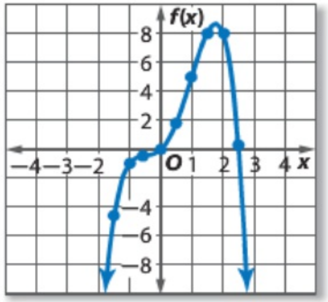
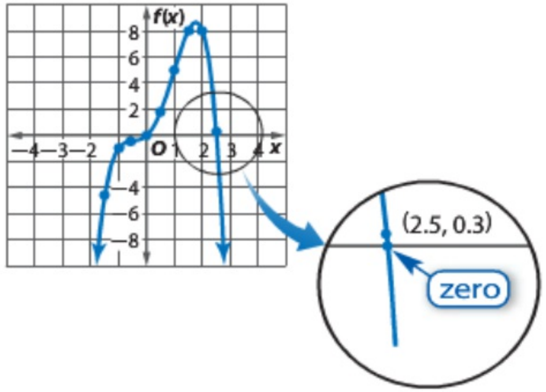
5-4 Analyzing Graphs of Polynomial Functions

Example 1 Graph of a Polynomial Function

Graph $f(x) = -x^4 + x^3 + 3x^2 + 2x$ by making a table of values.

x	$f(x)$	x	$f(x)$
−2.5	≈ −41	0.5	≈ 1.8
−2.0	−16	1.0	5.0
−1.5	≈ −4.7	1.5	≈ 8.1
−1.0	−1.0	2.0	8.0
−0.5	≈ −0.4	2.5	≈ 0.3
0.0	0.0	3.0	−21





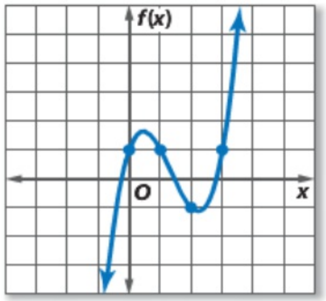
This is an even-degree polynomial with a negative leading coefficient, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. Notice that the graph intersects the x-axis at two points, indicating there are two zeros for this function.

Example 2 Locate Zeros of a Function

Determine consecutive integer values of x between which each real zero of $f(x) = x^3 - 4x^2 + 3x + 1$ is located. Then draw the graph.

Make a table of values. Since $f(x)$ is a third-degree polynomial function, it will have either 3 or 1 real zeros. Look at the values of $f(x)$ to locate the zeros. Then use the points to sketch a graph of the function.

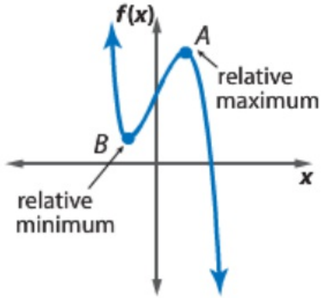
x	$f(x)$	
−2	−29	
−1	−7	
0	1	← change in sign
1	1	
2	−1	← change in sign
3	1	← change in sign
4	13	



The changes in sign indicate that there are zeros between $x = -1$ and $x = 0$, between $x = 1$ and $x = 2$, and between $x = 2$ and $x = 3$.

2 Maximum and Minimum Points

The graph below shows the general shape of a third-degree polynomial function.



Point A on the graph is a **relative maximum** of the function since no other nearby points have a greater y-coordinate. The graph is increasing as it approaches A and decreasing as it moves from A.

Likewise, point B is a **relative minimum** since no other nearby points have a lesser y-coordinate. The graph is decreasing as it approaches B and increasing as it moves from B. The maximum and minimum values of a function are called the **extrema**.

These points are often referred to as **turning points**. The graph of a polynomial function of degree n has at most $n - 1$ turning points.

StudyTip

Odd Functions Some odd functions, like $f(x) = x^3$, have no turning points.

Example 3 Maximum and Minimum Points

Graph $f(x) = x^3 - 4x^2 - 2x + 3$. Estimate the x-coordinates at which the relative maxima and relative minima occur.

Make a table of values and graph the function.

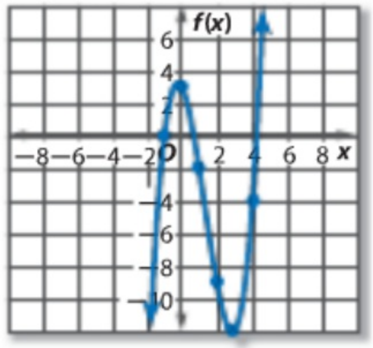
x	$f(x)$
−2	−17
−1	0
0	3
1	−2
2	−9
3	−12
4	−4
5	18

zero

indicates a relative maximum

indicates a relative minimum

zero between 4 and 5



Look at the table of values and the graph.

The value of $f(x)$ changes signs between $x = 4$ and $x = 5$, indicating a zero of the function.

The value of $f(x)$ at $x = 0$ is greater than the surrounding points, so there must be a relative maximum *near* $x = 0$.

The value of $f(x)$ at $x = 3$ is less than the surrounding points, so there must be a relative minimum *near* $x = 3$.

Example 1 Graph each polynomial equation by making a table of values. **1–4. See margin.**

1. $f(x) = 2x^4 - 5x^3 + x^2 - 2x + 4$
2. $f(x) = -2x^4 + 4x^3 + 2x^2 + x - 3$
3. $f(x) = 3x^4 - 4x^3 - 2x^2 + x - 4$
4. $f(x) = -4x^4 + 5x^3 + 2x^2 + 3x + 1$

Example 2 Determine the consecutive integer values of x between which each real zero of each function is located. Then draw the graph. **5–8. See Chapter 5 Answer Appendix.**

5. $f(x) = x^3 - 2x^2 + 5$
6. $f(x) = -x^4 + x^3 + 2x^2 + x + 1$
7. $f(x) = -3x^4 + 5x^3 + 4x^2 + 4x - 8$
8. $f(x) = 2x^4 - x^3 - 3x^2 + 2x - 4$

Example 3 Graph each polynomial function. Estimate the x-coordinates at which the relative maxima and relative minima occur. State the domain and range for each function. **9–12. See Chapter 5 Answer Appendix.**

9. $f(x) = x^3 + x^2 - 6x - 3$
10. $f(x) = 3x^3 - 6x^2 - 2x + 2$
11. $f(x) = -x^3 + 4x^2 - 2x - 1$
12. $f(x) = -x^3 + 2x^2 - 3x + 4$

Example 4 13. **CCSS SENSE-MAKING** Annual compact disc sales can be modeled by the quartic function $f(x) = 0.48x^4 - 9.6x^3 + 53x^2 - 49x + 599$, where x is the number of years after 1995 and $f(x)$ is annual sales in millions.

a. Graph the function for $0 \leq x \leq 10$. **See Chapter 5 Answer Appendix.**

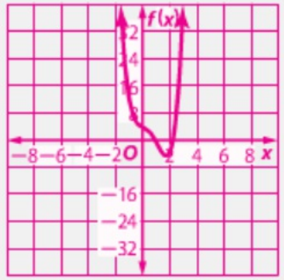
b. Describe the turning points of the graph, its end behavior, and the intervals on which the graph is increasing or decreasing.

c. Continue the graph for $x = 11$ and $x = 12$. What trends in compact disc sales does the graph suggest? **See Chapter 5 Answer Appendix.**

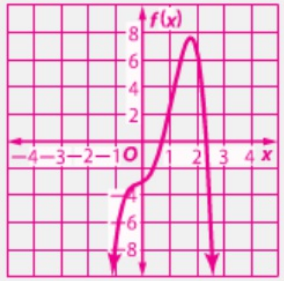
d. Is it reasonable that the trend will continue indefinitely? Explain. **See Chapter 5 Answer Appendix.**

13b. Sample answer: Relative maximum at $x = 5$ and relative minimum at $x \approx 9.5$ $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$. The graph increases when $x < 5$ and $x > 9.5$ and decreases when $5 < x < 9.5$.

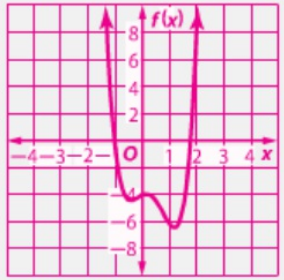
1.



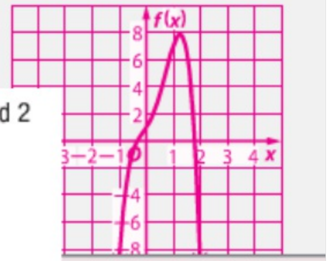
2.



3.

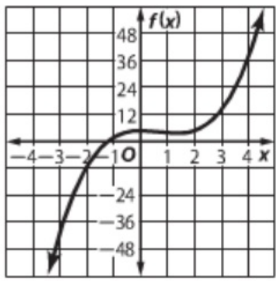


4.

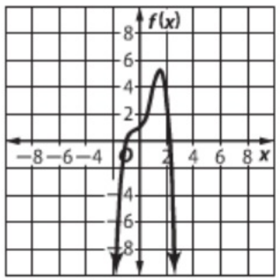


Lesson 5-4

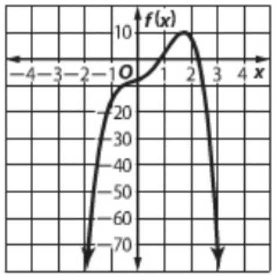
5. between −2 and −1



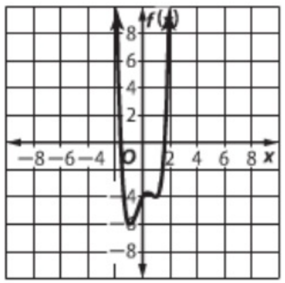
6. at −1 and between 2 and 3



7. between 0 and 1 and between 2 and 3



8. between −2 and −1 and between 1 and 2



Practice and Problem Solving

Extra Practice is on page R5.

Examples 1–3 Complete each of the following. **14–21. See Chapter 5 Answer Appendix.**

a. Graph each function by making a table of values.

b. Determine the consecutive integer values of x between which each real zero is located.

c. Estimate the x-coordinates at which the relative maxima and minima occur.

14. $f(x) = x^3 + 3x^2$

15. $f(x) = -x^3 + 2x^2 - 4$

16. $f(x) = x^3 + 4x^2 - 5x$

17. $f(x) = x^3 - 5x^2 + 3x + 1$

18. $f(x) = -2x^3 + 12x^2 - 8x$

19. $f(x) = 2x^3 - 4x^2 - 3x + 4$

20. $f(x) = x^4 + 2x - 1$

21. $f(x) = x^4 + 8x^2 - 12$

22b. Sample answer: The graph has a relative minimum at $x = 10$ and then increases as x increases.

Example 4

22. **FINANCIAL LITERACY** The average annual price of gasoline can be modeled by the cubic function $f(x) = 0.0007x^3 - 0.014x^2 + 0.08x + 0.96$, where x is the number of years after 1987 and $f(x)$ is the price in dollars. **a, d. See Chapter 5 Answer Appendix.**

a. Graph the function for $0 \le x \le 30$.

b. Describe the turning points of the graph and its end behavior.

c. What trends in gasoline prices does the graph suggest?

d. Is it reasonable that the trend will continue indefinitely? Explain.

22c. The graph suggests a fairly steep continuous increase and gas prices at $5 per gallon by 2012, which could be possible.

23. rel. max: $x = -2.73$; rel. min: $x = 0.73$

25. rel. max: $x = 1.34$; no rel. min

26. rel. max: $x = -1.87$; rel. min: $x = 1.52$

Use a graphing calculator to estimate the x-coordinates at which the maxima and minima of each function occur. Round to the nearest hundredth.

23. $f(x) = x^3 + 3x^2 - 6x - 6$

24. $f(x) = -2x^3 + 4x^2 - 5x + 8$ **no relative max or min**

25. $f(x) = -2x^4 + 5x^3 - 4x^2 + 3x - 7$

26. $f(x) = x^5 - 4x^3 + 3x^2 - 8x - 6$

Sketch the graph of polynomial functions with the following characteristics.

27. an odd function with zeros at $-5, -3, 0, 2$ and 4

28. an even function with zeros at $-2, 1, 3,$ and 5

29. a 4-degree function with a zero at -3, maximum at $x = 2$, and minimum at $x = -1$

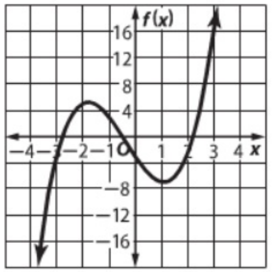
30. a 5-degree function with zeros at $-4, -1,$ and 3, maximum at $x = -2$

31. an odd function with zeros at $-1, 2,$ and 5 and a negative leading coefficient

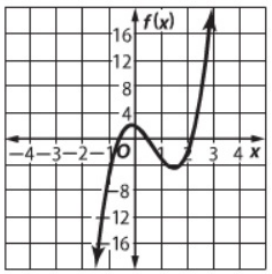
32. an even function with a minimum at $x = 3$ and a positive leading coefficient

27–32. See Chapter 5 Answer Appendix.

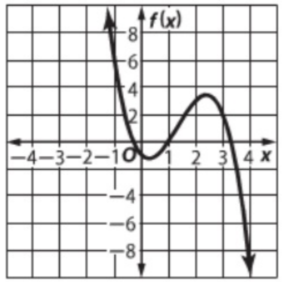
9. rel. max at $x \approx -1.8$; rel. min at $x \approx 1.1$; D = {all real numbers}, R = {all real numbers}



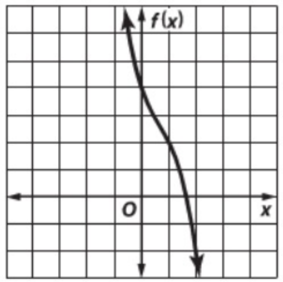
10. rel. max at $x \approx -0.1$; rel. min at $x \approx 1.5$; D = {all real numbers}, R = {all reals numbers}



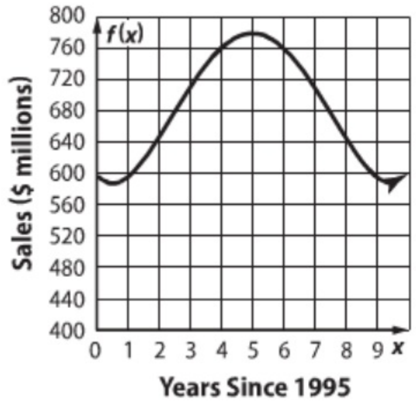
11. rel. max at $x \approx 2.4$; rel. min at $x \approx 0.3$; D = {all real numbers}, R = {all real numbers}



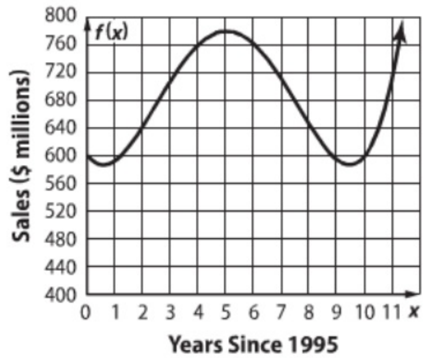
12. no relative max or min; D = {all real numbers}, R = {all real numbers}



13a.



13c.

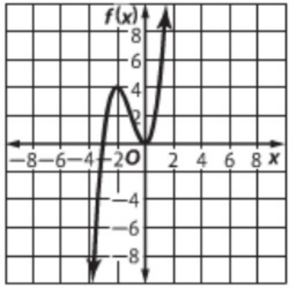


Sample answer: This suggests a dramatic increase in sales.

13d. Sample answer: No; with so many other forms of media on the market today, CD sales will not increase dramatically. In fact, the sales will probably decrease. The function appears to be accurate only until about 2005.

14a.

x	$f(x)$
−4	−16
−3	0
−2	4
−1	2
0	0
1	4
2	20
3	54
4	112

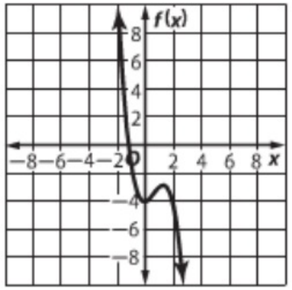


14b. at 0 and at −3

14c. rel. max: $x = -2$; rel. min: $x = 0$

15a.

x	$f(x)$
−4	92
−3	41
−2	12
−1	−1
0	−4
1	−3
2	−4
3	−13
4	−36

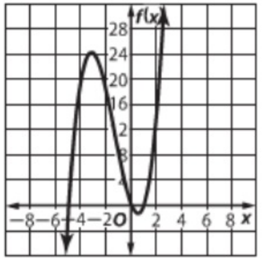


15b. between −2 and −1

15c. rel. min: $x = 0$; rel. max: $x = 1$

16a.

x	$f(x)$
−6	−42
−5	0
−4	20
−3	24
−2	18
−1	8
0	0
1	0
2	14
3	48

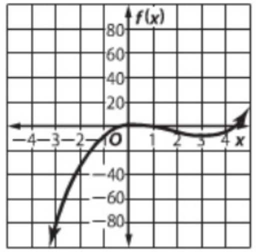


16b. −5, 0, and 1

16c. rel. max: $x = -3$; rel. min: between $x = 0$ and $x = 1$

17a.

x	$f(x)$
−4	−155
−3	−80
−2	−33
−1	−8
0	1
1	0
2	−5
3	−8
4	−3
5	16

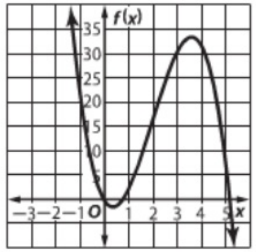


17b. at $x = 1$, between −1 and 0, and between $x = 4$ and $x =$

17c. rel. max: $x \approx \frac{1}{3}$, rel. min: $x \approx 3$

18a.

x	$f(x)$
−1	22
0	0
1	2
2	16
3	30
4	32
5	10
6	−48
7	−154

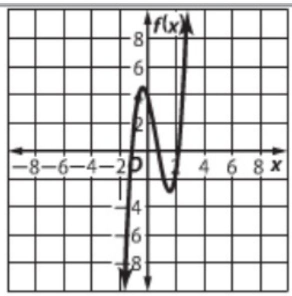


18b. at 0, between 0 and 1, and between 5 and 6

18c. rel. min: between $x = 0$ and $x = 1$; rel. max: near $x = 4$

19a.

x	$f(x)$
−4	−176
−3	−77
−2	−22
−1	1
0	4
1	−1
2	−2
3	13
4	56

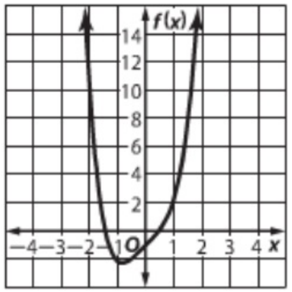


19b. between $x = -2$ and $x = -1$, between $x = 0$ and $x = 1$, and between $x = 2$ and $x = 3$

19c. rel. max: near $x = -0.3$; rel. min: near $x = 1.6$

20a.

x	$f(x)$
−4	247
−3	74
−2	11
−1	−2
0	−1
1	2
2	19
3	86
4	263



20b. between $x = -2$ and $x = -1$ and between $x = 0$ and $x = 1$

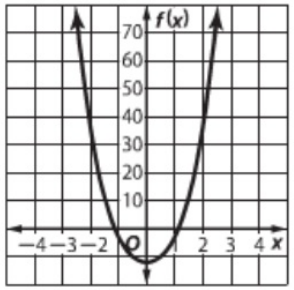
20c. rel. min: near $x = -1$; no rel. max

20b. between $x = -2$ and $x = -1$ and between $x = 0$ and $x = 1$

20c. rel. min: near $x = -1$; no rel. max

21a.

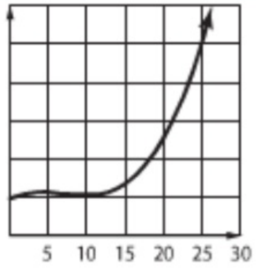
x	$f(x)$
−4	372
−3	141
−2	36
−1	−3
0	−12
1	−3
2	36
3	141
4	372



21b. between $x = -2$ and $x = -1$ and between $x = 1$ and $x = 2$

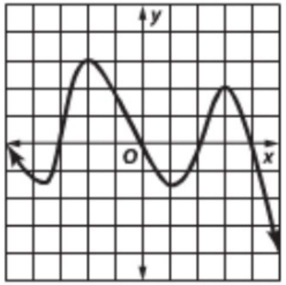
21c. min: near $x = 0$

22a.

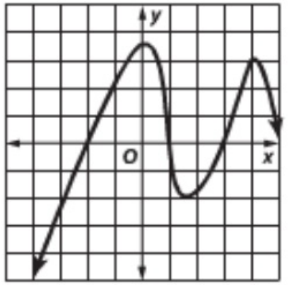


22d. Sample answer: While it is possible for gasoline prices to continue to soar at this rate, it is likely that alternate forms of transportation and fuel will slow down this rapid increase.

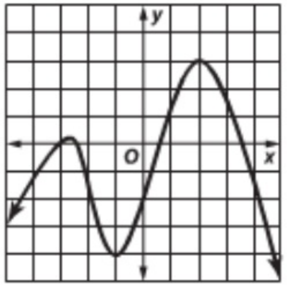
27.



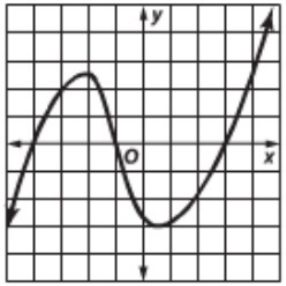
28.



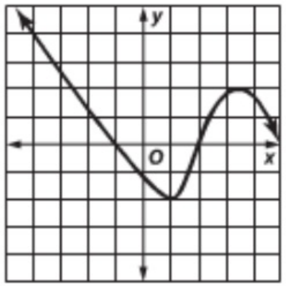
29.



30.



31.



32.