5-3 Polynomial Functions

5-2

2 Operations With Polynomials The **degree of a polynomial** is the degree of the monomial with the greatest degree.

Example 2 Degree of a Polynomial

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $\frac{1}{4}x^4y^3 - 8x^5$

This expression is a polynomial because each term is a monomial. The degree of the first term is 4 + 3 or 7, and the degree of the second term is 5. The degree of the polynomial is 7.

Today!

1 Polynomial Functions A **polynomial in one variable** is an expression of the form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$, where $a_n \neq 0$, a_{n-1}, a_2, a_1, and a_0 are real numbers, and n is a nonnegative integer.

The polynomial is written in standard form when the values of the exponents are in descending order. The degree of the polynomial is the value of the greatest exponent. The coefficient of the first term of a polynomial in standard form is called the **leading coefficient**.

Polynomial	Expression	Degree	Leading Coefficient
Constant	12	0	12
Linear	$4x - 9$	1	4
Quadratic	$5x^2 - 6x - 9$	2	5
Cubic	$8x^3 + 12x^2 - 3x + 1$	3	8
General	$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$	n	a_n

Example 1 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $8x^5 - 4x^3 + 2x^2 - x - 3$

This is a polynomial in one variable. The greatest exponent is 5, so the degree is 5 and the leading coefficient is 8.

b. $12x^2 - 3xy + 8x$

This is not a polynomial in one variable. There are two variables, x and y.

c. $3x^4 + 6x^3 - 4x^8 + 2x$

This is a polynomial in one variable. The greatest exponent is 8, so the degree is 8 and the leading coefficient is -4.

Example 1 **State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.**

1. $11x^6 - 5x^5 + 4x^2$

1. degree = 6, leading coefficient = 11

2. $-10x^7 - 5x^3 + 4x - 22$

2. degree = 7, leading coefficient = −10

3. $14x^4 - 9x^3 + 3x - 4y$

3. not in one variable because there are two variables, x and y

4. $8x^5 - 3x^2 + 4xy - 5$

4. not in one variable because there are two variables, x and y

Real-World Example 2 Evaluate a Polynomial Function

RESPIRATION **Refer to the beginning of the lesson. Find the volume of air in the lungs 2 seconds into the respiratory cycle.**

By substituting 2 into the function we can find $v(2)$, the volume of air in the lungs 2 seconds into the respiratory cycle.

$v(t) = -0.037t^3 + 0.152t^2 + 0.173t$	Original function
$v(2) = -0.037(2)^3 + 0.152(2)^2 + 0.173(2)$	Replace t with 2.
$= -0.296 + 0.608 + 0.346$	Simplify.
$= 0.658$ L	Add.

Example 2 **Find $w(5)$ and $w(-4)$ for each function.**

5. $w(x) = -2x^3 + 3x - 12$
$w(5) = -247$; $w(-4) = 104$

6. $w(x) = 2x^4 - 5x^3 + 3x^2 - 2x + 8$
$w(5) = 698$; $w(-4) = 896$

(5) $w(5) = -2(5)^3 + 3(5) - 12$
$w(5) = -2(125) + 15 - 12$
$= -250 + 3$
$= -247$

Example 3 Function Values of Variables

Find $f(3c - 4) - 5f(c)$ if $f(x) = x^2 + 2x - 3$.

To evaluate $f(3c - 4)$, replace the x in $f(x)$ with $3c - 4$.

$f(x) = x^2 + 2x - 3$ Original function

$f(3c - 4) = (3c - 4)^2 + 2(3c - 4) - 3$ Replace x with $3c - 4$.

$= 9c^2 - 24c + 16 + 6c - 8 - 3$ Multiply.

$= 9c^2 - 18c + 5$ Simplify.

To evaluate $5f(c)$, replace x with c in $f(x)$, then multiply by 5.

$f(x) = x^2 + 2x - 3$ Original function

$5f(c) = 5(c^2 + 2c - 3)$ Replace x with c.

$= 5c^2 + 10c - 15$ Distributive Property

Now evaluate $f(3c - 4) - 5f(c)$.

$f(3c - 4) - 5f(c) = (9c^2 - 18c + 5) - (5c^2 + 10c - 15)$

$= 9c^2 - 18c + 5 - 5c^2 - 10c + 15$ Distribute.

$= 4c^2 - 28c + 20$ Simplify.

$y^3 \cdot y^3 \cdot y^3 = y^9$

(7) $c(x) = 4x^3 - 5x^2 + 2$

$c(y^3) = 4(y^3)^3 - 5(y^3)^2 + 2$

$= 4y^9 - 5y^6 + 2$

Example 3 **If $c(x) = 4x^3 - 5x^2 + 2$ and $d(x) = 3x^2 + 6x - 10$, find each value.**

7. $c(y^3)$ $4y^9 - 5y^6 + 2$

8. $-4[d(3z)]$ $-108z^2 - 72z + 40$

9. $6c(4a) + 2d(3a - 5)$ $1536a^3 - 426a^2 - 144a + 82$

10. $-3c(2b) + 6d(4b - 3)$ $-96b^3 + 348b^2 - 288b - 12$

$6(c(4a))$

$c(x) = 4x^3 - 5x^2 + 2$

$c(x) = 4(4a)^3 - 5(4a)^2 + 2$

$c(4a) = 256a^3 - 80a^2 + 2$

$c(4a) = 1536a^3 + 480a^2 + 12$

$6c(4a) = 1536a^3 + 480a^2 + 12$

$d(x) = 3x^2 + 6x - 10$

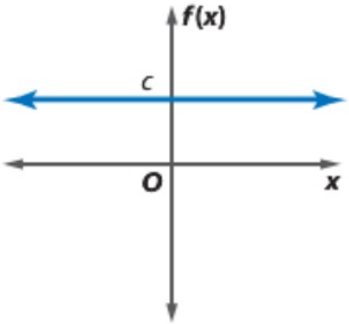
$d(3a - 5) = 3(3a - 5)^2 + 6(3a - 5) - 10$

just keep going...

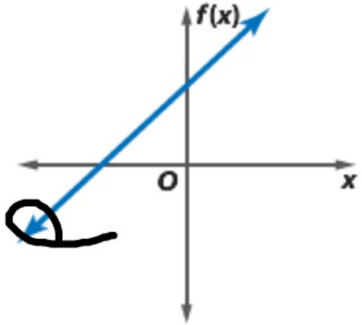
2 Graphs of Polynomial Functions

The general shapes of the graphs of several polynomial functions show the *maximum* number of times the graph of each function may intersect the x-axis. This is the same number as the degree of the polynomial.

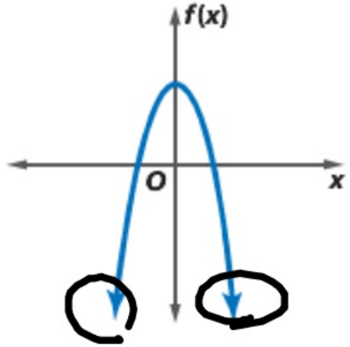
Constant function
Degree 0



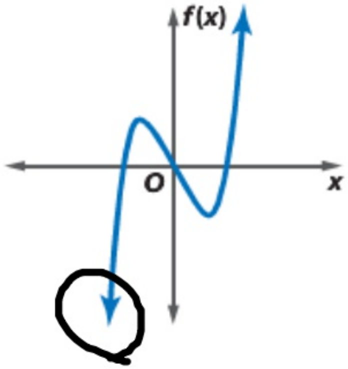
Linear function
Degree 1



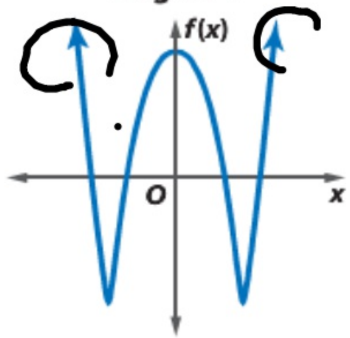
Quadratic function
Degree 2



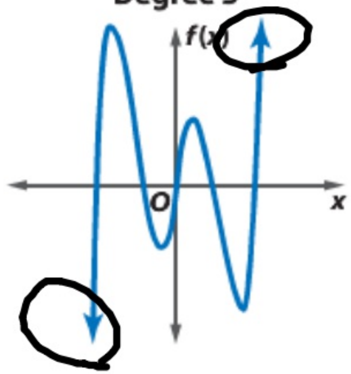
Cubic function
Degree 3

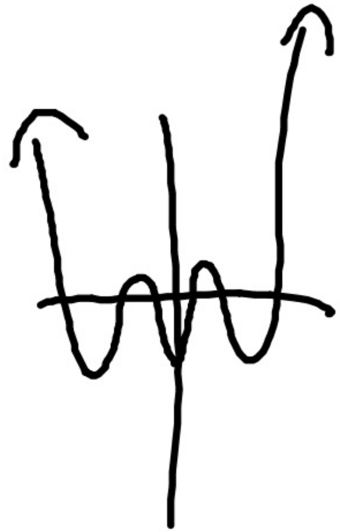


Quartic function
Degree 4



Quintic function
Degree 5



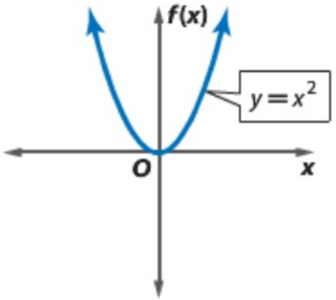


KeyConcept End Behavior of a Polynomial Function

Degree: even
Leading Coefficient: positive
End Behavior:

$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

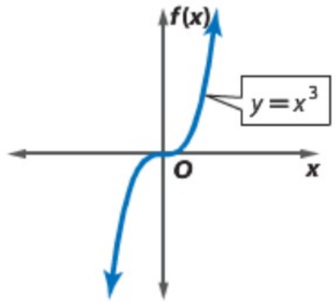


Domain: all real numbers
Range: all real numbers $\geq$ minimum

Degree: odd
Leading Coefficient: positive
End Behavior:

$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

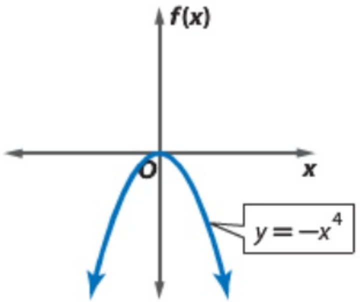


Domain: all real numbers
Range: all real numbers

Degree: even
Leading Coefficient: negative
End Behavior:

$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

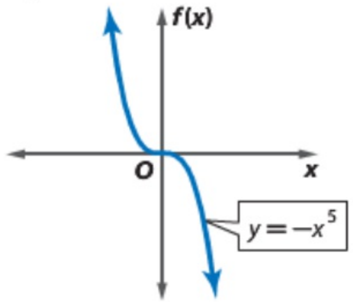


Domain: all real numbers
Range: all real numbers $\leq$ maximum

Degree: odd
Leading Coefficient: negative
End Behavior:

$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$



Domain: all real numbers
Range: all real numbers

ReviewVocabulary

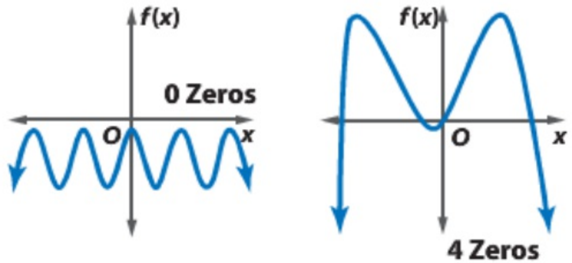
zero the *x*-coordinate of the point at which a graph intersects the *x*-axis

The number of real zeros of a polynomial function can be determined by examining its graph. Recall that real zeros occur at *x*-intercepts, so the number of times a graph crosses the *x*-axis equals the number of real zeros.

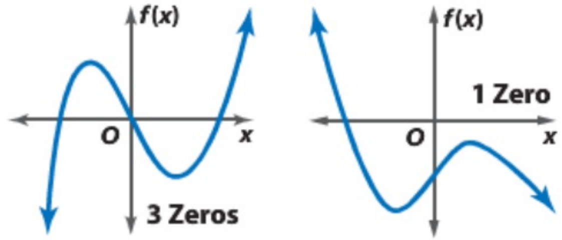
KeyConcept Zeros of Even- and Odd-Degree Functions

Odd-degree functions will always have an odd number of real zeros. Even-degree functions will always have an even number of real zeros or no real zeros at all.

Even-Degree Polynomials



Odd-Degree Polynomials



StudyTip

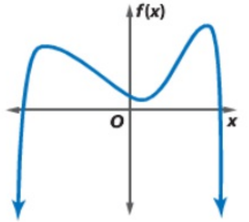
Double roots When a graph is tangent to the *x*-axis, there is a *double root*, which represents two of the same root.

Example 4 Graphs of Polynomial Functions

For each graph,

- **describe the end behavior,**
- **determine whether it represents an odd-degree or an even-degree polynomial function, and**
- **state the number of real zeros.**

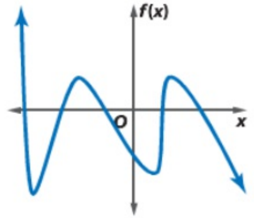
a.



$f(x) \to -\infty$ as $x \to -\infty$.
$f(x) \to -\infty$ as $x \to +\infty$.

Since the end behavior is in the same direction, it is an even-degree function. The graph intersects the *x*-axis at two points, so there are two real zeros.

b.



$f(x) \to +\infty$ as $x \to -\infty$.
$f(x) \to -\infty$ as $x \to +\infty$.

Since the end behavior is in opposite directions, it is an odd-degree function. The graph intersects the *x*-axis at five points, so there are five real zeros.

Additional Answers

11a. $f(x) \to -\infty$ as $x \to -\infty$. $f(x) \to +\infty$ as $x \to +\infty$.

11b. Since the end behavior is in opposite directions, it is an odd-degree function.

11c. The graph intersects the *x*-axis at three points, so there are three real zeros.

12a. $f(x) \to -\infty$ as $x \to -\infty$. $f(x) \to -\infty$ as $x \to +\infty$.

12b. Since the end behavior is in the same direction, it is an even-degree function.

12c. The graph does not intersect the *x*-axis, so there are no real zeros.

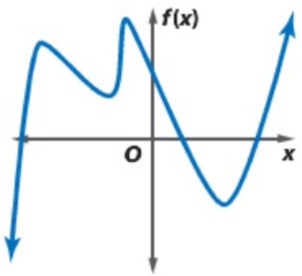
Example 4 For each graph,

a. describe the end behavior,

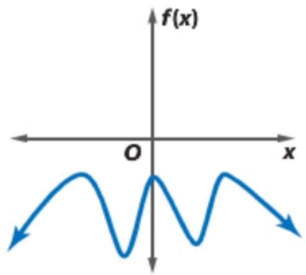
b. determine whether it represents an odd-degree or an even-degree function, and

c. state the number of real zeros. **11, 12. See margin.**

11.



12.



13. not in one variable because there are two variables, x and y

14. not a polynomial because there is a negative exponent

15. degree = 6, leading coefficient = −12

16. degree = 7, leading coefficient = −21

17. degree = 4, leading coefficient = −5

18. degree = 5, leading coefficient = 3

19. degree = 2, leading coefficient = 3

20. degree = 2, leading coefficient = −6

21. degree = 9, leading coefficient = 2

22. degree = 8, leading coefficient = −2

23. $p(-6) = 1227$; $p(3) = 66$

24. $p(-6) = 546$; $p(3) = -93$

25. $p(-6) = -156$; $p(3) = 78$

26. $p(-6) = 2322$; $p(3) = 9$

27. $p(-6) = 319$; $p(3) = -5$

28. $p(-6) = 2232$; $p(3) = 153$

Practice and Problem Solving

Extra Practice is

Example 1

CCSS PERSEVERANCE State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why. **13–22. See margin.**

13. $-6x^6 - 4x^5 + 13xy$

14. $3a^7 - 4a^4 + \frac{3}{a}$

15. $8x^5 - 12x^6 + 14x^3 - 9$

16. $-12 - 8x^2 + 5x - 21x^7$

17. $15x - 4x^3 + 3x^2 - 5x^4$

18. $13b^3 - 9b + 3b^5 - 18$

19. $(d + 5)(3d - 4)$

20. $(5 - 2y)(4 + 3y)$

21. $6x^5 - 5x^4 + 2x^9 - 3x^2$

22. $7x^4 + 3x^7 - 2x^8 + 7$

Example 2

Find $p(-6)$ and $p(3)$ for each function. **23–28. See margin.**

23. $p(x) = x^4 - 2x^2 + 3$

24. $p(x) = -3x^3 - 2x^2 + 4x - 6$

25. $p(x) = 2x^3 + 6x^2 - 10x$

26. $p(x) = x^4 - 4x^3 + 3x^2 - 5x + 24$

27. $p(x) = -x^3 + 3x^2 - 5$

28. $p(x) = 2x^4 + x^3 - 4x^2$

Example 3

If $c(x) = 2x^2 - 4x + 3$ and $d(x) = -x^3 + x + 1$, find each value.

29. $c(3a)$ $18a^2 - 12a + 3$

30. $5d(2a)$ $-40a^3 + 10a + 5$

31. $c(b^2)$ $2b^4 - 4b^2 + 3$

32. $d(4a^2)$ $-64a^6 + 4a^2 + 1$

33. $d(4y - 3)$ $-64y^3 + 144y^2 - 104y + 25$

34. $c(y^2 - 1)$ $2y^4 - 8y^2 + 9$