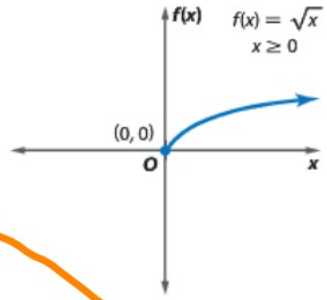
6-3 Square Root Functions and Inequalities

1 Square Root Functions If a function contains the square root of a variable, it is called a **square root function**. The square root function is a type of **radical function**.

KeyConcept Parent Function of Square Root Functions

Parent function:	$f(x) = \sqrt{x}$
Domain:	$\{x \mid x \geq 0\}$
Range:	$\{f(x) \mid f(x) \geq 0\}$
Intercepts:	$x = 0, f(x) = 0$
Not defined:	$x < 0$
End behavior:	$x \to 0, f(x) \to 0$
	$x \to +\infty, f(x) \to +\infty$



$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4
−1	i

complex

Example 1 Identify Domain and Range

Identify the domain and range of $f(x) = \sqrt{x+4}$.

The domain only includes values for which the radicand is nonnegative.

$x + 4 \geq 0$ Write an inequality.

$x \geq -4$ Subtract 4 from each side.

Thus, the domain is $\{x \mid x \geq -4\}$.

Find $f(-4)$ to determine the lower limit of the range.

$f(-4) = \sqrt{-4+4}$ or 0

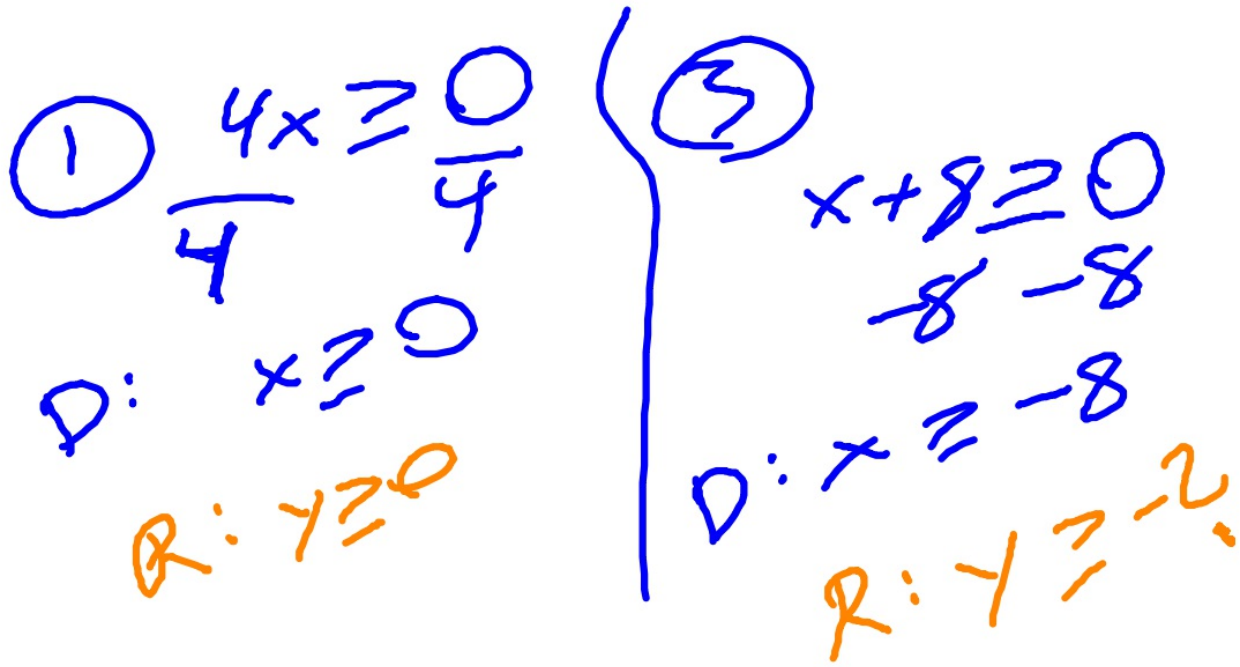
So, the range is $\{f(x) \mid f(x) \geq 0\}$.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Identify the domain and range of each function.**

1. $f(x) = \sqrt{4x}$
   D = {x | x ≥ 0}; R = {f(x) | f(x) ≥ 0}
2. $f(x) = \sqrt{x-5}$
   D = {x | x ≥ 5}; R = {f(x) | f(x) ≥ 0}
3. $f(x) = \sqrt{x+8} - 2$
   D = {x | x ≥ −8}; R = {f(x) | f(x) ≥ −2}



KeyConcept Transformations of Square Root Functions

$$f(x) = a\sqrt{x - h} + k$$

h—Horizontal Translation

h units right if *h* is positive
| *h* | units left if *h* is negative

The domain is $\{x \mid x \geq h\}$.

k—Vertical Translation

k units up if *k* is positive
| *k* | units down if *k* is negative

If $a > 0$, then the range is $\{f(x) \mid f(x) \geq k\}$.
If $a < 0$, then the range is $\{f(x) \mid f(x) \leq k\}$.

a—Orientation and Shape

- If $a < 0$, the graph is reflected across the *x*-axis.
- If $|a| > 1$, the graph is stretched vertically.
- If $0 < |a| < 1$, the graph is compressed vertically.

Example 2 Graph each function. State the domain and range. **4–7. See margin.**

4. $f(x) = \sqrt{x} - 2$

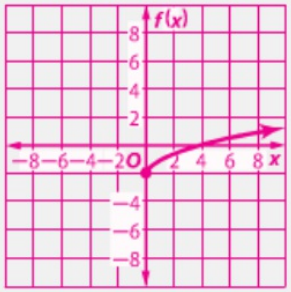
5. $f(x) = 3\sqrt{x - 1}$

6. $f(x) = \frac{1}{2}\sqrt{x + 4} - 1$

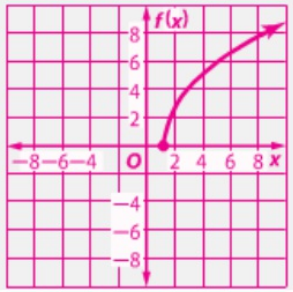
7. $f(x) = -\sqrt{3x - 5} + 5$

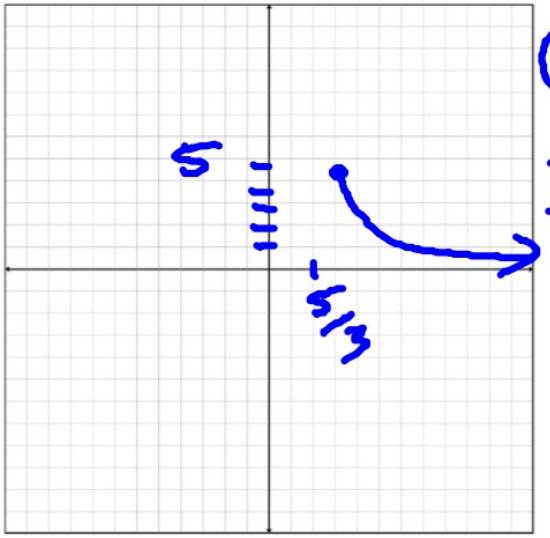
Additional Answers

4. D = {x | x ≥ 0};
R = {f(x) | f(x) ≥ −2}

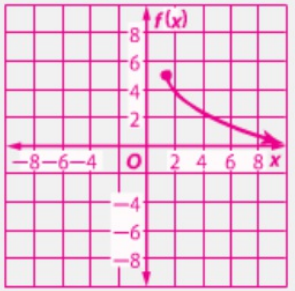


5. D = {x | x ≥ 1};
R = {f(x) | f(x) ≥ 0}





7. D = $\left\{x \mid x \geq \frac{5}{3}\right\}$;
R = {f(x) | f(x) ≤ 5}



Real-World Example 3 Use Graphs to Analyze Square Root Functions

MUSIC Refer to the application at the beginning of the lesson. The pitch, or frequency, measured in hertz (Hz) of a certain string can be modeled by $f(T) = \frac{1}{1.28}\sqrt{\frac{T}{0.0000708}}$, where T is tension in kilograms.

a. Graph the function for tension in the domain $\{T \mid 0 \leq T \leq 10\}$.

b. How much tension is needed for a pitch of over 200 Hz?

Example 3

8. **OCEAN** The speed that a tsunami, or tidal wave, can travel is modeled by the equation $v = 356\sqrt{d}$, where v is the speed in kilometers per hour and d is the average depth of the water in kilometers. A tsunami is found to be traveling at 145 kilometers per hour. What is the average depth of the water? Round to the nearest hundredth of a kilometer. **0.17 km**

13. D = {x | x ≥ 0}; R = {f(x) | f(x) ≤ 2} 14. D = {x | x ≥ 0}; R = {f(x) | f(x) ≥ −6}

Practice and Problem Solving

Extra Practice is on page R6.

Example 1 Identify the domain and range of each function.

15. D = {x | x ≥ 2}; R = {f(x) | f(x) ≥ −8}

13. $f(x) = -\sqrt{2x} + 2$
14. $f(x) = \sqrt{x} - 6$
15. $f(x) = 4\sqrt{x-2} - 8$
16. $f(x) = \sqrt{x+2} + 5$
17. $f(x) = \sqrt{x-4} - 6$
18. $f(x) = -\sqrt{x-6} + 5$

Example 2 Graph each function. State the domain and range.

19–28. See Chapter 6 Answer Appendix.

19. $f(x) = \sqrt{6x}$
20. $f(x) = -\sqrt{5x}$
21. $f(x) = \sqrt{x-8}$
22. $f(x) = \sqrt{x+1}$
23. $f(x) = \sqrt{x+3} + 2$
24. $f(x) = \sqrt{x-4} - 10$
25. $f(x) = 2\sqrt{x-5} - 6$
26. $f(x) = \frac{3}{4}\sqrt{x+12} + 3$
27. $f(x) = -\frac{1}{5}\sqrt{x-1} - 4$
28. $f(x) = -3\sqrt{x+7} + 9$

16. D = {x | x ≥ −2}; R = {f(x) | f(x) ≥ 5}
17. D = {x | x ≥ 4}; R = {f(x) | f(x) ≥ −6}
18. D = {x | x ≥ 6}; R = {f(x) | f(x) ≤ 5}

Example 3

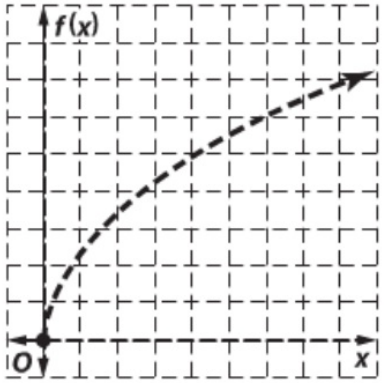
29. **SKYDIVING** The approximate time t in seconds that it takes an object to fall a distance of d feet is given by $t = \sqrt{\frac{d}{16}}$. Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time? 1936 ft

30. CCSS **MODELING** The velocity of a roller coaster as it moves down a hill is $V = \sqrt{v^2 + 64h}$, where v is the initial velocity in feet per second and h is the vertical drop in feet. The designer wants the coaster to have a velocity of 90 feet per second when it reaches the bottom of the hill.

   a. If the initial velocity of the coaster at the top of the hill is 10 feet per second, write an equation that models the situation. $90 = \sqrt{100 + 64h}$

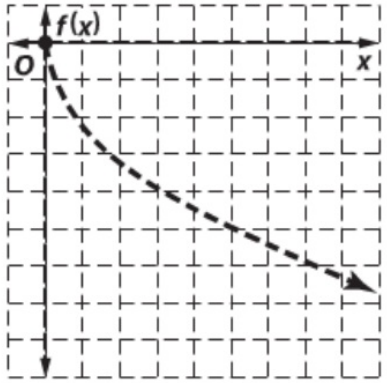
   b. How high should the designer make the hill? 125 ft

19.



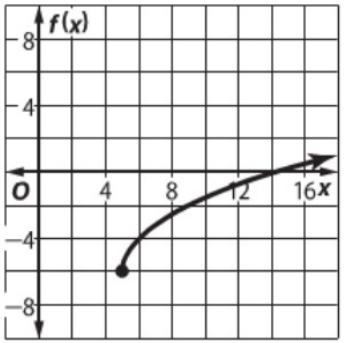
$D = \{x \mid x \geq 0\}$;

20.



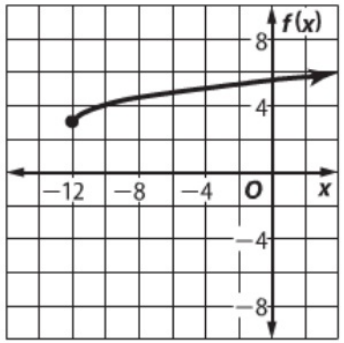
$D = \{x \mid x \geq 0\}$;

25.



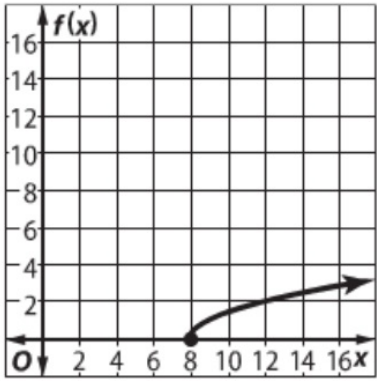
$D = \{x \mid x \geq 5\}$;
$R = \{f(x) \mid f(x) \geq -6\}$

26.



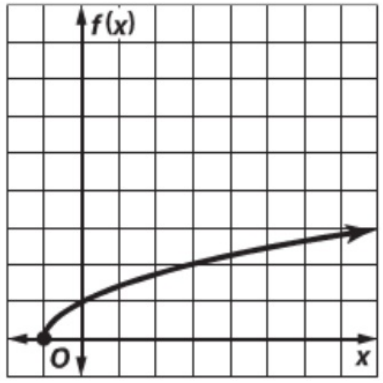
$D = \{x \mid x \geq -12\}$;
$R = \{f(x) \mid f(x) \geq 3\}$

21.



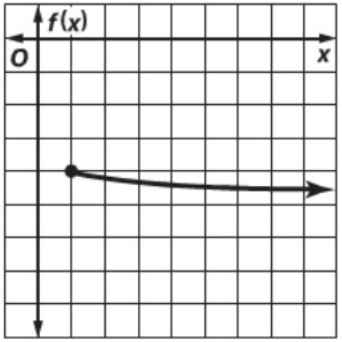
$D = \{x \mid x \geq 8\}$;
$R = \{f(x) \mid f(x) \geq 0\}$

22.



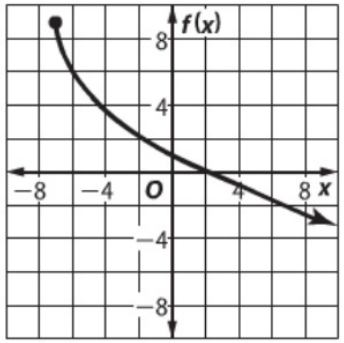
$D = \{x \mid x \geq -1\}$;
$R = \{f(x) \mid f(x) \geq 0\}$

27.



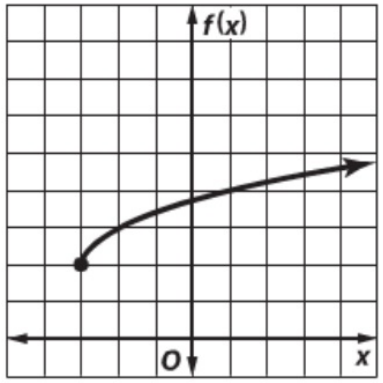
$D = \{x \mid x \geq 1\}$;
$R = \{f(x) \mid f(x) \leq -4\}$

28.



$D = \{x \mid x \geq -7\}$;
$R = \{f(x) \mid f(x) \leq 9\}$

23.



24.