

5-1 Operations with Polynomials

ConceptSummary Properties of Exponents		
For any real numbers x and y , integers a and b :		
Property	Definition	Examples
Product of Powers	$x^a \cdot x^b = x^{a+b}$	$3^2 \cdot 3^4 = 3^{2+4}$ or 3^6 $p^2 \cdot p^9 = p^{2+9}$ or p^{11}
Quotient of Powers	$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$	$\frac{9^5}{9^2} = 9^{5-2}$ or 9^3 $\frac{b^6}{b^4} = b^{6-4}$ or b^2
Negative Exponent	$x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-a}} = x^a, x \neq 0$	$3^{-5} = \frac{1}{3^5}$ $\frac{1}{b^{-7}} = b^7$
Power of a Power	$(x^a)^b = x^{ab}$	$(3^3)^2 = 3^{3 \cdot 2}$ or 3^6 $(d^2)^4 = d^{2 \cdot 4}$ or d^8
Power of a Product	$(xy)^a = x^a y^a$	$(2k)^4 = 2^4 k^4$ or $16k^4$ $(ab)^3 = a^3 b^3$
Power of a Quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}, y \neq 0$, and $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$ or $\frac{y^a}{x^a}, x \neq 0, y \neq 0$	$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$ $\left(\frac{a}{b}\right)^{-5} = \frac{b^5}{a^5}$
Zero Power	$x^0 = 1, x \neq 0$	$7^0 = 1$

Example 1 Simplify Expressions

Simplify each expression. Assume that no variable equals 0.

a. $(2a^{-2})(3a^3b^2)(c^{-2})$

$$(2a^{-2})(3a^3b^2)(c^{-2})$$

Original expression

$$= 2\left(\frac{1}{a^2}\right)(3a^3b^2)\left(\frac{1}{c^2}\right)$$

Definition of negative exponents

$$= \left(\frac{2}{a \cdot a}\right)(3 \cdot a \cdot a \cdot a \cdot b \cdot b)\left(\frac{1}{c \cdot c}\right)$$

Definition of exponents

$$= \left(\frac{2}{\cancel{a} \cdot \cancel{a}}\right)(3 \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot b \cdot b)\left(\frac{1}{\cancel{c} \cdot \cancel{c}}\right)$$

Divide out common factors.

$$= \frac{6ab^2}{c^2}$$

Simplify.

b. $\frac{q^2r^4}{q^7r^3}$

$$\frac{q^2r^4}{q^7r^3} = q^{2-7} \cdot r^{4-3}$$

Quotient of powers

$$= q^{-5}r$$

Subtract powers.

$$= \frac{r}{q^5}$$

Simplify.

c. $\left(\frac{-2a^4}{b^2}\right)^3$

$$\left(\frac{-2a^4}{b^2}\right)^3 = \frac{(-2a^4)^3}{(b^2)^3}$$

Power of a quotient

$$= \frac{(-2)^3(a^4)^3}{(b^2)^3}$$

Power of a product

$$= \frac{-8a^{12}}{b^6}$$

Power of a power

Example 1

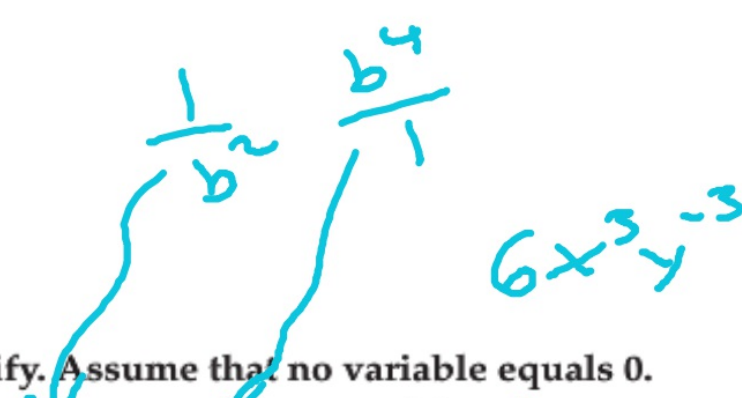
Simplify. Assume that no variable equals 0.

$$1. (2a^3b^{-2})(-4a^2b^4) \quad 2. \frac{12x^4y^2}{2xy^5} \frac{6x^3}{y^3}$$

$$-8a^5b^2$$

$$3. \left(\frac{2a^2}{3b}\right)^3 \frac{8a^6}{27b^3}$$

$$4. (6g^5h^{-4})^3 \frac{216g^{15}}{h^{12}}$$



2 Operations With Polynomials The **degree of a polynomial** is the degree of the monomial with the greatest degree.



Example 2 Degree of a Polynomial

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $\frac{1}{4}x^4y^3 - 8x^5$

This expression is a polynomial because each term is a monomial. The degree of the first term is $4 + 3$ or 7 , and the degree of the second term is 5 . The degree of the polynomial is 7 .

b. $\sqrt{x} + x + 4$

This expression is not a polynomial because \sqrt{x} is not a monomial.

c. $x^{-3} + 2x^{-2} + 6$

This expression is not a polynomial because x^{-3} and x^{-2} are not monomials:

$x^{-3} = \frac{1}{x^3}$ and $x^{-2} = \frac{1}{x^2}$. Monomials cannot contain variables in the denominator.

Guided Practice

2A. $\frac{x}{y} + 3x^2$ **No; $\frac{x}{y}$ is not a monomial.**

2B. $x^5y + 9x^4y^3 - 2xy$ **yes; 7**

Example 2

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

5. $3x + 4y$ **yes; 1**

6. $\frac{1}{2}x^2 - 7y$ **yes; 2**

7. $x^2 + \sqrt{x}$ **no**

8. $\frac{ab^3 - 1}{az^4 + 3}$ **no**



Example 3 Simplify Polynomial Expressions

Simplify each expression.

a. $(4x^2 - 5x + 6) - (2x^2 + 3x - 1)$

Remove parentheses, and group like terms together.

$$(4x^2 - 5x + 6) - (2x^2 + 3x - 1)$$

$$= 4x^2 - 5x + 6 - 2x^2 - 3x + 1$$

Distribute the -1 .

$$= (4x^2 - 2x^2) + (-5x - 3x) + (6 + 1)$$

Group like terms.

$$= 2x^2 - 8x + 7$$

Combine like terms.

b. $(6x^2 - 7x + 8) + (-4x^2 + 9x - 5)$

Align like terms vertically and add.

$$6x^2 - 7x + 8$$

$$\begin{array}{r} (+) -4x^2 + 9x - 5 \\ \hline 2x^2 + 2x + 3 \end{array}$$

Examples 3-4, and 6Simplify. **9. $-2x^2 - 6x + 3$**

9. $(x^2 - 5x + 2) - (3x^2 + x - 1)$

11. $2a(4b + 5)$ **$8ab + 10a$**

13. $(n - 9)(n + 7)$ **$n^2 - 2n - 63$**

Example 4 Simplify by Using the Distributive PropertyFind $3x(2x^2 - 4x + 6)$.

$$\begin{aligned} 3x(2x^2 - 4x + 6) &= 3x(2x^2) + 3x(-4x) + 3x(6) \\ &= 6x^3 - 12x^2 + 18x \end{aligned}$$

Distributive Property

Multiply the monomials.

Example 6 Multiply PolynomialsFind $(n^2 + 4n - 6)(n + 2)$.

$$(n^2 + 4n - 6)(n + 2)$$

$$= n^2(n + 2) + 4n(n + 2) + (-6)(n + 2)$$

Distributive Property

$$= n^2 \cdot n + n^2 \cdot 2 + 4n \cdot n + 4n \cdot 2 + (-6) \cdot n + (-6) \cdot 2$$

Distributive Property

$$= n^3 + 2n^2 + 4n^2 + 8n - 6n - 12$$

Multiply monomials.

$$= n^3 + 6n^2 + 2n - 12$$

Combine like terms.

10. $(3a + 4b) + (6a - 6b)$ **$9a - 2b$**

12. $3x^2(2xy - 3xy^2 + 4x^2y^3)$ **$6x^3y - 9x^3y^2 + 12x^4y^3$**

14. $(a + 4)(a - 6)$ **$a^2 - 2a - 24$**



Real-World Example 5 Write a Polynomial Expression

DRIVING The U.S. Department of Transportation limits the time a truck driver can work between periods of rest to ten hours. For the first part of his shift, Tom drives at a speed of 60 miles per hour, and for the second part of the shift, he drives at a speed of 70 miles per hour. Write a polynomial to represent the distance driven.

Words 60 mph for some time and 70 mph for the rest

Variable Let x = the number of hours he drives at 60 miles per hour.

Expression $60x + 70(10 - x)$

$$\begin{aligned}60x + 70(10 - x) & \text{ Original expression} \\= 60x + 700 - 70x & \text{ Distributive Property} \\= 700 - 10x & \text{ Combine like terms.}\end{aligned}$$

The polynomial is $700 - 10x$.

- Example 5** 15. **EXERCISE** Tara exercises 75 minutes a day. She does cardio, which burns an average of 10 Calories per minute, and weight training, which burns an average of 7.5 Calories per minute. Write a polynomial to represent the amount of Calories Tara burns in one day if she does x minutes of weight training. $750 - 2.5x$

$$7.5x + 10(75 - x)$$

Example 1Simplify. Assume that no variable equals 0. **17. $-8b^5c^3$**

16. $\frac{20x^4}{y^2}$

16. $(5x^3y^{-5})(4xy^3)$

17. $(-2b^3c)(4b^2c^2)$

18. $\frac{a^3n^7}{an^4}$ **a^2n^3**

19. $\frac{-y^3z^5}{y^2z^3}$ **$-yz^2$**

20. $\frac{-7x^5y^5z^4}{21x^7y^5z^2}$ **$\frac{z^2}{-3x^2}$**

21. $\frac{9a^7b^5c^5}{18a^5b^9c^3}$ **$\frac{a^2c^2}{2b^4}$**

22. $(n^5)^4$ **n^{20}**

23. $(z^3)^6$ **z^{18}**

Example 2

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

24. $2x^2 - 3x + 5$
yes; 2

25. $a^3 - 11$ **yes; 3**

26. $\frac{5np}{n^2} - \frac{2g}{h}$ **no**

27. $\sqrt{m-7}$ **no**

28. $2a^2 - a - 2$ **29.** $3b^2 + 6b - 5$

CCSS REGULARITY Simplify. **32.** $x^3 + x^2y - xy^2 - y^3$

33. $a^4 + a^3b - 3a^2b - 4ab^2 - b^3$

28. $(6a^2 + 5a + 10) - (4a^2 + 6a + 12)$

29. $(7b^2 + 6b - 7) - (4b^2 - 2)$

34. $-6a^3 + 4a^2 + 8a - 9$

30. $3p(np - z)$ **$3np^2 - 3pz$**

31. $4x(2x^2 + y)$ **$8x^3 + 4xy$**

35. $10c^3 - c^2 + 4c$

32. $(x - y)(x^2 + 2xy + y^2)$

33. $(a + b)(a^3 - 3ab - b^2)$

36. $10x^2y - 5xy^2 + 6x^2y^2 + 36y^2$

34. $4(a^2 + 5a - 6) - 3(2a^3 + 4a - 5)$

35. $5c(2c^2 - 3c + 4) + 2c(7c - 8)$

37. $12a^2b + 8a^2b^2 - 15ab^2 + 4b^2$

36. $5xy(2x - y) + 6y^2(x^2 + 6)$

37. $3ab(4a - 5b) + 4b^2(2a^2 + 1)$

38. $(x - y)(x + y)(2x + y)$
 $2x^3 + x^2y - 2xy^2 - y^3$

39. $(a + b)(2a + 3b)(2x - y)$
 $4a^2x - 2a^2y + 10abx - 5aby + 6b^2x - 3b^2y$

Example 5**40. PAINTING** Connor has hired two painters to paint his house. The first painter charges \$12 per hour and the second painter charges \$11 per hour. It will take 15 hours of labor to paint the house.

- Write a polynomial to represent the total cost of the job if the first painter does x hours of the labor. **$x + 165$**
- Write a polynomial to represent the total cost of the job if the second painter does y hours of the labor. **$180 - y$**