4-6 The Quadratic Formula and the Discriminant

$Ax^2 + Bx + C = 0$

uadratic Formula

lutions of a quadratic equation of the form $ax^2 + bx + c = 0$
ven by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$5x + 6 = 0 \rightarrow x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 Two Rational Roots

Solve $x^2 - 10x = 11$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$
\begin{array}{ccc}
 & & ax^2 + bx + c = 0 \\
 & & \downarrow \quad \downarrow \quad \downarrow \\
x^2 - 10x = 11 & \rightarrow & 1x^2 - 10x - 11 = 0
\end{array}
$$

Then, substitute these values into the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic Formula

$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-11)}}{2(1)}$ — Replace a with 1, b with -10, and c with -11.

$= \frac{10 \pm \sqrt{100 + 44}}{2}$ — Multiply.

$= \frac{10 \pm \sqrt{144}}{2}$ — Simplify.

$= \frac{10 \pm 12}{2}$ — $\sqrt{144} = 12$

$x = \frac{10 + 12}{2}$ or $x = \frac{10 - 12}{2}$ — Write as two equations.

$= 11$ $\quad$ $= -1$ — Simplify.

Agenda!

1) quadratic formula.
2) The discriminant.

$(x-11)(x+1) = 0$

Example 2 One Rational Root

Solve $x^2 + 8x + 16 = 0$ by using the Quadratic Formula. $(x+4)(x+4) = 0$

Identify a, b, and c. Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(16)}}{2(1)}$$ Replace a with 1, b with 8, and c with 16.

$$= \frac{-8 \pm \sqrt{0}}{2}$$ Simplify.

$$= \frac{-8}{2} \text{ or } -4$$ $\sqrt{0} = 0$

The solution is -4.

CHECK A graph of the related function shows that there is one solution at $x = -4$.

Example 3 Irrational Roots

Solve $2x^2 + 6x - 7 = 0$ by using the Quadratic Formula.

factor?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-7)}}{2(2)}$$ Replace a with 2, b with 6, and c with -7.

$$= \frac{-6 \pm \sqrt{92}}{4}$$ Simplify.

$$= \frac{-6 \pm 2\sqrt{23}}{4} \text{ or } \frac{-3 \pm \sqrt{23}}{2}$$ $\sqrt{92} = \sqrt{4 \cdot 23}$ or $2\sqrt{23}$

The approximate solutions are -3.9 and 0.9.

Example 4 Complex Roots

Solve $x^2 - 6x = -10$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$ Replace a with 1, b with −6, and c with 10.

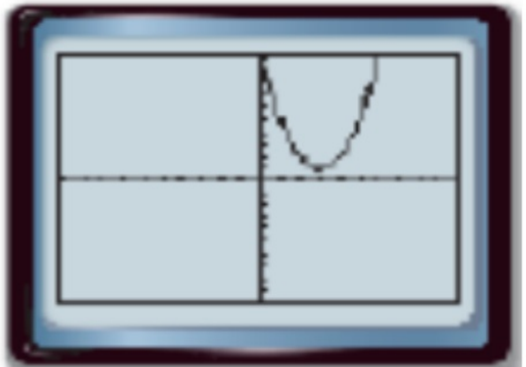
$$= \frac{6 \pm \sqrt{-4}}{2}$$ Simplify.

$$= \frac{6 \pm 2i}{2}$$ $\sqrt{-4} = \sqrt{4 \cdot (-1)}$ or $2i$

$$= 3 \pm i$$ Simplify.

The solutions are the complex numbers $3 + i$ and $3 - i$.

CHECK A graph of the related function shows that the solutions are complex, but it cannot help you find them. To check complex solutions, substitute them into the original equation.



[−10, 10] scl: 1 by [−10, 10] scl: 1

Check Your Understanding

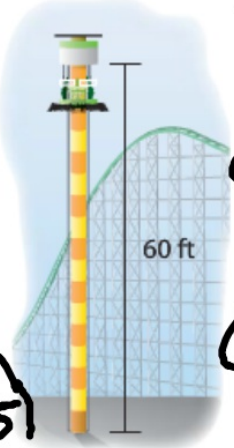
● = Step-by-Step Solutions begin on page R14.

Examples 1–4 Solve each equation by using the Quadratic Formula.

1. $x^2 + 12x - 9 = 0$ $-6 \pm 3\sqrt{5}$
2. $x^2 + 8x + 5 = 0$ $-4 \pm \sqrt{11}$
3. $4x^2 - 5x - 2 = 0$ $\frac{5 \pm \sqrt{57}}{8}$
4. $9x^2 + 6x - 4 = 0$ $\frac{-1 \pm \sqrt{5}}{3}$
5. $10x^2 - 3 = 13x$ $(1.5, -0.2)$
6. $22x = 12x^2 + 6$ $\left(\frac{3}{2}, \frac{1}{3}\right)$
7. $-3x^2 + 4x = -8$ $\frac{2 \pm 2\sqrt{7}}{3}$
8. $x^2 + 3 = -6x + 8$ $-3 \pm \sqrt{14}$

$+6x - 8 \quad +6x - 8$

Examples 3–4 9. **CCSS MODELING** An amusement park ride takes riders to the top of a tower and drops them at speeds reaching 80 feet per second. A function that models this ride is $h = -16t^2 - 64t + 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop from 60 feet to 0 feet? **about 0.78 second**



56
$8 \cdot 7$
$4 \cdot 2$
$2 \cdot 2$

8) $x^2 + 6x - 5 = 0$

$x = \frac{-6 \pm \sqrt{36 + 4(1)(5)}}{2}$

$= \frac{-6 \pm \sqrt{36 + 20}}{2} = \frac{-6 \pm \sqrt{56}}{2}$

$= \frac{-6 \pm 2\sqrt{14}}{2}$

1 $x^2 + 12x - 9 = 0$ $-6 \pm 3\sqrt{5}$

$$= \frac{-12 \pm \sqrt{180}}{2}$$

$$= \frac{-12 \pm 6\sqrt{5}}{2}$$

$$= -6 \pm 3\sqrt{5}$$

180

6 · 30

6 5

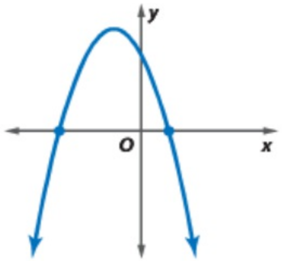
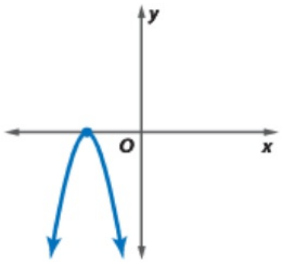
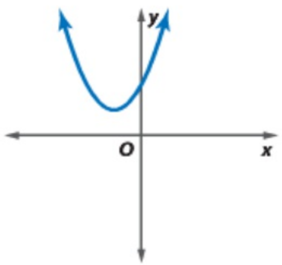
2 Roots and the Discriminant

In the previous examples, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{discriminant}$$

The value of the discriminant can be used to determine the number and type of roots of a quadratic equation. The table on the following page summarizes the possible types of roots.

The discriminant can also be used to confirm the number and type of solutions after you solve the quadratic equation.

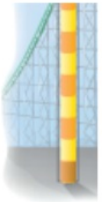
Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

Example 5 Complete parts a and b for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

10a. 40
10b. 2 irrational roots
11a. −36
11b. 2 complex roots



10. $3x^2 + 8x + 2 = 0$

11. $2x^2 - 6x + 9 = 0$

12. $-16x^2 + 8x - 1 = 0$ **12a. 0** **12b. 1 rational root**

13. $5x^2 + 2x + 4 = 0$ **13a. −76** **13b. 2 complex roots**

10

$b^2 - 4ac$

~~a)~~

$a = 3$

$b = 8$

$c = 2$

$8^2 - 4(3)(2)$

$64 - 24 = 40$

b) 2 roots

11

$(-6)^2 - 4(2)(9)$

$36 - 72 = -36$

0 roots

Here's a summary of what we accomplished so far;

ConceptSummary Solving Quadratic Equations

Method	Can be Used	When to Use
graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 7x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x - 5)^2 = 18$
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 6x - 14 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. **Example** $2.3x^2 - 1.8x + 9.7 = 0$

Examples 1–4 **Solve each equation by using the Quadratic Formula.**

14. $x^2 + 45x = -200$ **−5, −40**

15. $4x^2 - 6 = -12x$ $\frac{-3 \pm \sqrt{15}}{2}$

16. $3x^2 - 4x - 8 = -6$ **16.** $\frac{2 \pm \sqrt{10}}{3}$

17. $4x^2 - 9 = -7x - 4$ $\frac{-7 \pm \sqrt{129}}{8}$

18. $5x^2 - 9 = 11x$ $\frac{11 \pm \sqrt{301}}{10}$

19. $12x^2 + 9x - 2 = -17$ $\frac{-3 \pm i\sqrt{71}}{8}$

20. **DIVING** Competitors in the 10-meter platform diving competition jump upward and outward before diving into the pool below. The height h of a diver in meters above the pool after t seconds can be approximated by the equation $h = -4.9t^2 + 3t + 10$.

a. Determine a domain and range for which this function makes sense. $D = \{t \mid 0 \leq t \leq 2\}$, $R = \{h \mid 0 \leq h \leq 10\}$

b. When will the diver hit the water? **about 1.77 seconds**

Example 5 **Complete parts a–c for each quadratic equation.**

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula. **21–32. See margin.**

21. $2x^2 + 3x - 3 = 0$

22. $4x^2 - 6x + 2 = 0$

23. $6x^2 + 5x - 1 = 0$

24. $6x^2 - x - 5 = 0$

25. $3x^2 - 3x + 8 = 0$

26. $2x^2 + 4x + 7 = 0$

27. $-5x^2 + 4x + 1 = 0$

28. $x^2 - 6x = -9$

29. $-3x^2 - 7x + 2 = 6$

30. $-8x^2 + 5 = -4x$

31. $x^2 + 2x - 4 = -9$

32. $-6x^2 + 5 = -4x + 8$

Additional Answers

21a. 33

21b. 2 irrational

21c. $\frac{-3 \pm \sqrt{33}}{4}$

22a. 4

22b. 2 rational

22c. $\frac{1}{2}, 1$

23a. 49

23b. 2 rational

23c. $\frac{1}{6}, -1$

24a. 121

24b. 2 rational

24c. $1, -\frac{5}{6}$

25a. −87

25b. 2 complex

25c. $\frac{3 \pm i\sqrt{87}}{6}$

26a. −40

26b. 2 complex

26c. $\frac{-2 \pm i\sqrt{10}}{2}$

27a. 36

27b. 2 rational

27c. $1, -\frac{1}{5}$

28a. 0

28b. 1 rational

28c. 3

29a. 1

29b. 2 rational

29c. $-1, -\frac{4}{3}$

30a. 176

30b. 2 irrational

30c. $\frac{1 \pm \sqrt{11}}{4}$