

LESSON **5-3** Solving Multi-Step Inequalities

 **5-Minute Check**

Over Lesson 5-2

2 Solve $\frac{5}{7}p > -20$.

A. $\{p \mid p > 28\}$

B. $\{p \mid p < 28\}$

C. $\left\{p \mid p > -19\frac{2}{7}\right\}$

D. $\{p \mid p > -28\}$

LESSON 5-3 Solving Multi-Step Inequalities

✓ 5-Minute Check

Over Lesson 5-2

2 Solve $\frac{5}{7}p > -20$.
 Handwritten notes: A yellow 'S' is written under the fraction. A yellow '4' is written above the denominator '7'. A yellow '1' is written below the denominator '7'. A yellow arrow points from the '4' to the '1'. A yellow arrow points from the '1' to the '-20'. A yellow '-28' is written to the right of the inequality.

A. $\{p \mid p > 28\}$

B. $\{p \mid p < 28\}$

C. $\left\{p \mid p > -19\frac{2}{7}\right\}$

→ D. $\{p \mid p > -28\}$

LESSON 5-3 Solving Multi-Step Inequalities

✓ 5-Minute Check

Over Lesson 5-2

3 Solve $\frac{-9v}{-9} \geq \frac{-108}{-9}$.

A. $\{v \mid v \geq 99\}$

B. $\{v \mid v \leq 12\}$

C. $\{v \mid v \geq 12\}$

D. $\{v \mid v \geq -12\}$

LESSON 5-3 Solving Multi-Step Inequalities

✓ 5-Minute Check

Over Lesson 5-2

3 Solve $-9v \geq -108$.

A. $\{v \mid v \geq 99\}$

→ B. $\{v \mid v \leq 12\}$

C. $\{v \mid v \geq 12\}$

D. $\{v \mid v \geq -12\}$

LESSON 5-3 Solving Multi-Step Inequalities

✓ 5-Minute Check

Over Lesson 5-2

4 Solve $\frac{2c}{-3} \leq \frac{5}{9}$.

A. $\{c \mid c \leq -5\}$

B. $\{c \mid c \leq -2\}$

C. $\left\{c \mid c \leq \frac{5}{6}\right\}$

D. $\left\{c \mid c \geq -\frac{5}{6}\right\}$

 5-Minute Check

Over Lesson 5-2

4 Solve $\frac{2c}{-3} \leq \frac{5}{9}$.

A. $\{c \mid c \leq -5\}$

B. $\{c \mid c \leq -2\}$

C. $\left\{c \mid c \leq \frac{5}{6}\right\}$

 D. $\left\{c \mid c \geq -\frac{5}{6}\right\}$

Real-World Example 1 Solve a Multi-Step Inequality

SALES Write and solve an inequality to find the sales Mrs. Jones needs if she earns a monthly salary of \$2000 plus a 10% commission on her sales. Her goal is to make at least \$4000 per month. What sales does she need to meet her goal?

base salary + (commission \times sales) \geq income needed

$$\begin{aligned} 2000 + 0.10x &\geq 4000 && \text{Substitution} \\ 0.10x &\geq 2000 && \text{Subtract 2000 from each side.} \\ x &\geq 20,000 && \text{Divide each side by 0.10.} \end{aligned}$$

She must make at least \$20,000 in sales to meet her monthly goal.

$$4x + 60 \leq 800$$

Guided Practice

- FINANCIAL LITERACY** The Print Shop advertises a special to print 400 flyers for less than the competition. The price includes a \$3.50 set-up fee. If the competition charges \$35.50, what does the Print Shop charge for each flyer?

$$11.95 + 9.99x + 10 \leq 50$$

less than 8¢

Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



Example 1

- CANOEING** If four people plan to use the canoe with 60 pounds of supplies, write and solve an inequality to find the allowable average weight per person.



800-lb capacity

$$4n + 60 \leq 800; n \leq 185; \text{at most 185 lb per person}$$

- SHOPPING** Rita is ordering a movie for \$11.95 and a few CDs. She has \$50 to spend. Shipping and sales tax will be \$10. If each CD costs \$9.99, write and solve an inequality to find the greatest number of CDs that she can buy.

$$9.99x + 11.95 + 10 \leq 50; x \leq 2.8; \text{Rita can buy 2 CDs.}$$



EXAMPLE 2

Inequality Involving a Negative Coefficient

Solve $13 - 11d \geq 79$.

$$13 - 11d \geq 79$$

Original inequality

$$13 - 11d - 13 \geq 79 - 13$$

Subtract 13 from each side.

$$-11d \geq 66$$

Simplify.

$$\frac{-11d}{-11} \leq \frac{66}{-11}$$

Divide each side by -11 and change \geq to \leq .

$$d \leq -6$$

Simplify.

Answer: The solution set is $\{d \mid d \leq -6\}$.

EXAMPLE 2



Check Your Progress

Solve $-8y + 3 > -5$.

A. $\{y \mid y < -1\}$

B. $\{y \mid y > 1\}$

C. $\{y \mid y > -1\}$

D. $\{y \mid y < 1\}$

LESSON 5-3 Solving Multi-Step Inequalities

EXAMPLE 2

 Check Your Progress

Solve $-8y + 3 > -5$.

A. $\{y \mid y < -1\}$

B. $\{y \mid y > 1\}$

C. $\{y \mid y > -1\}$

D. $\{y \mid y < 1\}$

Example 2

CCSS STRUCTURE Solve each inequality. Graph the solution on a number line.

3. $6h - 10 \geq 32$ ($h | h \geq 7$)

4. $-3 \leq \frac{2}{3}r + 9$ ($r | r \geq -18$)

5. $-3x + 7 > 43$

6. $4m - 17 < 6m + 25$

3

$$\begin{array}{r} 6h - 10 \geq 32 \\ +10 \quad +10 \\ \hline 6h \geq 42 \\ \hline 6 \quad 6 \\ \hline h \geq 7 \end{array}$$

4

$$\begin{array}{r} -3 \leq \frac{2}{3}r + 9 \\ -9 \quad -9 \\ \hline -3 \leq \frac{2}{3}r \\ \hline -18 \leq 2r \\ \hline -9 \leq r \end{array}$$

~~$-12 \leq r$~~

Example 2**STRUCTURE**

Solve each inequality. Graph the solution on a number line.

3. $6h - 10 \geq 32$ ($h \mid h \geq 7$)

4. $-3 \leq \frac{2}{3}r + 9$ ($r \mid r \geq -18$)

5. $-3x + 7 > 43$ ($x \mid x < -12$)

6. $4m - 17 < 6m + 25$ ($m \mid m > -21$)

5. $-3x + 7 > 43$

$$\begin{array}{r} -3x + 7 > 43 \\ -7 \quad -7 \\ \hline -3x > 36 \\ \hline -3 \quad -3 \\ \hline x < -12 \end{array}$$

6. $4m - 17 < 6m + 25$

$$\begin{array}{r} 4m - 17 < 6m + 25 \\ +17 \quad +17 \\ \hline 4m < 6m + 42 \\ -6m \quad -6m \\ \hline -2m < 42 \\ \hline -2 \quad -2 \\ \hline m > -21 \end{array}$$

EXAMPLE 3 Write and Solve an Inequality

Define a variable, write an inequality, and solve the problem below.

Four times a number plus twelve is less than the number minus three.

LESSON 5-3 Solving Multi-Step Inequalities

EXAMPLE 3 Write and Solve an Inequality

$$4n + 12 < n - 3 \quad \text{Original inequality}$$



EXAMPLE 3

 **Check Your Progress**

Write an inequality for the sentence below. Then solve the inequality.

6 times a number is greater than 4 times the number minus 2.

A. $6n > 4n - 2; \{n \mid n > -1\}$

B. $6n < 4n - 2; \{n \mid n < -1\}$

C. $6n > 4n + 2; \{n \mid n > 1\}$

D. $6n > 2 - 4n; \left\{n \mid n < -\frac{1}{5}\right\}$

EXAMPLE 3

 **Check Your Progress**

Write an inequality for the sentence below. Then solve the inequality.

6 times a number is greater than 4 times the number minus 2.

A. $6n > 4n - 2; \{n \mid n > -1\}$

B. $6n < 4n - 2; \{n \mid n < -1\}$

C. $6n > 4n + 2; \{n \mid n > 1\}$

D. $6n > 2 - 4n; \left\{n \mid n < -\frac{1}{5}\right\}$

Example 3

Define a variable, write an inequality, and solve each problem. Then check your solution. 7. Sample answer: Let $n =$ the number; $4n - 6 > 8 + 2n$; $\{n \mid n > 7\}$.

7. Four times a number minus 6 is greater than eight plus two times the number.

8. Negative three times a number plus 4 is less than five times the number plus 8.

Sample answer: Let $n =$ the number; $-3n + 4 < 5n + 8$; $\{n \mid n > -\frac{1}{2}\}$.

$$\begin{array}{r} \textcircled{7} \quad 4x - 6 > 8 + 2x \\ \quad -2x \quad +6 \quad \quad +6 \quad -2x \\ \hline \quad \quad 2x > 14 \\ \quad \quad \underline{2} \quad \quad \underline{2} \\ \quad \quad x > 7 \end{array}$$

EXAMPLE 4 Distributive Property

Solve $6c + 3(2 - c) \geq -2c + 1$.

LESSON 5-3 Solving Multi-Step Inequalities

EXAMPLE 4



Check Your Progress

Solve $3p - 2(p - 4) < p - (2 - 3p)$.

A. $\left\{ p \mid p > \frac{10}{3} \right\}$

B. $\left\{ p \mid p < \frac{10}{3} \right\}$

C. $\left\{ p \mid p > -\frac{10}{3} \right\}$

D. $\left\{ p \mid p < -\frac{10}{3} \right\}$

LESSON 5-3 Solving Multi-Step Inequalities

EXAMPLE 4



Check Your Progress

Solve $3p - 2(p - 4) < p - (2 - 3p)$.

A. $\left\{ p \mid p > \frac{10}{3} \right\}$

B. $\left\{ p \mid p < \frac{10}{3} \right\}$

C. $\left\{ p \mid p > -\frac{10}{3} \right\}$

D. $\left\{ p \mid p < -\frac{10}{3} \right\}$

EXAMPLE 5

Empty Set and All Reals

A. Solve $-7(s + 4) + 11s \geq 8s - 2(2s + 1)$.

$-7(s + 4) + 11s \geq 8s - 2(2s + 1)$	Original inequality
$-7s - 28 + 11s \geq 8s - 4s - 2$	Distributive Property
$4s - 28 \geq 4s - 2$	Combine like terms.
$4s - 28 - 4s \geq 4s - 2 - 4s$	Subtract $4s$ from each side.
$-28 \geq -2$	Simplify.

Answer: Since the inequality results in a false statement, the solution set is the empty set, \emptyset .

EXAMPLE 5 Empty Set and All Reals

B. Solve $2(4r + 3) \leq 22 + 8(r - 2)$.

$$2(4r + 3) \leq 22 + 8(r - 2) \quad \text{Original inequality}$$

$$8r + 6 \leq 22 + 8r - 16 \quad \text{Distributive Property}$$

$$8r + 6 \leq 6 + 8r \quad \text{Simplify.}$$

$$8r + 6 - 8r \leq 6 + 8r - 8r \quad \text{Subtract } 8r \text{ from each side.}$$

$$6 \leq 6 \quad \text{Simplify.}$$

Answer: All values of r make the inequality true.
 All real numbers are the solution.
 $\{r \mid r \text{ is a real number.}\}$

Examples 4-5 Solve each inequality. Graph the solution on a number line. 9-11. See margin for graphs.

9. $-6 \leq 3(5v - 2)$
($v | v \geq 0$)

10. $-5(g + 4) > 3(g - 4)$
($g | g < -1$)

11. $3 - 8x \geq 9 + 2(1 - 4x)$
 \emptyset

9. $-6 \leq 15v - 6$
 $+6 \quad +6$

$0 \leq 15v$

$0 \leq v$
 $v \geq 0$

11. $3 - 8x \geq 9 + 2 - 8x$
 $3 - 8x \geq 11 - 8x$
 $+8x \quad +8x$
 $3 \geq 11$ false

No Solution
 \emptyset

Examples 1
and 2

- CCSS STRUCTURE** Solve each inequality. Graph the solution on a number line.
12. $5b - 1 \geq -11$ $\{b \mid b \geq -2\}$
13. $21 > 15 + 2a$ $\{a \mid a < 3\}$
14. $-9 \geq \frac{2}{5}m + 7$ $\{m \mid m \leq -40\}$
15. $\frac{w}{8} - 13 > -6$ $\{w \mid w > 56\}$
16. $-a + 6 \leq 5$ $\{a \mid a \geq 1\}$
17. $37 < 7 - 10w$ $\{w \mid w < -3\}$
18. $8 - \frac{z}{3} \geq 11$ $\{z \mid z \leq -9\}$
19. $-\frac{5}{4}p + 6 < 12$ $\{p \mid p > -\frac{24}{5}\}$
20. $3b - 6 \geq 15 + 24b$ $\{b \mid b \leq -1\}$
21. $15h + 30 < 10h - 45$ $\{h \mid h < -15\}$
22. Sample answer:
Let $n =$ the number;
 $\frac{3}{4}n - 9 \geq 42$;
 $\{n \mid n \geq 68\}$
23. Sample answer:
Let $n =$ the number;
 $\frac{2}{3}n + 6 \geq 22$;
 $\{n \mid n \geq 24\}$

12–21. See Ch. 5 Answer Appendix for graphs.

22. Three fourths of a number decreased by nine is at least forty-two.
23. Two thirds of a number added to six is at least twenty-two.
24. Seven tenths of a number plus 14 is less than forty-nine.
25. Eight times a number minus twenty-seven is no more than the negative of that number plus eighteen. **Sample answer:** Let $n =$ the number; $8n - 27 \leq -n + 18$; $\{n \mid n \leq 5\}$
26. Ten is no more than 4 times the sum of twice a number and three. **See margin.**
27. Three times the sum of a number and seven is greater than five times the number less thirteen. **Sample answer:** Let $n =$ the number; $3(n + 7) > 5n - 13$; $\{n \mid n < 17\}$
28. The sum of nine times a number and fifteen is less than or equal to the sum of twenty-four and ten times the number. **Sample answer:** Let $n =$ the number; $9n + 15 \leq 24 + 10n$; $\{n \mid n \geq -9\}$

Examples 4
and 5

- CCSS STRUCTURE** Solve each inequality. Graph the solution on a number line.
29. $-3(7n + 3) < 6n$ $\{n \mid n > -\frac{1}{3}\}$
30. $21 \geq 3(a - 7) + 9$ $\{a \mid a \leq 11\}$
31. $2y + 4 > 2(3 + y)$ \emptyset
32. $3(2 - b) < 10 - 3(b - 6)$ $\{b \mid b \text{ is a real number.}\}$
33. $7 + t \leq 2(t + 3) + 2$ $\{t \mid t \geq -1\}$
34. $8a + 2(1 - 5a) \leq 20$ $\{a \mid a \geq -9\}$
- 29–34. See margin for graphs.