# 6-6 Rational Exponents

## KeyConcept $b^{\frac{1}{n}}$

**Words** For any real number $b$ and any positive integer $n$, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and $n$ is even. When $b < 0$ and $n$ is even, a complex root may exist.

**Examples** $27^{\frac{1}{3}} = \sqrt[3]{27}$ or 3 $\quad$ $(-16)^{\frac{1}{2}} = \sqrt{-16}$ or $4i$

## Example 1 Radical and Exponential Forms

**Simplify.**

**a. Write $x^{\frac{1}{6}}$ in radical form.**

$x^{\frac{1}{6}} = \sqrt[6]{x}$ Definition of $b^{\frac{1}{n}}$

**b. Write $\sqrt[4]{z}$ in exponential form.**

$\sqrt[4]{z} = z^{\frac{1}{4}}$ Definition of $b^{\frac{1}{n}}$

## Check Your Understanding

= Step-by-Step Solutions begin on page R14.

**Example 1** Write each expression in radical form, or write each radical in exponential form.

1. $10^{\frac{1}{4}}$ $\sqrt[4]{10}$
2. $x^{\frac{3}{5}}$ $\sqrt[5]{x^3}$
3. $\sqrt[3]{15}$ $15^{\frac{1}{3}}$
4. $\sqrt[4]{7x^6y^9}$ $7^{\frac{1}{4}}x^{\frac{3}{2}}y^{\frac{9}{4}}$

## Example 2 Evaluate Expressions with Rational Exponents

**Evaluate each expression.**

**a.** $81^{-\frac{1}{4}}$

$$81^{-\frac{1}{4}} = \frac{1}{81^{\frac{1}{4}}} \qquad b^{-n} = \frac{1}{b^n}$$

$$= \frac{1}{\sqrt[4]{81}} \qquad 81^{\frac{1}{4}} = \sqrt[4]{81}$$

$$= \frac{1}{\sqrt[4]{3^4}} \qquad 81 = 3^4$$

$$= \frac{1}{3} \qquad \text{Simplify.}$$

**b.** $216^{\frac{2}{3}}$

$(216^2)^{\frac{1}{3}}$

OR

$(216^{\frac{1}{3}})^2$

**Example 2** **Evaluate each expression.**

**5.** $343^{\frac{1}{3}}$ 7

$\sqrt[3]{343}$

$49 \cdot 7$

$7 \cdot 7$

**6.** $32^{-\frac{1}{5}}$ $\frac{1}{2}$

$\frac{1}{\sqrt[5]{32}}$

**7.** $125^{\frac{2}{3}}$ 25

$(\sqrt[3]{125})^2$

$= 5^2$

**8.** $\frac{24}{4^{\frac{3}{2}}}$ 3

$\frac{24}{(\sqrt{4})^3} = \frac{24}{8}$

## KeyConcept Rational Exponents

**Words** For any real nonzero number $b$, and any integers $x$ and $y$, with $y > 1$, $b^{\frac{x}{y}} = \sqrt[y]{b^x} = \left(\sqrt[y]{b}\right)^x$, except when $b < 0$ and $y$ is even. When $b < 0$ and $y$ is even, a complex root may exist.

**Examples** $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2$ or 9 $\qquad (-16)^{\frac{3}{2}} = \left(\sqrt{-16}\right)^3 = (4i)^3$ or $-64i$

## Real-World Example 3 Solve Equations with Rational Exponents

**FINANCIAL LITERACY** **Refer to the beginning of the lesson. Suppose a video game system costs $390 now. How much would the price increase in six months with an annual inflation rate of 5.3%?**

$C = c(1 + r)^n$ Original formula

$= 390(1 + 0.053)^{\frac{1}{2}}$ $c = 390$, $r = 0.053$, and $n = \frac{6 \text{ months}}{12 \text{ months}}$ or $\frac{1}{2}$

$\approx 400.20$ Use a calculator.

In six months the price of the video game system will be $400.20 − $390.00 or $10.20 more than its current price.

**Example 3**

**9** **GARDENING** If the area $A$ of a square is known, then the lengths of its sides $\ell$ can be computed using $\ell = A^{\frac{1}{2}}$. You have purchased a 169 ft² share in a community garden for the season. What is the length of one side of your square garden? 13 ft

$169^{1/2} = \sqrt{169}$

## Example 4 Simplify Expressions with Rational Exponents

**Simplify each expression.**

**a.** $a^{\frac{2}{7}} \cdot a^{\frac{4}{7}}$

$a^{\frac{2}{7}} \cdot a^{\frac{4}{7}} = a^{\frac{2}{7}+\frac{4}{7}}$ Add powers.

$= a^{\frac{6}{7}}$ Add exponents.

$x^a \cdot x^b = x^{a+b}$

**b.** $b^{-\frac{5}{6}}$

$b^{-\frac{5}{6}} = \frac{1}{b^{\frac{5}{6}}}$ $b^{-n} = \frac{1}{b^n}$

$= \frac{1}{b^{\frac{5}{6}}} \cdot \frac{b^{\frac{1}{6}}}{b^{\frac{1}{6}}}$ $\frac{b^{\frac{1}{6}}}{b^{\frac{1}{6}}} = 1$

$= \frac{b^{\frac{1}{6}}}{b^{\frac{6}{6}}}$ $b^{\frac{5}{6}} \cdot b^{\frac{1}{6}} = b^{\frac{5}{6}+\frac{1}{6}}$

$= \frac{b^{\frac{1}{6}}}{b}$ $b^{\frac{6}{6}} = b^1$ or $b$

$\frac{x^a}{x^b} = x^{a-b}$

**Examples 4–5** CCSS **PRECISION** Simplify each expression.

**10.** $a^{\frac{3}{4}} \cdot a^{\frac{1}{2}}$ $a^{\frac{5}{4}}$

$\frac{3}{4}+\frac{1}{2}$

$a^{\frac{3}{4}+\frac{2}{4}}$

$a$

**11.** $\frac{x^{\frac{4}{5}}}{x^{\frac{1}{5}}}$ $x^{\frac{3}{5}}$

$x^{4/5 - 1/5}$

**12.** $\frac{b^3}{c^{\frac{1}{2}}} \cdot \frac{c}{b^{\frac{1}{3}}}$ $b^{\frac{8}{3}} c^{\frac{1}{2}}$

$b^{9/3} c^{2/2}$

$\frac{b^{9/3}}{c^{1/2}} \cdot \frac{c^{2/2}}{b^{1/3}}$

## Example 5 Simplify Radical Expressions

**Simplify each expression.**

**a.** $\dfrac{\sqrt[4]{27}}{\sqrt{3}}$

$\dfrac{\sqrt[4]{27}}{\sqrt{3}} = \dfrac{27^{\frac{1}{4}}}{3^{\frac{1}{2}}}$ Rational exponents

$= \dfrac{(3^3)^{\frac{1}{4}}}{3^{\frac{1}{2}}}$ $27 = 3^3$

$= \dfrac{3^{\frac{3}{4}}}{3^{\frac{1}{2}}}$ Power of a Power

$= 3^{\frac{3}{4} - \frac{1}{2}}$ Quotient of Powers

$= 3^{\frac{1}{4}}$ Simplify.

$= \sqrt[4]{3}$ Rewrite in radical form.

**b.** $\sqrt[3]{64z^6}$

$\sqrt[3]{64z^6} = (64z^6)^{\frac{1}{3}}$ Rational exponents

$= (8^2 \cdot z^6)^{\frac{1}{3}}$ $64 = 8^2$

$= 8^{\frac{2}{3}} \cdot z^{\frac{6}{3}}$ Power of a Power

$= 4z^2$ $8^{\frac{2}{3}} = 4$

13. $\sqrt[4]{9g^2}$ $\sqrt{3g}$

$(3^2 g^2)^{\frac{1}{4}}$

$3^{\frac{1}{2}} g^{\frac{1}{2}}$

14. $\dfrac{\sqrt[5]{64}}{\sqrt[5]{4}}$ $2\frac{4}{5}$ or $\sqrt[5]{16}$

$64^{4/5}$

$= \dfrac{(4^3)^{1/5}}{4^{1/5}} = \dfrac{4^{3/5}}{4^{1/5}}$

$4^{2/5}$

**c.** $\dfrac{x^{\frac{1}{2}}-2}{3x^{\frac{1}{2}}+2}$

$$\frac{x^{\frac{1}{2}}-2}{3x^{\frac{1}{2}}+2}=\frac{x^{\frac{1}{2}}-2}{3x^{\frac{1}{2}}+2}\cdot\frac{3x^{\frac{1}{2}}-2}{3x^{\frac{1}{2}}-2}$$

$3x^{\frac{1}{2}}-2$ is the conjugate of $3x^{\frac{1}{2}}+2$.

$$=\frac{3x^{\frac{2}{2}}-8x^{\frac{1}{2}}+4}{9x^{\frac{2}{2}}-4}$$

Multiply.

$$=\frac{3x-8x^{\frac{1}{2}}+4}{9x-4}$$

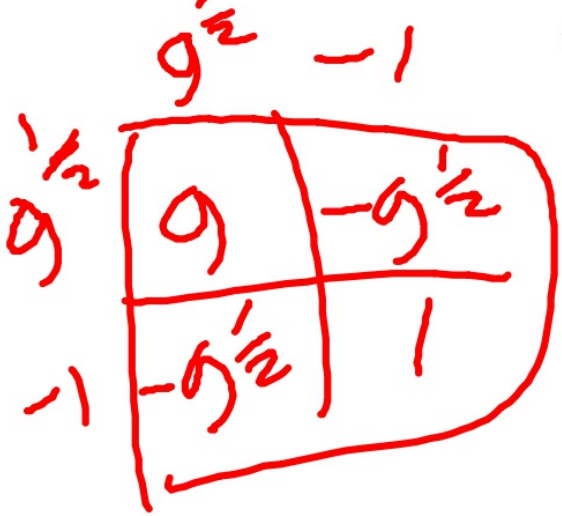
Simplify.

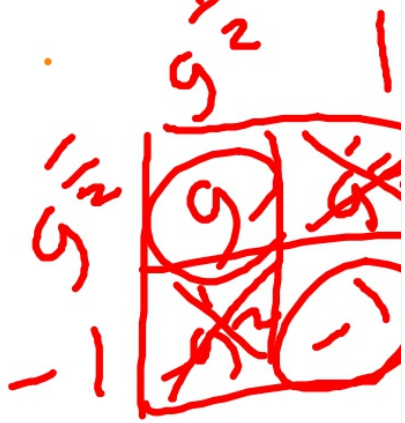
**15.** $\dfrac{g^{\frac{1}{2}}-1}{g^{\frac{1}{2}}+1}$ $\dfrac{g-2g^{\frac{1}{2}}+1}{g-1}$

$g^{\frac{1}{2}}+\frac{1}{2}$

15 $\dfrac{(g^{\frac{1}{2}}-1)(g^{\frac{1}{2}}-1)}{(g^{\frac{1}{2}}+1)(g^{\frac{1}{2}}-1)}$

$\dfrac{g-2g^{\frac{1}{2}}+1}{(g^{\frac{1}{2}})^2-1^2}$



**Example 1** Write each expression in radical form, or write each radical in exponential form.

16. $8^{\frac{1}{5}}$ $\sqrt[5]{8}$

17. $4^{\frac{2}{7}}$ $\sqrt[7]{16}$

18. $a^{\frac{3}{4}}$ $\sqrt[4]{a^3}$

19. $(x^3)^{\frac{3}{2}}$ $\sqrt{x^9}$

20. $\sqrt{17}$ $17^{\frac{1}{2}}$

21. $\sqrt[4]{63}$ $63^{\frac{1}{4}}$

22. $\sqrt[3]{5xy^2}$ $5^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{2}{3}}$

23. $\sqrt[4]{625x^2}$ $5x^{\frac{1}{2}}$

**Example 2** Evaluate each expression.

24. $27^{\frac{1}{3}}$ 3

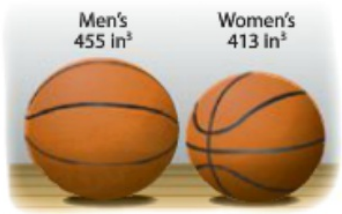
25. $256^{\frac{1}{4}}$ 4

26. $16^{-\frac{1}{2}}$ $\frac{1}{4}$

27. $81^{-\frac{1}{4}}$ $\frac{1}{3}$

**Example 3**

28. CCSS **SENSE-MAKING** A women's regulation-sized basketball is slightly smaller than a men's basketball. The radius $r$ of the ball that holds $V$ cubic units of air is $\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$.




a. Find the radius of a women's basketball. about 4.62 in.

b. Find the radius of a men's basketball. about 4.77 in.

29. **GEOMETRY** The radius $r$ of a sphere with volume $V$ is given by $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$. Find the radius of a ball with a volume of 77 cm³. about 2.64 cm

**Examples 4–5** Simplify each expression.

39. $\frac{x + 4x^{\frac{3}{4}} + 8x^{\frac{1}{2}} + 16x^{\frac{1}{4}} + 16}{x - 16}$

30. $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$ $x^{\frac{11}{15}}$

31. $a^{\frac{4}{9}} \cdot a^{\frac{1}{4}}$ $a^{\frac{25}{36}}$

32. $b^{-\frac{3}{4}}$ $\frac{b^{\frac{1}{4}}}{b}$

33. $y^{-\frac{4}{5}}$ $\frac{y^{\frac{1}{5}}}{y}$

34. $\frac{\sqrt[8]{81}}{\sqrt[6]{3}}$ $\sqrt[3]{3}$

35. $\frac{\sqrt[4]{27}}{\sqrt[4]{3}}$ $\sqrt{3}$

36. $\sqrt[4]{25x^2}$ $\sqrt{5x}$

37. $\sqrt[6]{81g^3}$ $\sqrt[3]{9} \cdot \sqrt{g}$

38. $\frac{h^{\frac{1}{2}} + 1}{h^{\frac{1}{2}} - 1}$ $\frac{h + 2h^{\frac{1}{2}} + 1}{h - 1}$

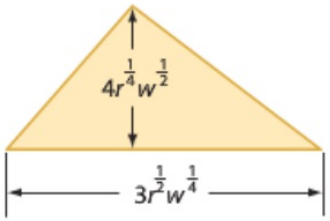
39. $\frac{x^{\frac{1}{4}} + 2}{x^{\frac{1}{4}} - 2}$

**GEOMETRY** Find the area of each figure.

$h - 1$

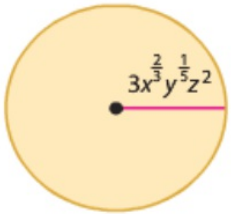
**B** **GEOMETRY** **Find the area of each figure.**

**40.**




$6r^{\frac{3}{4}}w^{\frac{3}{4}}$ units$^2$

**41.**




$28.27x^{\frac{4}{3}}y^{\frac{2}{5}}z^4$ units$^2$

**42.** Find the simplified form of $18^{\frac{1}{2}} + 2^{\frac{1}{2}} - 32^{\frac{1}{2}}$. $0$

**43.** What is the simplified form of $64^{\frac{1}{3}} - 32^{\frac{1}{3}} + 8^{\frac{1}{3}}$? $6 - 2 \cdot 4^{\frac{1}{3}}$

**Simplify each expression.**

**44.** $a^{\frac{7}{4}} \cdot a^{\frac{5}{4}}$ $a^3$

**45.** $x^{\frac{2}{3}} \cdot x^{\frac{8}{3}}$ $x^{\frac{10}{3}}$

**46.** $\left(b^{\frac{3}{4}}\right)^{\frac{1}{3}}$ $b^{\frac{1}{4}}$

**47.** $\left(y^{-\frac{3}{5}}\right)^{-\frac{1}{4}}$ $y^{\frac{3}{20}}$

**48.** $\sqrt[4]{64}$ $2\sqrt{2}$

**49.** $\sqrt[6]{216}$ $\sqrt{6}$

**50.** $d^{-\frac{5}{6}}$ $\frac{d^{\frac{1}{6}}}{d}$

**51.** $w^{-\frac{7}{8}}$ $\frac{w^{\frac{1}{8}}}{w}$