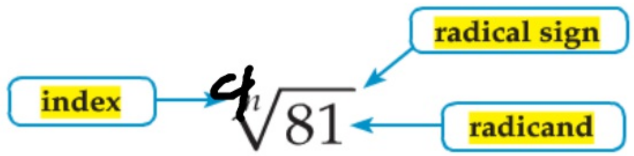
6-4 *n*th Roots

KeyConcept Definition of *n*th Root

Words	For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
Example	Because $(-3)^4 = 81$, -3 is a fourth root of 81 and 3 is a principal root.

The symbol $\sqrt[n]{}$ indicates an nth root.



Some numbers have more than one real nth root. For example, 64 has two square roots, 8 and -8, since 8^2 and $(-8)^2$ both equal 64. When there is more than one real root and n is even, the nonnegative root is called the **principal root**.

Some examples of nth roots are listed below.

$\sqrt{25} = 5$	$\sqrt{25}$ indicates the principal square root of 25.
$-\sqrt{25} = -5$	$-\sqrt{25}$ indicates the opposite of the principal square root of 25.
$\pm\sqrt{25} = \pm 5$	$\pm\sqrt{25}$ indicates both square roots of 25.

KeyConcept Real *n*th Roots

Suppose n is an integer greater than 1, and a is a real number.

a	n is even.	n is odd.
$a > 0$	1 unique positive and 1 unique negative real root: $\pm\sqrt[n]{a}$; positive root is principal root	1 unique positive and 0 negative real roots: $\sqrt[n]{a}$
$a < 0$	0 real roots	0 positive and 1 negative real root: $\sqrt[n]{a}$
$a = 0$	1 real root: $\sqrt[n]{0} = 0$	1 real root: $\sqrt[n]{0} = 0$

Example 1 Find Roots

Simplify.

a. $\pm\sqrt{16y^4}$

$$\pm\sqrt{16y^4} = \pm\sqrt{(4y^2)^2}$$
$$= \pm 4y^2$$

The square roots of $16y^4$ are $\pm 4y^2$.

b. $-\sqrt{(x^2-6)^8}$

$$-\sqrt{(x^2-6)^8} = -\sqrt{[(x^2-6)^4]^2}$$
$$= -(x^2-6)^4$$

The opposite of the principal square root of $(x^2-6)^8$ is $-(x^2-6)^4$.

c. $\sqrt[5]{243a^{20}b^{25}}$

$$\sqrt[5]{243a^{20}b^{25}} = \sqrt[5]{(3a^4b^5)^5}$$
$$= 3a^4b^5$$

The fifth root of $243a^{20}b^{25}$ is $3a^4b^5$

d. $\sqrt{-16x^4y^8}$

$$\sqrt[2]{-16x^4y^8}$$

n is even.

b is negative.

There are no real roots since $\sqrt{-16}$ is not a real number. However, there are two imaginary roots, $4ix^2y^4$ and $-4ix^2y^4$.

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Examples 1–2 **Simplify.**

1. $\pm\sqrt{100y^8}$ $\pm 10y^4$

2. $-\sqrt{49u^8v^{12}}$ $-7u^4v^6$

3. $\sqrt{(y-6)^8}$ $(y-6)^4$

4. $\sqrt[4]{16g^{16}h^{24}}$ $2g^4h^6$

When you find an even root of an even power and the result is an odd power, you must use the absolute value of the result to ensure that the answer is nonnegative.

Example 2 Simplify Using Absolute Value

Simplify.

a. $\sqrt[4]{y^4}$

$\sqrt[4]{y^4} = |y|$

Since y could be negative, you must take the absolute value of y to identify the principal root.

b. $\sqrt[6]{64(x^2 - 3)^{18}}$

$\sqrt[6]{64(x^2 - 3)^{18}} = 2\left|(x^2 - 3)^3\right|$

Since the index 6 is even and the exponent 3 is odd, you must use absolute value.

5. $\sqrt{-16y^4}$ $\pm 4iy^2$

Why no absolute for #5?

6. $\sqrt[6]{64(2y + 1)^{18}}$ $2\left|(2y + 1)^3\right|$

$-3^7 = -$

$(-3)^8 = +$

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5. $\sqrt{-16y^4}$ $\pm 4iy^2$

6. $\sqrt[6]{64(2y+1)^{18}}$ $2|(2y+1)^3|$

Example 3 **Use a calculator to approximate each value to three decimal places.**

7. $\sqrt{58}$ 7.616

8. $-\sqrt{76}$ −8.718

9. $\sqrt[5]{-43}$ −2.122

10. $\sqrt[4]{71}$ 2.903

11. CCSS **PERSEVERANCE** The radius r of the orbit of a television satellite is given by $\sqrt[3]{\frac{GMt^2}{4\pi^2}}$, where G is the universal gravitational constant, M is the mass of Earth, and t is the time it takes the satellite to complete one orbit. Find the radius of the satellite's orbit if G is $6.67 \times 10^{-11} N \cdot m^2/kg^2$, M is 5.98×10^{24} kg, and t is 2.6×10^6 seconds. about 4.088×10^8 m

Real-World Example 3 Approximate Radicals

INJURY PREVENTION Refer to the beginning of the lesson.

a. If $c = \sqrt[5]{b^2}$ represents the number of collisions and b represents the number of bicycle riders per intersection, estimate the number of collisions at an intersection that has 1000 bicycle riders per week.

Understand You want to find out how many collisions there are.

Plan Let 1000 be the number of bicycle riders. The number of collisions is c.

Solve

$c = \sqrt[5]{b^2}$ Original formula

$= \sqrt[5]{1000^2}$ $b = 1000$

≈ 15.85 Use a calculator.

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Examples 1–2 **Simplify.**

12. $\pm\sqrt{121x^4y^{16}}$ $\pm 11x^2y^8$

13. $\pm\sqrt{225a^{16}b^{36}}$ $\pm 15a^8b^{18}$

14. $\pm\sqrt{49x^4}$ $\pm 7x^2$

15. $-\sqrt{16c^4d^2}$ $-4c^2|d|$

16. $-\sqrt{81a^{16}b^{20}c^{12}}$ $-9a^8b^{10}c^6$

17. $-\sqrt{400x^{32}y^{40}}$ $-20x^{16}y^{20}$

18. $\sqrt{(x+15)^4}$ $(x+15)^2$

19. $\sqrt{(x^2+6)^{16}}$ $(x^2+6)^8$

20. $\sqrt{(a^2+4a)^{12}}$ $(a^2+4a)^6$

21. $\sqrt[3]{8a^6b^{12}}$ $2a^2b^4$

22. $\sqrt[6]{d^{24}x^{36}}$ d^4x^6

23. $\sqrt[3]{27b^{18}c^{12}}$ $3b^6c^4$

24. $-\sqrt{(2x+1)^6}$ $|(2x+1)^3|$

25. $\sqrt{-(x+2)^8}$ $i(x+2)^4$

26. $\sqrt[3]{-(y-9)^9}$ $-(y-9)^3$

27. $\sqrt[6]{x^{18}}$ $|x^3|$

28. $\sqrt[4]{a^{12}}$ $|a^3|$

29. $\sqrt[3]{a^{12}}$ a^4

30. $\sqrt[4]{81(x+4)^4}$ $3|(x+4)|$

31. $\sqrt[3]{(4x-7)^{24}}$ $(4x-7)^8$

32. $\sqrt[3]{(y^3+5)^{18}}$ $(y^3+5)^6$

33. $\sqrt[4]{256(5x-2)^{12}}$ $4|(5x-2)^3|$

34. $\sqrt[8]{x^{16}y^8}$ $x^2|y|$

35. $\sqrt[5]{32a^{15}b^{10}}$ $2a^3b^2$

Example 3

36. **SHIPPING** An online book store wants to increase the size of the boxes it uses to ship orders. The new volume N is equal to the old volume V times the scale factor F cubed, or $N = V \cdot F^3$. What is the scale factor if the old volume was 0.8 cubic feet and the new volume is 21.6 cubic feet? **3**

37. **GEOMETRY** The side length of a cube is determined by $r = \sqrt[3]{V}$, where V is the volume in cubic units. Determine the side length of a cube with a volume of 512 cm^3. **8 cm**

Use a calculator to approximate each value to three decimal places.

38. $\sqrt{92}$ 9.592

39. $-\sqrt{150}$ −12.247

40. $\sqrt{0.43}$ 0.656

41. $\sqrt{0.62}$ 0.787

42. $\sqrt[3]{168}$ 5.518

43. $\sqrt[5]{-4382}$ −5.350

44. $\sqrt[6]{(8912)^2}$ 20.733

45. $\sqrt[5]{(4756)^2}$ 29.573