If $a=b$, then $a^2=b^2$

6-7 Solving Radical Equations and Inequalities

KeyConcept Solving Radical Equations

Step 1 Isolate the radical on one side of the equation.

Step 2 Raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Step 3 Solve the resulting polynomial equation. Check your results.

Example 1 Solve Radical Equations

Solve each equation.

a. $\left(\sqrt{x+2}+4\right)^2=(7)^2$ ← No!

$$\sqrt{x+2}+4=7 \quad \text{Original equation}$$
$$\sqrt{x+2}=3 \quad \text{Subtract 4 from each side to isolate the radical.}$$
$$\left(\sqrt{x+2}\right)^2=3^2 \quad \text{Square each side to eliminate the radical.}$$
$$x+2=9 \quad \text{Find the squares.}$$
$$x=7 \quad \text{Subtract 2 from each side.}$$

CHECK
$$\sqrt{x+2}+4=7 \quad \text{Original equation}$$
$$\sqrt{7+2}+4\stackrel{?}{=}7 \quad \text{Replace } x \text{ with 7.}$$
$$7=7 \checkmark \quad \text{Simplify.}$$

b. $\sqrt{x-12}=2-\sqrt{x}$

$$\sqrt{x-12}=2-\sqrt{x} \quad \text{Original equation}$$
$$\left(\sqrt{x-12}\right)^2=\left(2-\sqrt{x}\right)^2 \quad \text{Square each side.}$$
$$x-12=4-4\sqrt{x}+x \quad \text{Find the squares.}$$
$$-16=-4\sqrt{x} \quad \text{Isolate the radical.}$$
$$4=\sqrt{x} \quad \text{Divide each side by } -4.$$
$$16=x \quad \text{Square each side.}$$

CHECK
$$\sqrt{x-12}=2-\sqrt{x}$$
$$\sqrt{16-12}\stackrel{?}{=}2-\sqrt{16}$$
$$\sqrt{4}\stackrel{?}{=}2-4$$
$$2\neq-2 \ \times$$

Example 2 Solve a Cube Root Equation

Solve $2(6x - 3)^{\frac{1}{3}} - 4 = 0$.

Check Your Understanding

Examples 1–2 Solve each equation.

1. $\sqrt{x-4}+6=10$ **20**
2. $\sqrt{x+13}-8=-2$ **23**
3. $8-\sqrt{x+12}=3$ **13**
4. $\sqrt{x-8}+5=7$ **12**
5. $\sqrt[3]{x-2}=3$ **29**
6. $(x-5)^{\frac{1}{3}}-4=-2$ **13**
7. $(4y)^{\frac{1}{3}}+3=5$ **2**
8. $\sqrt[3]{n+8}-6=-3$ **19**
9. $\sqrt{y}-7=0$ **49**
10. $2+4z^{\frac{1}{2}}=0$ **no solution**
11. $5+\sqrt{4y-5}=12$ $\frac{27}{2}$
12. $\sqrt{2t-7}=\sqrt{t+2}$ **9**

13. CCSS **REASONING** The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T=2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared.

13a. about 9.5 seconds

   a. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?

   b. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be? **about 324 ft**

Example 3

14. **MULTIPLE CHOICE** Solve $(2y+6)^{\frac{1}{4}}-2=0$. **B**

   A $y=1$ **B** $y=5$ **C** $y=11$ **D** $y=15$

Examples 1–2 **Solve each equation.**

1. $\sqrt{x-4}+6=10$ **20**
2. $\sqrt{x+13}-8=-2$ **23**
3. $8-\sqrt{x+12}=3$ **13**
4. $\sqrt{x-8}+5=7$ **12**

① $\sqrt{x-4}+6=10$

$-6 \quad -6$

$(\sqrt{x-4})^2=(4)^2$

$x-4=16$

$+4 \quad +4$

$x=20$

Examples 1–2 **Solve each equation.**

1. $\sqrt{x-4}+6=10$ **20**
2. $\sqrt{x+13}-8=-2$ **23**
3. $8-\sqrt{x+12}=3$ **13**
4. $\sqrt{x-8}+5=7$ **12**

2

$$\sqrt{x+13}-8=-2$$
$$+8 \quad +8$$
$$\left(\sqrt{x+13}\right)^2=(6)^2$$
$$x+13=36$$
$$-13 \quad -13$$
$$x=23$$

Examples 1–2 **Solve each equation.**

1. $\sqrt{x-4}+6=10$ 20
2. $\sqrt{x+13}-8=-2$ 23
3. $8-\sqrt{x+12}=3$ 13
4. $\sqrt{x-8}+5=7$ 12

3

$$8-\sqrt{x+12}=3$$
$$-8 \qquad\qquad -8$$
$$-\sqrt{x+12}=-5$$
$$\overline{-1} \qquad \overline{-1}$$
$$\left(\sqrt{x+12}\right)^2=(5)^2$$
$$x+12=25$$
$$x=13$$

5 $\sqrt[3]{x-2}=3$ 29

6. $(x-5)^{\frac{1}{3}}-4=-2$ 13

7. $(4y)^{\frac{1}{3}}+3=5$ 2

8. $\sqrt[3]{n+8}-6=-3$ 19

5 $\left(\sqrt[3]{x-2}\right)^3=(3)^3$

$x-2=27$

$+2 \quad +2$

$x=29$

5. $\sqrt[3]{x-2}=3$ **29**

6. $(x-5)^{\frac{1}{3}}-4=-2$ **13**

7. $(4y)^{\frac{1}{3}}+3=5$ **2**

8. $\sqrt[3]{n+8}-6=-3$ **19**

1

7

$$(4y)^{1/3}+3=5$$
$$-3 \quad -3$$
$$\left((4y)^{1/3}\right)^3=(2)^3$$
$$\frac{4y}{4}=\frac{8}{4}$$
$$y=2$$

Standardized Test Example 3 Solve a Radical Equation

What is the solution of $3\left(\sqrt[4]{2n+6}\right) - 6 = 0$?

A -1 **B** 1 **C** 5 **D** 11

$3\left(\sqrt[4]{2n+6}\right) - 6 = 0$ Original equation

$3\left(\sqrt[4]{2n+6}\right) = 6$ Add 6 to each side.

$\frac{\;}{3}$ $\frac{\;}{3}$

$\sqrt[4]{2n+6} = 2$ Divide each side by 3.

$\left(\sqrt[4]{2n+6}\right)^4 = 2^4$ Raise each side to the fourth power.

$2n + 6 = 16$ Evaluate each side.

$2n = 10$ Subtract 6 from each side.

$n = 5$ The answer is C.

Example 3

14. MULTIPLE CHOICE Solve $(2y+6)^{\frac{1}{4}} - 2 = 0$. **B**

$+2 \; +2$

A $y = 1$ **B** $y = 5$ **C** $y = 11$ **D** $y = 15$

$\left((2y+6)^{1/4}\right)^4 = (2)^4$

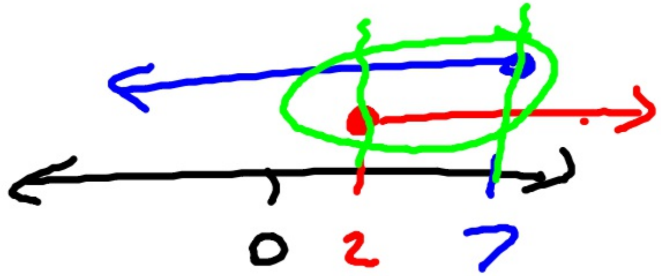
$2y + 6 = 16$

Example 4 Solve a Radical Inequality

Solve $3 + \sqrt{5x - 10} \leq 8$.

Step 1 Since the radicand of a square root must be greater than or equal to zero, first solve $5x - 10 \geq 0$ to identify the values of x for which the left side of the inequality is defined.

$5x - 10 \geq 0$	Set the radicand ≥ 0.
$5x \geq 10$	Add 10 to each side.
$x \geq 2$	Divide each side by 5.



Step 2 Solve $3 + \sqrt{5x - 10} \leq 8$.

$3 + \sqrt{5x - 10} \leq 8$	Original inequality
$\sqrt{5x - 10} \leq 5$	Isolate the radical.
$5x - 10 \leq 25$	Eliminate the radical.
$5x \leq 35$	Add 10 to each side.
$x \leq 7$	Divide each side by 5.

step 3:

$2 \leq x \leq 7$

15

$3x+4 \geq 0$
$\quad -4 \quad -4$
$3x \geq -4$
$\frac{3x}{3} \quad \frac{-4}{3}$

$\frac{-4}{3} \leq x$

$\sqrt{3x+4} - 5 \leq 4$
$\quad +5 \quad +5$
$\sqrt{3x+4} \leq 9$
$3x+4 \leq 81$
$x \leq \frac{77}{3}$

Example 4 **Solve each inequality.**

15. $\sqrt{3x+4} - 5 \leq 4$ $-\frac{4}{3} \leq x \leq \frac{77}{3}$

16. $\sqrt{b-7} + 6 \leq 12$ $7 \leq b \leq 43$

17. $2 + \sqrt{4y-4} \leq 6$

18. $\sqrt{3a+3} - 1 \leq 2$

19. $1 + \sqrt{7x-3} > 3$ $x > 1$

20. $\sqrt{3x+6} + 2 \leq 5$

21. $-2 + \sqrt{9-5x} \geq 6$

22. $6 - \sqrt{2y+1} < 3$

16. $\sqrt{b-7}+6 \le 12$ $7 \le b \le 43$

16

$$b-7 \ge 0$$
$$+7 \quad +7$$
$$b \ge 7$$

$$\sqrt{b-7}+6 \le 12$$
$$-6 \quad -6$$
$$\sqrt{b-7}^{\,2} \le 6^2$$
$$b-7 \le 36$$
$$+7 \quad +7$$
$$\rightarrow b \le 43$$

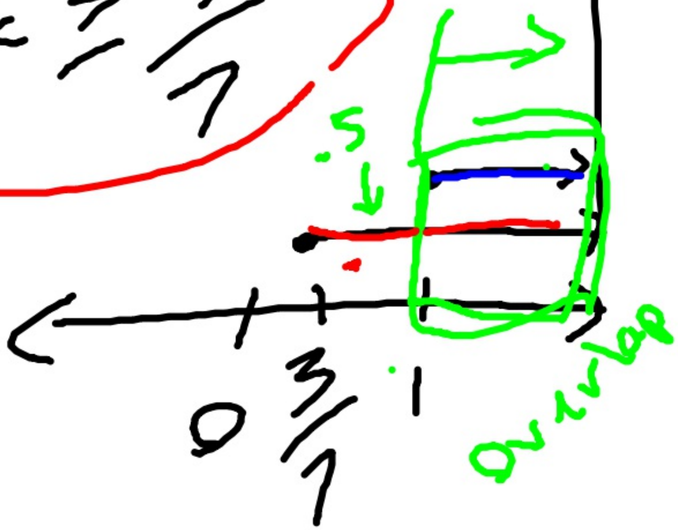
19. $1+\sqrt{7x-3}>3$ $x>1$

$1+\sqrt{\frac{7}{2}-3}$

$1+\sqrt{.5}>3$.

$$7x-3\geq 0$$
$$\underline{\quad +3 \qquad +3}$$
$$7x\geq 3$$
$$\frac{7x}{7}\geq\frac{3}{7}$$
$$x\geq\frac{3}{7}$$

$$1+\sqrt{7x-3}>3$$
$$\underline{-1 \qquad\qquad -1}$$
$$\sqrt{7x-3}>2$$
$$7x-3>4$$
$$\underline{\quad +3 \quad +3}$$
$$7x>7$$
$$\frac{7x}{7}>\frac{7}{7}$$
$$x>1$$



Example 1 **Solve each equation. Confirm by using a graphing calculator.**

23. $\sqrt{2x+5}-4=3$ **22**

24. $6+\sqrt{3x+1}=11$ **8**

25. $\sqrt{x+6}=5-\sqrt{x+1}$ **3**

26. $\sqrt{x-3}=\sqrt{x+4}-1$ **12**

27. $\sqrt{x-15}=3-\sqrt{x}$ **no real solution**

28. $\sqrt{x-10}=1-\sqrt{x}$ **no real solution**

29. $6+\sqrt{4x+8}=9$ $\frac{1}{4}$

30. $2+\sqrt{3y-5}=10$ **23**

31. $\sqrt{x-4}=\sqrt{2x-13}$ **9**

32. $\sqrt{7a-2}=\sqrt{a+3}$ $\frac{5}{6}$

33. $\sqrt{x-5}-\sqrt{x}=-2$ $\frac{81}{16}$

34. $\sqrt{b-6}+\sqrt{b}=3$ $\frac{25}{4}$

35. CCSS **SENSE-MAKING** Isabel accidentally dropped her keys from the top of a Ferris wheel. The formula $t=\frac{1}{4}\sqrt{d-h}$ describes the time t in seconds at which the keys are h meters above the ground and Isabel is d meters above the ground. If Isabel was 65 meters high when she dropped the keys, how many meters above the ground will the keys be after 2 seconds? **1 m**

Example 2 **Solve each equation.**

36. $(5n-6)^{\frac{1}{3}}+3=4$ $\frac{7}{5}$

37. $(5p-7)^{\frac{1}{3}}+3=5$ **3**

38. $(6q+1)^{\frac{1}{4}}+2=5$ $\frac{40}{3}$

39. $(3x+7)^{\frac{1}{4}}-3=1$ **83**

40. $(3y-2)^{\frac{1}{5}}+5=6$ **1**

41. $(4z-1)^{\frac{1}{5}}-1=2$ **61**

42. $2(x-10)^{\frac{1}{3}}+4=0$ **2**

43. $3(x+5)^{\frac{1}{3}}-6=0$ **3**

44. $\sqrt[3]{5x+10}-5=0$ **23**

45. $\sqrt[3]{4n-8}-4=0$ **18**

46. $\frac{1}{7}(14a)^{\frac{1}{3}}=1$ **24.5**

47. $\frac{1}{4}(32b)^{\frac{1}{3}}=1$ **2**

Example 3 **48.** **MULTIPLE CHOICE** Solve $\sqrt[4]{y+2}+9=14$. **D**

A 23 **B** 53 **C** 123 **D** 623

$$5 - \sqrt{x+1}$$

	5	$-\sqrt{x+1}$
5	25	$-5\sqrt{x+1}$
$-\sqrt{x+1}$	$-5\sqrt{x+1}$	$(\sqrt{x+1})^2$

25. $(\sqrt{x+6})^2 = (5 - \sqrt{x+1})^2$ **3**

$$x+6 = 25 - 10\sqrt{x+1} + (\sqrt{x+1})^2$$

$$x+6 = 25 - 10\sqrt{x+1} + x + 1$$

$$-x \qquad\qquad -x$$

$$6 = 26 - 10\sqrt{x+1}$$

$$\frac{10\sqrt{x+1}}{10} = \frac{20}{10}$$

$$\sqrt{x+1} = 2$$

$$x+1 = 4$$

$$-1 \quad -1$$

$$x = 3$$

25. $\left(\sqrt{x+6}\right)^2 = \left(5 - \sqrt{x+1}\right)^2$ 3

$$x+6 = (5-\sqrt{x+1})(5-\sqrt{x+1})$$

$$x+6 = 25 - 5\sqrt{x+1} - 5\sqrt{x+1} + (\sqrt{x+1})^2$$

$$x+6 = 25 - 10\sqrt{x+1} + x + 1.$$

$$x+6 = 26 + x - 10\sqrt{x+1}$$

$$\cancel{x}+6 = 26 \cancel{+x} - 10\sqrt{x+1}$$

$$-26 \quad -26$$

$$\frac{-20}{-10} = \frac{-10\sqrt{x+1}}{-10}$$

$$2 = \sqrt{x+1}$$

$$4 = x+1$$

$$-1 \quad -1$$

$$x = 3$$

56. $-3 \le x < 24$

Example 4 **Solve each inequality.** 53. no real solution 54. $x > 4$ 55. $d > -\frac{3}{4}$ 57. $-\frac{5}{2} \le y \le 2$

50. $1 + \sqrt{5x - 2} > 4$ $x > \frac{11}{5}$ **51.** $\sqrt{2x + 14} - 6 \ge 4$ $x \ge 43$ **52.** $10 - \sqrt{2x + 7} \le 3$ $x \ge 21$

53. $6 + \sqrt{3y + 4} < 6$ **54.** $\sqrt{2x + 5} - \sqrt{9 + x} > 0$ **55.** $\sqrt{d + 3} + \sqrt{d + 7} > 4$

56. $\sqrt{3x + 9} - 2 < 7$ **57.** $\sqrt{2y + 5} + 3 \le 6$ **58.** $-2 + \sqrt{8 - 4z} \ge 8$ $z \le -23$

59. $-3 + \sqrt{6a + 1} > 4$ $a > 8$ **60.** $\sqrt{2} - \sqrt{b + 6} \le -\sqrt{b}$ **61.** $\sqrt{c + 9} - \sqrt{c} > \sqrt{3}$

$0 \le b \le 2$ $0 \le c < 3$