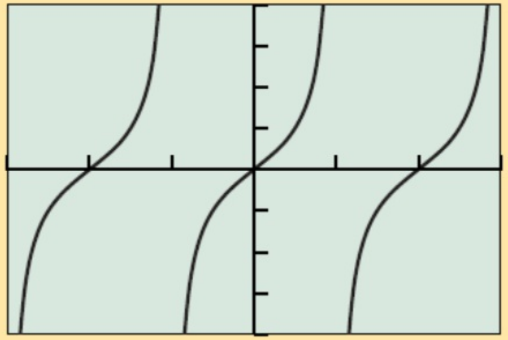
What you'll learn about

- The Tangent Function
- The Cotangent Function
- The Secant Function
- The Cosecant Function

4.5 Graphs of Tangent, Cotangent, Secant, and Cosecant



$[-3\pi/2, 3\pi/2]$ by $[-4, 4]$

FIGURE 4.44A

THE TANGENT FUNCTION

$f(x) = \tan x$
Domain: All reals except odd multiples of $\pi/2$
Range: All reals
Continuous (i.e., continuous on its domain)
Increasing on each interval in its domain
Symmetric with respect to the origin (odd)
Not bounded above or below
No local extrema
No horizontal asymptotes
Vertical asymptotes: $x = k \cdot (\pi/2)$ for all odd integers k
End behavior: $\lim_{x \to -\infty} \tan x$ and $\lim_{x \to \infty} \tan x$ do not exist. (The function values continually oscillate between $-\infty$ and ∞ and approach no limit.)

Why does the graph of tangent behave like that...?

$$\tan x = \frac{\sin x}{\cos x}.$$

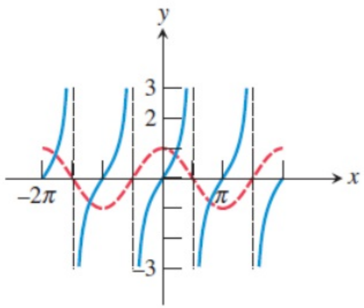


FIGURE 4.45 The tangent function has asymptotes at the zeros of cosine.

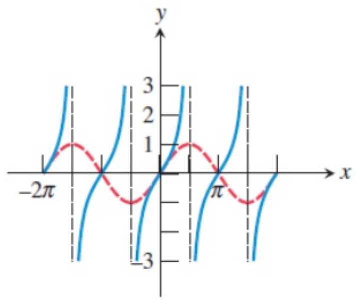


FIGURE 4.46 The tangent function has zeros at the zeros of sine.

let's explore the values that come from the quotient of tangent...

The Cotangent Function

The cotangent function is the reciprocal of the tangent function. Thus,

$$\cot x = \frac{\cos x}{\sin x}.$$

The graph of $y = \cot x$ will have asymptotes at the zeros of the sine function (Figure 4.48) and zeros at the zeros of the cosine function (Figure 4.49).

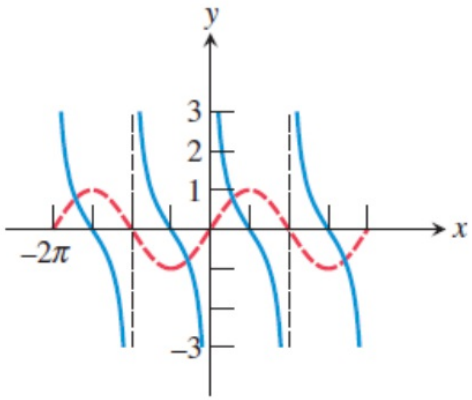


FIGURE 4.48 The cotangent has asymptotes at the zeros of the sine function.

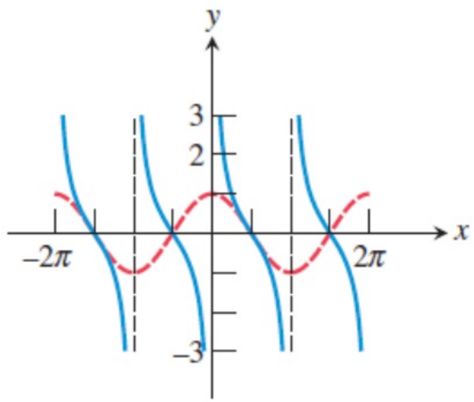


FIGURE 4.49 The cotangent has zeros at the zeros of the cosine function.

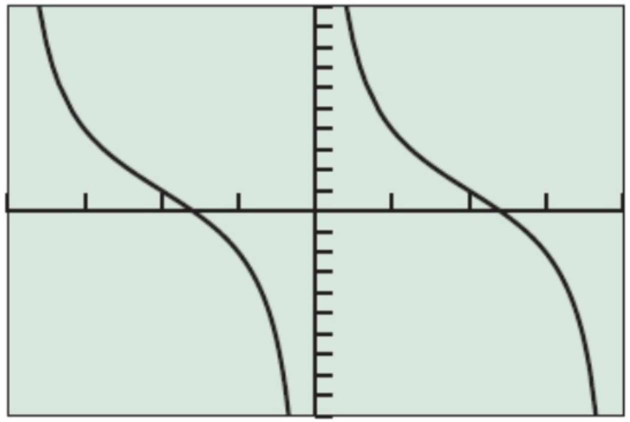
EXAMPLE 2 Graphing a Cotangent Function

Describe the graph of $f(x) = 3 \cot(x/2) + 1$ in terms of a basic trigonometric function. Locate the vertical asymptotes and graph two periods.

SOLUTION The graph is obtained from the graph of $y = \cot x$ by effecting a horizontal stretch by a factor of 2, a vertical stretch by a factor of 3, and a vertical translation up 1 unit. The horizontal stretch makes the period of the function 2π (twice the period of $y = \cot x$), and the asymptotes are at the even multiples of π. Figure 4.50 shows two periods of the graph of f. *Now try Exercise 9.*

All tricks used to transform graphs apply here too!

(remember horizontal and vertical stretches/ shrinks?)



$[-2\pi, 2\pi]$ by $[-10, 10]$

FIGURE 4.50 Two periods of $f(x) = 3 \cot(x/2) + 1$. (Example 2)

Check out the graphs of secant and cosecant...

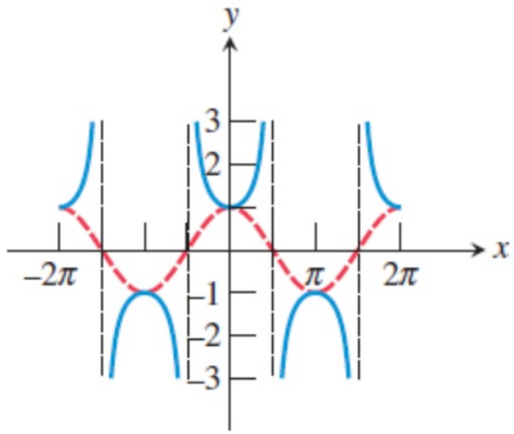


FIGURE 4.51 Characteristics of the secant function are inferred from the fact that it is the reciprocal of the cosine function.

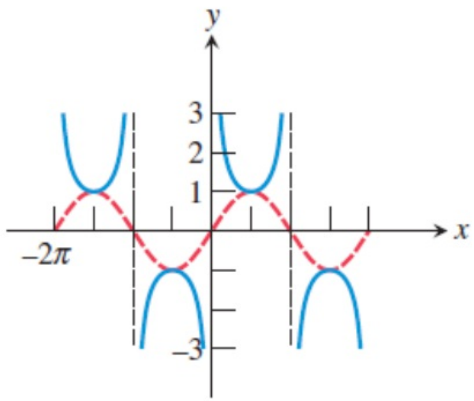
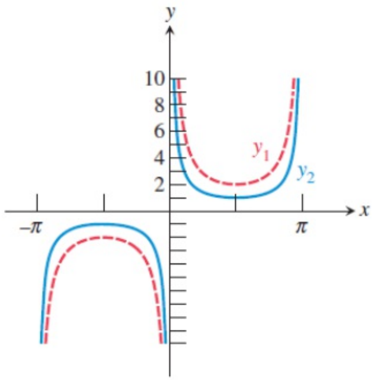


FIGURE 4.54 Characteristics of the cosecant function are inferred from the fact that it is the reciprocal of the sine function.

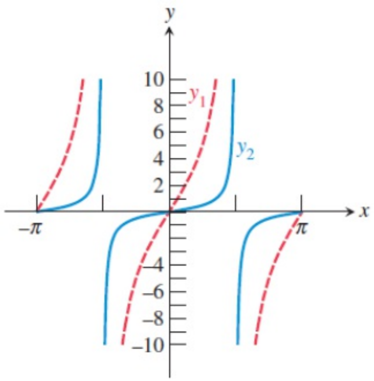
Remember, they are defined as reciprocols, and their calculations are reciprocols, too!

In Exercises 1–4, identify the graph of each function. Use your unde
standing of transformations, not your graphing calculator.

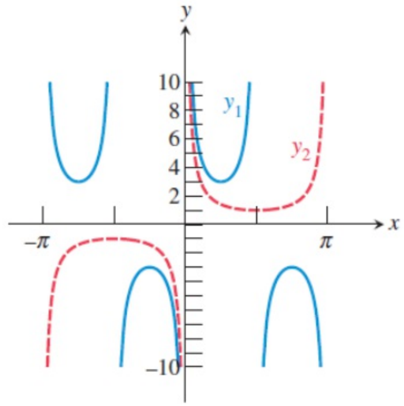
1. Graphs of one period of csc x and 2 csc x are shown.



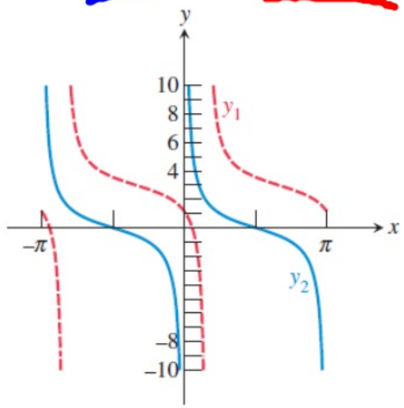
2. Graphs of two periods of 0.5 tan x and 5 tan x are shown.



3. Graphs of csc x and 3 csc 2x are shown.



4. Graphs of cot x and cot $(x - 0.5) + 3$ are shown.



Summary: Basic Trigonometric Functions

Function	Period	Domain	Range	Asymptotes	Zeros	Even/Odd
$\sin x$	2π	All reals	$[-1, 1]$	None	$n\pi$	Odd
$\cos x$	2π	All reals	$[-1, 1]$	None	$\pi/2 + n\pi$	Even
$\tan x$	π	$x \neq \pi/2 + n\pi$	All reals	$x = \pi/2 + n\pi$	$n\pi$	Odd
$\cot x$	π	$x \neq n\pi$	All reals	$x = n\pi$	$\pi/2 + n\pi$	Odd
$\sec x$	2π	$x \neq \pi/2 + n\pi$	$(-\infty, -1] \cup [1, \infty)$	$x = \pi/2 + n\pi$	None	Even
$\csc x$	2π	$x \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$	$x = n\pi$	None	Odd

There are patterns between the calculations of reciprocols, and they are just swell :)

In Exercises 5–12, describe the graph of the function in terms of a basic trigonometric function. Locate the vertical asymptotes and graph two periods of the function.

5. $y = \tan 2x$

6. $y = -\cot 3x$

7. $y = \sec 3x$

8. $y = \csc 2x$

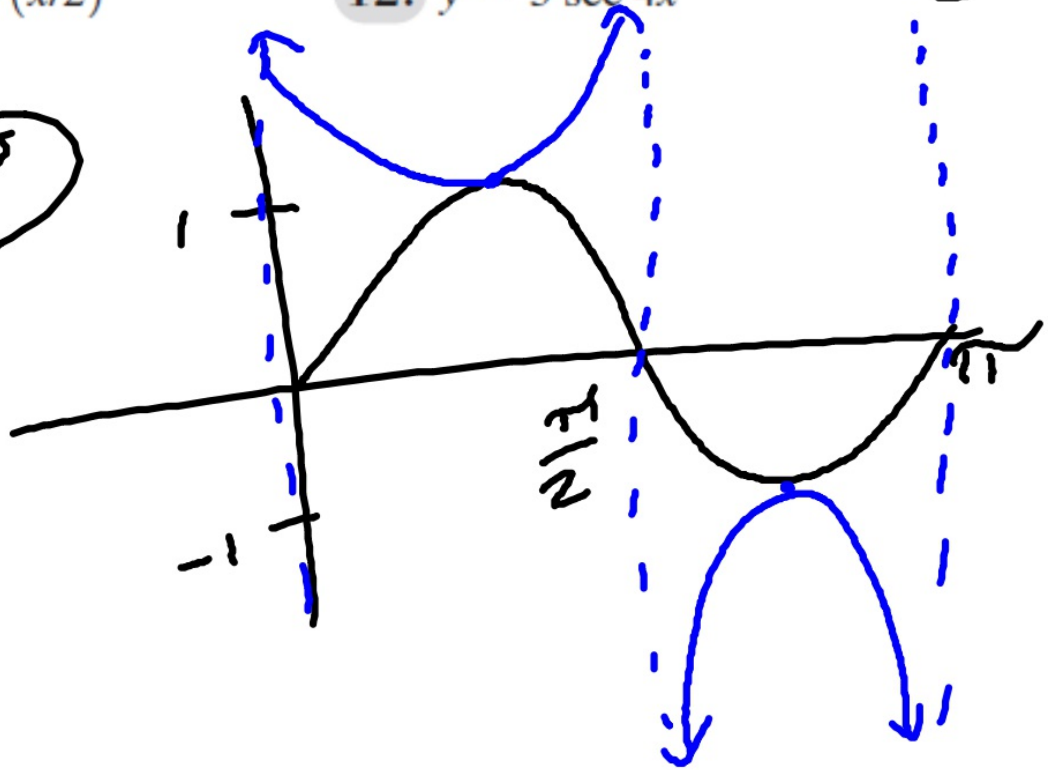
9. $y = 2 \cot 2x$

10. $y = 3 \tan (x/2)$

11. $y = \csc (x/2)$

12. $y = 3 \sec 4x$

8



1 period...

y=sin 2x

$y = \csc 2x$

7. $y = \sec 3x$

8. $y = \csc 2x$

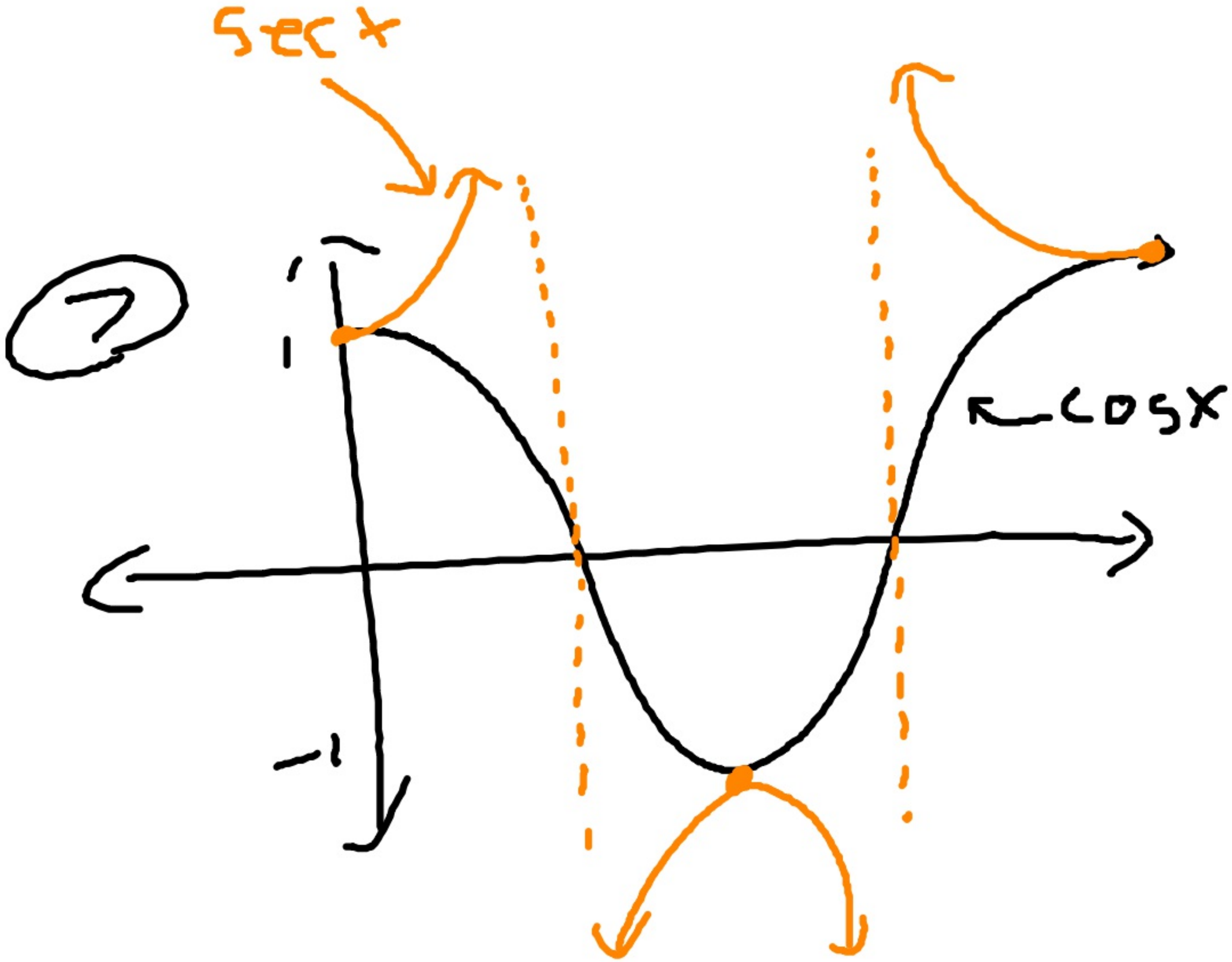
9. $y = 2 \cot 2x$

10. $y = 3 \tan (x/2)$

11. $y = \csc (x/2)$

12. $y = 3 \sec 4x$

$$\frac{2\pi}{3}$$



7. $y = \sec 3x$

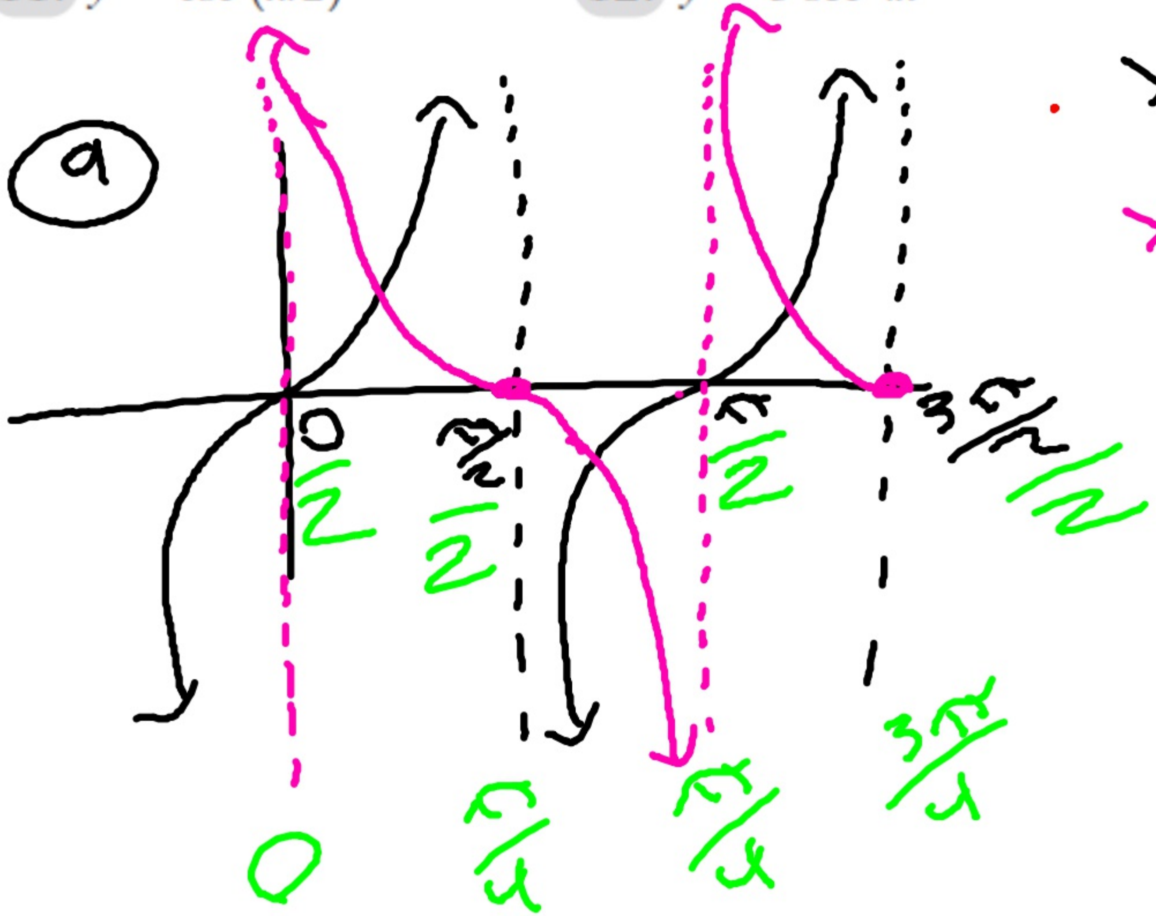
8. $y = \csc 2x$

9. $y = 2 \cot 2x$

10. $y = 3 \tan (x/2)$

11. $y = \csc (x/2)$

12. $y = 3 \sec 4x$



$y = 2\tan(2x)$

$y = \cot(x)$

7. $y = \sec 3x$

8. $y = \csc 2x$

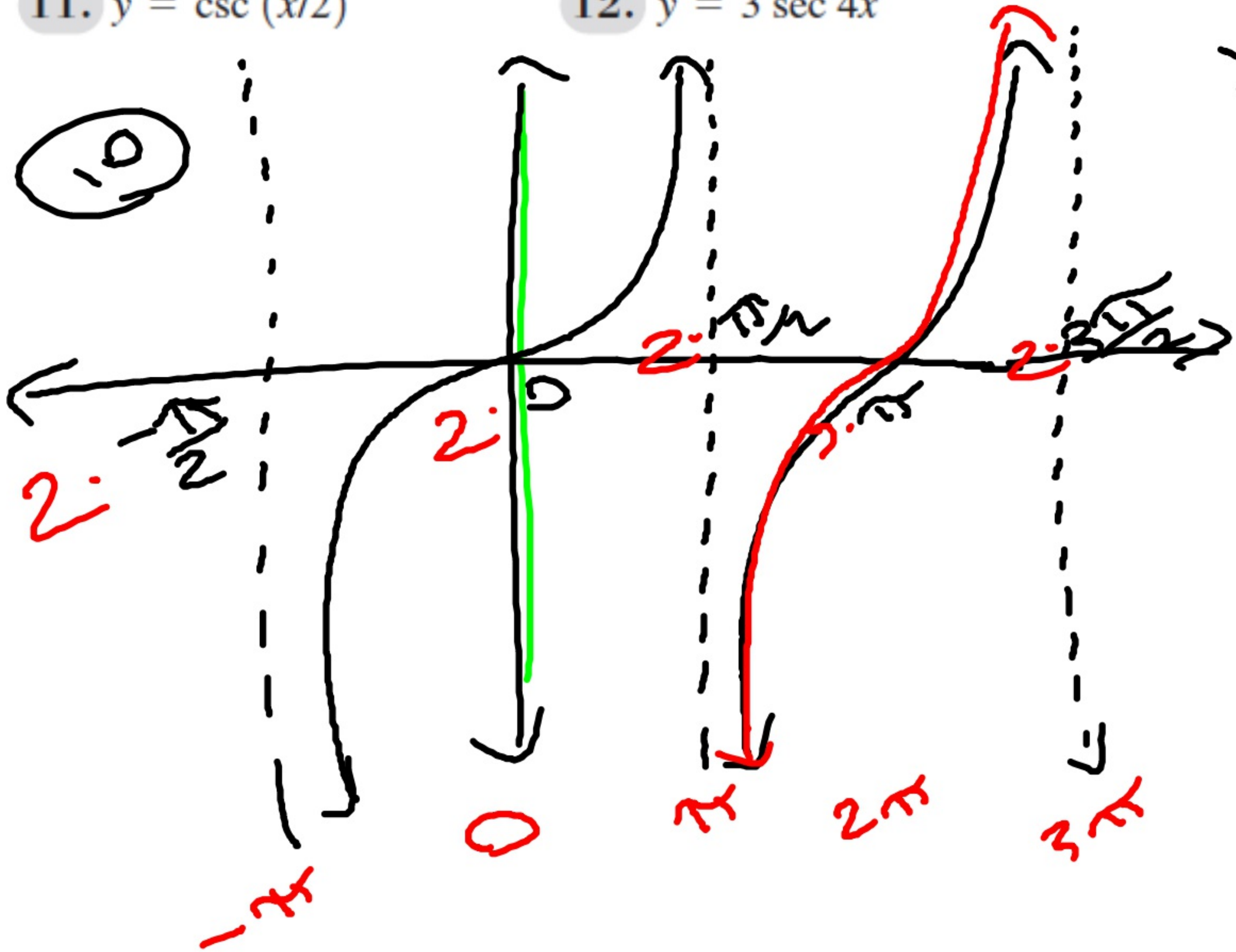
9. $y = 2 \cot 2x$

10. $y = 3 \tan (x/2)$

11. $y = \csc (x/2)$

12. $y = 3 \sec 4x$

10



$y = 3 \tan 2x)$

$\frac{\pi}{1/2} = 2\pi$

In Exercises 21–28, describe the transformations required to obtain the graph of the given function from a basic trigonometric graph.

21. $y = 3 \tan x$

22. $y = -\tan x$

23. $y = 3 \csc x$

24. $y = 2 \tan x$

25. $y = -3 \cot \frac{1}{2} x$

26. $y = -2 \sec \frac{1}{2} x$

27. $y = -\tan \frac{\pi}{2} x + 2$

28. $y = 2 \tan \pi x - 2$

$\frac{2\pi}{\frac{1}{2}}$

$\frac{\pi}{\frac{\pi}{2}} \Rightarrow \pi \div \pi/2$

$= \pi \times \frac{2}{\pi}$

$= 2$

$\frac{\text{period}}{|b|}$