Simplify $\dfrac{6x^4y^3 + 12x^3y^2 - 18x^2y}{3xy}$.

$$\frac{6x^4y^3 + 12x^3y^2 - 18x^2y}{3xy} = \frac{6x^4y^3}{3xy} + \frac{12x^3y^2}{3xy} - \frac{18x^2y}{3xy}$$

Sum of quotients

$$= \frac{6}{3} \cdot x^{4-1}y^{3-1} + \frac{12}{3} \cdot x^{3-1}y^{2-1} - \frac{18}{3} \cdot x^{2-1}y^{1-1}$$

Divide.

$$= 2x^3y^2 + 4x^2y - 6x$$

$y^{1-1} = y^0$ or 1

please note that the divisor is a monomial...

1. $\dfrac{4xy^2 - 2xy + 2x^2y}{xy}$ **4y + 2x − 2**

2. $(3a^2b - 6ab + 5ab^2)(ab)^{-1}$ **3a + 5b − 6**

Example 2 Division Algorithm

Use long division to find $(x^2 + 3x - 40) \div (x - 5)$.

$$
\begin{array}{rl}
 & x + 8 \\
x - 5 \overline{)\, x^2 + 3x - 40} & \\
\underline{(-)\, x^2 - 5x} & \text{Multiply the divisor by } x \text{ since } \frac{x^2}{x} = x. \\
8x - 40 & \text{Subtract. Bring down the next term.} \\
\underline{(-)\, 8x - 40} & \text{Multiply the divisor by 8 since } \frac{8x}{x} = 8. \\
0 & \text{Subtract.}
\end{array}
$$

The quotient is $x + 8$. The remainder is 0.

Please note that the divisor is NOT a monomial...

3. $(x^2 - 6x - 20) \div (x + 2)$ $x - 8 - \frac{4}{x+2}$

4. $(2a^2 - 4a - 8) \div (a + 1)$ $2a - 6 - \frac{2}{a+1}$

5. $(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z + 3)$

6. $(y^5 - 3y^2 - 20) \div (y - 2)$

5. $3z^3 - 15z^2 + 36z - 105 + \frac{309}{z+3}$ **6.** $y^4 + 2y^3 + 4y^2 + 5y + 10$

Standardized Test Example 3 Divide Polynomials

Which expression is equal to $(a^2 + 7a - 11)(3 - a)^{-1}$?

A $a + 10 - \frac{19}{3 - a}$

B $-a + 10$

C $-a - 10 + \frac{19}{3 - a}$

D $-a - 10 - \frac{19}{3 - a}$

Read the Test Item

Since the second factor has an exponent of -1, this is a division problem.

$(a^2 + 7a - 11)(3 - a)^{-1} = \frac{a^2 + 7a - 11}{3 - a}$

Solve the Test Item

$$
\begin{array}{r|l}
 & -a - 10 \\
\hline
-a + 3 & a^2 + 7a - 11 \\
 & (-)\ a^2 - 3a \qquad \text{For ease in dividing, rewrite } 3 - a \text{ as } -a + 3. \\
\hline
 & 10a - 11 \qquad -a(-a + 3) = a^2 - 3a \\
 & \qquad\qquad 7a - (-3a) = 10a \\
 & (-)\ 10a - 30 \qquad -10(-a + 3) = 10a - 30 \\
\hline
 & 19 \qquad\qquad -11 - (-30) = 19
\end{array}
$$

The quotient is $-a - 10$, and the remainder is 19.

Therefore, $(a^2 + 7a - 11)(3 - a)^{-1} = -a - 10 + \frac{19}{3 - a}$. The answer is **C**.

Example 3

7. MULTIPLE CHOICE Which expression is equal to $(x^2 + 3x - 9)(4 - x)^{-1}$? ☐

A $-x - 7 + \frac{19}{4 - x}$ **B** $-x - 7$ **C** $x + 7 - \frac{19}{4 - x}$ **D** $-x - 7 - \frac{19}{4 - x}$

etic division to find $(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$.

Write the coefficients of the dividend. Write the constant r in the box. In this case, $r = 4$. Bring the first coefficient, 2, down.

4	2	−13	26	−24
	2			|

Multiply the first coefficient by r: $2 \cdot 4 = 8$. Write the product under the second coefficient.

4	2	−13	26	−24
		8		
	2 ↗			|

Add the product and the second coefficient: $-13 + 8 = -5$.

4	2	−13	26	−24
		8		
	2	−5		|

Multiply the sum, -5, by r: $-5 \times 4 = -20$. Write the product under the next coefficient, and add: $26 + (-20) = 6$. Multiply the sum, -5, by r: $6 \cdot 4 = 24$. Write the product under the next coefficient and add: $-24 + 24 = 0$.

4	2	−13	26	−24
		8	−20	24
	2	−5 ↗	6 ↗	| 0

$$
\begin{array}{r}
2x^2 - 5x + 6 \\
x - 4 \overline{)\,2x^3 - 13x^2 + 26x - 24} \\
\underline{(-)\ 2x^3 - 8x^2} \\
-5x^2 + 26x \\
\underline{(-)\ -5x^2 + 20x} \\
6x - 24 \\
\underline{(-)\ 6x - 24} \\
0
\end{array}
$$

$$\overline{)\,2x^3 - 4x^2 + 0x + 6}$$

3. $(x^2 - 6x - 20) \div (x + 2)$ $x - 8 - \frac{4}{x+2}$

4. $(2a^2 - 4a - 8) \div (a + 1)$ $2a - 6 - \frac{2}{a+1}$

5. $(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z + 3)$

6. $(y^5 - 3y^2 - 20) \div (y - 2)$

$y^5 + 0y^4 + 0y^3 - 3y^2 + 0y - 20$

5. $3z^3 - 15z^2 + 36z - 105 + \frac{309}{z+3}$ 6. $y^4 + 2y^3 + 4y^2 + 5y + 10$

(3) -2 | $1 \quad -6 \quad -20$

$\downarrow \quad -2 \quad 16$

$1 \quad -8 \quad -4$

(4) -1 | $2 \quad -4 \quad -8$

$\downarrow \quad -2 \quad 6$

$2 \quad -6 \quad -2$

Example 5 Divisor with First Coefficient Other than 1

Use synthetic division to find $(3x^4 - 5x^3 + x^2 + 7x) \div (3x + 1)$.

$$\frac{3x^4 - 5x^3 + x^2 + 7x}{3x + 1} = \frac{(3x^4 - 5x^3 + x^2 + 7x) \div 3}{(3x + 1) \div 3}$$

Rewrite the divisor with a leading coefficient of 1. Then divide the numerator and denominator by 3.

$$= \frac{x^4 - \frac{5}{3}x^3 + \frac{1}{3}x^2 + \frac{7}{3}x}{x + \frac{1}{3}}$$

Simplify the numerator and the denominator.

Since the numerator does not have a constant term, use a coefficient of 0 for the constant term.

$x - r = x + \frac{1}{3}$, so $r = -\frac{1}{3}$. $\rightarrow$

$-\frac{1}{3}$	1	$-\frac{5}{3}$	$\frac{1}{3}$	$\frac{7}{3}$	0
		$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$
	1	-2	1	2	$-\frac{2}{3}$

The result is $x^3 - 2x^2 + x + 2 - \frac{\frac{2}{3}}{x + \frac{1}{3}}$. Now simplify the fraction.

$$\frac{\frac{2}{3}}{x + \frac{1}{3}} = \frac{2}{3} \div \left(x + \frac{1}{3}\right)$$

Rewrite as a division expression.

$$= \frac{2}{3} \div \frac{3x + 1}{3}$$

$x + \frac{1}{3} = \frac{3x}{3} + \frac{1}{3} = \frac{3x+1}{3}$

$$= \frac{2}{3} \cdot \frac{3}{3x + 1}$$

Multiply by the reciprocal.

$$= \frac{2}{3x + 1}$$

Simplify.

The solution is $x^3 - 2x^2 + x + 2 - \frac{2}{3x + 1}$.

Example 5 **Simplify.**

8. $(10x^2 + 15x + 20) \div (5x + 5)$

9. $(18a^2 + 6a + 9) \div (3a - 2)$ $6a + 6 + \frac{21}{3a - 2}$

10. $\frac{12b^2 + 23b + 15}{3b + 8}$

11. $\frac{27y^2 + 27y - 30}{9y - 6}$ $3y + 5$

(8)

$-1 \mid 10 \quad 15 \quad 20$

$2/3 \mid 18 \quad 6 \quad 9$

$\quad\quad\quad 12 \quad 12$

$18 \quad 18 \quad 21$

(9)

$3a - 2 = 0$

$+2 \quad +2$

$\frac{3a}{3} = \frac{2}{3}$

$a = 2/3$

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 Simplify. **12.** $3a^2b - 2ab^2$ **13.** $x + 3y - 2$ **14.** $7g^2h + 3g - 2h^2$

12. $\frac{24a^3b^2 - 16a^2b^3}{8ab}$

13. $\frac{5x^2y - 10xy + 15xy^2}{5xy}$

14. $\frac{7g^3h^2 + 3g^2h - 2gh^3}{gh}$

15. $\frac{4a^3b - 6ab + 2ab^2}{2ab}$ $2a^2 + b - 3$

16. $\frac{16c^4d^4 - 24c^2d^2}{4c^2d^2}$ $4c^2d^2 - 6$

17. $\frac{9n^3p^3 - 18n^2p^2 + 21n^2p^3}{3n^2p^2}$ $3np - 6 + 7p$

18. ENERGY Compact fluorescent light (CFL) bulbs reduce energy waste. The amount of energy waste that is reduced each day in a certain community can be estimated by $-b^2 + 8b$, where b is the number of bulbs. Divide by b to find the average amount of energy saved per CFL bulb. $-b + 8$

19. BAKING The number of cookies produced in a factory each day can be estimated by $-w^2 + 16w + 1000$, where w is the number of workers. Divide by w to find the average number of cookies produced per worker. $-w + 16 + \frac{1000}{w}$

Examples 2, 4, and 5 Simplify. **21.** $b^2 - 5b + 6 - \frac{8}{b+1}$ **23.** $x^4 + 4x^3 + 12x^2 + 52x + 208 + \frac{832}{x-4}$

20. $(a^2 - 8a - 26) \div (a + 2)$ $a - 10 - \frac{6}{a+2}$

21. $(b^3 - 4b^2 + b - 2) \div (b + 1)$

22. $(z^4 - 3z^3 + 2z^2 - 4z + 4)(z - 1)^{-1}$

23. $(x^5 - 4x^3 + 4x^2) \div (x - 4)$

24. $\frac{y^3 + 11y^2 - 10y + 6}{y + 2}$

25. $(g^4 - 3g^2 - 18) \div (g - 2)$ $g + 2 - \frac{14}{g-2}$

26. $(6a^2 - 3a + 9) \div (3a - 2)$ $2a + \frac{1}{3} + \frac{29}{9a-6}$

27. $\frac{6x^5 + 5x^4 + x^3 - 3x^2 + x}{3x + 1}$ $2x^4 + x^3 - x + \frac{2}{3} - \frac{2}{9x+3}$

28. $\frac{4g^4 - 6g^3 + 3g^2 - g + 12}{4g - 4}$ $g^3 - \frac{1}{2}g^2 + \frac{1}{4}g + \frac{3}{g-1}$

29. $(2b^3 - 6b^2 + 8b) \div (2b + 2)$ $b^2 - 4b + 8 - \frac{8}{b+1}$

30. $(6z^6 + 3z^4 - 9z^2)(3z - 6)^{-1}$ $2z^5 + 4z^4 + 9z^3 + 18z^2 + 33z + 66 + \frac{132}{z-2}$

31. $(10y^6 + 5y^5 + 10y^3 - 20y - 15)(5y + 5)^{-1}$ $2y^5 - y^4 + y^3 + y^2 - y - 3$

32. CCSS REASONING A rectangular box for a new product is designed in such a way that the three dimensions always have a particular relationship defined by the variable x. The volume of the box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always $x + 2$. What are the width and length of the box? $2x + 3$, $3x + 5$

33. PHYSICS The voltage V is related to current I and power P by the equation $V = \frac{P}{I}$. The power of a generator is modeled by $P(t) = t^3 + 9t^2 + 26t + 24$. If the current of the generator is $I = t + 4$, write an expression that represents the voltage. $V(t) = t^2 + 5t + 6$