$	Since $\left(\frac{10}{2}\right)^2 = 25$, add 25 to each side.
$(x + 5)^2 = 36$	Write the left side as a perfect square.
$x + 5 = \pm 6$	Square Root Property
$x = -5 \pm 6$	Subtract 5 from each side.
$x = -5 + 6$ or $x = -5 - 6$	Write as two equations.
$= 1$ $\quad = -11$	Simplify.

8. $x^2 + 2x - 8 = 0$ **{−4, 2}**

Example 5 Equation with $a \neq 1$

Solve $2x^2 - 7x + 5 = 0$ by completing the square.

$2x^2 - 7x + 5 = 0$	Notice that $2x^2 - 7x + 5$ is not a perfect square.
$x^2 - \frac{7}{2}x + \frac{5}{2} = 0$	Divide by the coefficient of the quadratic term, 2.
$x^2 - \frac{7}{2}x = -\frac{5}{2}$	Subtract $\frac{5}{2}$ from each side.
$x^2 - \frac{7}{2}x + \frac{49}{16} = -\frac{5}{2} + \frac{49}{16}$	Since $\left(-\frac{7}{2} \div 2\right)^2 = \frac{49}{16}$, add $\frac{49}{16}$ to each side.
$\left(x - \frac{7}{4}\right)^2 = \frac{9}{16}$	Write the left side as a perfect square by factoring. Simplify the right side.
$x - \frac{7}{4} = \pm\frac{3}{4}$	Square Root Property
$x = \frac{7}{4} \pm \frac{3}{4}$	Add $\frac{7}{4}$ to each side.
$x = \frac{7}{4} + \frac{3}{4}$ or $x = \frac{7}{4} - \frac{3}{4}$	Write as two equations.
$= \frac{5}{2}$ $\qquad = 1$	

10. $2x^2 - 3x - 3 = 0$ **{−0.69, 2.19}**

$x^2 - \frac{3}{2} - \frac{3}{2}$

11. $2x^2 + 6x - 12 = 0$

{−4.37, 1.37}

$x^2 + 3x - 6 = 0$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$x^2 + 3x + \frac{9}{4} = 6 + \frac{9}{4}$

$(x + \frac{3}{2})^2 =$

Example 6 Equation with Imaginary Solutions

Solve $x^2 + 8x + 22 = 0$ by completing the square.

$x^2 + 8x + 22 = 0$	Notice that $x^2 + 8x + 22$ is not a perfect square.
$x^2 + 8x = -22$	Rewrite so the left side is of the form $x^2 + bx$.
$x^2 + 8x + 16 = -22 + 16$	Since $\left(\frac{8}{2}\right)^2 = 16$, add 16 to each side.
$(x + 4)^2 = -6$	Write the left side as a perfect square.
$x + 4 = \pm\sqrt{-6}$	Square Root Property
$x + 4 = \pm i\sqrt{6}$	$\sqrt{-1} = i$
$x = -4 \pm i\sqrt{6}$	Subtract 4 from each side.

9. $\{2 - i\sqrt{5}, 2 + i\sqrt{5}\}$

9. $x^2 - 4x + 9 = 0$

12. $x^2 + 4x + 6 = 0$

$\{-2 - i\sqrt{2}, -2 + i\sqrt{2}\}$

Examples 1–2 Solve each equation by using the Square Root Property. Round to the nearest hundredth if necessary. **14. {−5.16, 1.16} 15. {−1.47, 7.47} 16. {−8.24, 0.24} 17. {−7.65, −2.35} 18. {−8.24, −3.76}**

14. $x^2 + 4x + 4 = 10$
15. $x^2 - 6x + 9 = 20$
16. $x^2 + 8x + 16 = 18$
17. $x^2 + 10x + 25 = 7$
18. $x^2 + 12x + 36 = 5$
19. $x^2 - 2x + 1 = 4$ **{−1, 3}**
20. $x^2 - 5x + 6.25 = 4$
21. $x^2 - 15x + 56.25 = 8$
22. $x^2 + 32x + 256 = 1$ **{−17, −15}**
23. $x^2 - 3x + \frac{9}{4} = 6$ **{−0.95, 3.95}**
24. $x^2 + 7x + \frac{49}{4} = 4$ **{−5.5, −1.5}**
25. $x^2 - 9x + \frac{81}{4} = \frac{1}{4}$ **{4, 5}**

20. {0.5, 4.5} 21. {4.67, 10.33}

Example 3 Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

26. $x^2 + 8x + c$ **16; $(x + 4)^2$**
27. $x^2 + 16x + c$ **64; $(x + 8)^2$**
28. $x^2 - 11x + c$ **$\frac{121}{4}$; $\left(x - \frac{11}{2}\right)^2$**
29. $x^2 + 9x + c$ **20.25; $(x + 4.5)^2$**

Examples 4–6 Solve each equation by completing the square. **30. $\{2 - 2i\sqrt{2}, 2 + 2i\sqrt{2}\}$ 31. {−4.61, 2.61}**

30. $x^2 - 4x + 12 = 0$
31. $x^2 + 2x - 12 = 0$
32. $x^2 + 6x + 8 = 0$ **{−4, −2}**
33. $x^2 - 4x + 3 = 0$ **{1, 3}**
34. $2x^2 + x - 3 = 0$
35. $2x^2 - 3x + 5 = 0$
36. $2x^2 + 5x + 7 = 0$
37. $3x^2 - 6x - 9 = 0$
38. $x^2 - 2x + 3 = 0$
39. $x^2 + 4x + 11 = 0$
40. $x^2 - 6x + 18 = 0$
41. $x^2 - 10x + 29 = 0$
42. $3x^2 - 4x = 2$
43. $2x^2 - 7x = -12$
44. $x^2 - 2.4x = 2.2$
45. $x^2 - 5.3x = -8.6$
46. $x^2 - \frac{1}{5}x - \frac{11}{5} = 0$ **{−1.39, 1.59}**
47. $x^2 - \frac{9}{2}x - \frac{24}{5} = 0$ **{−0.89, 5.39}**

34. $\left\{-\frac{3}{2}, 1\right\}$

35–45. See Chapter 4 Answer Appendix.

Lesson 4-5

35. $\left\{\frac{3 - i\sqrt{31}}{4}, \frac{3 + i\sqrt{31}}{4}\right\}$
36. $\left\{\frac{-5 - i\sqrt{31}}{4}, \frac{-5 + i\sqrt{31}}{4}\right\}$
37. $\{-1, 3\}$
38. $\left\{1 - i\sqrt{2}, 1 + i\sqrt{2}\right\}$
39. $\left\{-2 - i\sqrt{7}, -2 + i\sqrt{7}\right\}$
40. $\{3 - 3i, 3 + 3i\}$
41. $\{5 - 2i, 5 + 2i\}$
42. $\{-0.39, 1.72\}$
43. $\left\{\frac{7 - i\sqrt{47}}{4}, \frac{7 + i\sqrt{47}}{4}\right\}$
44. $\{-0.71, 3.11\}$
45. $\left\{2.65 - i\sqrt{1.5775}, 2.65 + i\sqrt{1.5775}\right\}$