KeyConcept Solving by Substitution

Step 1 When necessary, solve at least one equation for one variable.

Step 2 Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

EXAMPLE 1   Solve a System by Substitution

Use substitution to solve the system of equations.
$$y = -4x + 12$$
$$2x + y = 2$$

Substitute $-4x + 12$ for y in the second equation.

$2x + y = 2$	Second equation
$2x + (-4x + 12) = 2$	$y = -4x + 12$
$2x - 4x + 12 = 2$	Simplify.
$-2x + 12 = 2$	Combine like terms.
$-2x = -10$	Subtract 12 from each side.
$x = 5$	Divide each side by -2.

EXAMPLE 1 **Solve a System by Substitution**

Substitute 5 for x in either equation to find y.

$y = -4x + 12$	First equation
$y = -4(5) + 12$	Substitute 5 for x.
$y = -8$	Simplify.

Answer: The solution is $(5, -8)$.

EXAMPLE 1 ✓ **Check Your Progress**

Use substitution to solve the system of equations.
$y = 2x$
$3x + 4y = 11$

A. $\left(1, \dfrac{1}{2}\right)$

B. $(1, 2)$

C. $(2, 1)$

D. $(0, 0)$

EXAMPLE 1 ✓ **Check Your Progress**

Use substitution to solve the system of equations.

$y = 2x$

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A. $\left(1, \dfrac{1}{2}\right)$

B. $(1, 2)$

C. $(2, 1)$

D. $(0, 0)$

EXAMPLE 2   Solve and then Substitute

Use substitution to solve the system of equations.
$x - 2y = -3$
$3x + 5y = 24$

Step 1   Solve the first equation for x since the coefficient is 1.

$$x - 2y = -3 \qquad \text{First equation}$$

$$x - 2y + 2y = -3 + 2y \qquad \text{Add } 2y \text{ to each side.}$$

$$x = -3 + 2y \qquad \text{Simplify.}$$

EXAMPLE 2 **Solve and then Substitute**

Step 2 Substitute $-3 + 2y$ for x in the second equation to find the value of y.

$3x + 5y = 24$	Second equation
$3(-3 + 2y) + 5y = 24$	Substitute $-3 + 2y$ for x.
$-9 + 6y + 5y = 24$	Distributive Property
$-9 + 11y = 24$	Combine like terms.
$-9 + 11y + 9 = 24 + 9$	Add 9 to each side.
$11y = 33$	Simplify.
$y = 3$	Divide each side by 11.

EXAMPLE 2   Solve and then Substitute

Step 3   Find the value of x.

$$x - 2y = -3 \qquad \text{First equation}$$
$$x - 2(3) = -3 \qquad \text{Substitute 3 for } y.$$
$$x - 6 = -3 \qquad \text{Simplify.}$$
$$x = 3 \qquad \text{Add 6 to each side.}$$

EXAMPLE 2 Solve and then Substitute

Step 3 Find the value of x.

$x - 2y$	$= -3$	First equation
$x - 2(3)$	$= -3$	Substitute 3 for y.
$x - 6$	$= -3$	Simplify.
x	$= 3$	Add 6 to each side.

Examples 1–3 Use substitution to solve each system of equations.

$y = -3x + 6$

1. $y = x + 5$
$3x + y = 25$ **(5, 10)**

2. $x = y - 2$
$4x + y = 2$ **(0, 2)**

3. $3x + y = 6$
$4x + 2y = 8$ **(2, 0)**

4. $2x + 3y = 4$
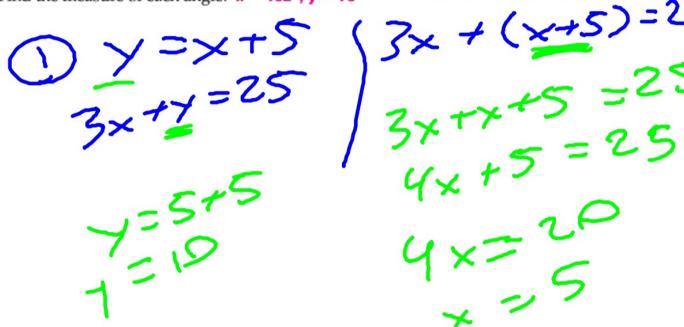$4x + 6y = 9$ **no solution**

5. $x - y = 1$
$3x = 3y + 3$ **infinitely many**

6. $2x - y = 6$
$-3y = -6x + 18$ **infinitely many**

Example 4   **7. GEOMETRY** The sum of the measures of angles X and Y is 180°. The measure of angle X is 24° greater than the measure of angle Y.

    a. Define the variables, and write equations for this situation. $x = m\angle X$, $y = m\angle Y$; $x + y = 180$, $x = 24 + y$

    b. Find the measure of each angle. $x = 102°$, $y = 78°$

① $y = x + 5$
$3x + y = 25$

$3x + (x + 5) = 25$
$3x + x + 5 = 25$
$4x + 5 = 25$
$4x = 20$
$x = 5$

$y = 5 + 5$
$y = 10$

Examples 1–3 Use substitution to solve each system of equations.

8. $y = 5x + 1$
$4x + y = 10$ **(1, 6)**

9 $y = 4x + 5$
$2x + y = 17$ **(2, 13)**

10. $y = 3x - 34$
$y = 2x - 5$ **(29, 53)**

11. $y = 3x - 2$
$y = 2x - 5$ **(−3, −11)**

12. $2x + y = 3$
$4x + 4y = 8$ **(1, 1)**

13. $3x + 4y = -3$
$x + 2y = -1$ **(−1, 0)**

14. $y = -3x + 4$
$-6x - 2y = -8$
infinitely many

15. $-1 = 2x - y$
$8x - 4y = -4$
infinitely many

16. $x = y - 1$
$-x + y = -1$ **no solution**

17. $y = -4x + 11$
$3x + y = 9$ **(2, 3)**

18. $y = -3x + 1$
$2x + y = 1$ **(0, 1)**

19. $3x + y = -5$
$6x + 2y = 10$ **no solution**

20. $5x - y = 5$
$-x + 3y = 13$ **(2, 5)**

21. $2x + y = 4$
$-2x + y = -4$ **(2, 0)**

22. $-5x + 4y = 20$
$10x - 8y = -40$ **infinitely many**