New Vocabulary

- monomial

- constant

1 **Multiply Monomials** A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. It has only one term. In the formula to calculate the horsepower of a car, the term $w\left(\frac{v}{234}\right)^3$ is a monomial.

An expression that involves division by a variable, like $\frac{ab}{c}$, is not a monomial.

A **constant** is a monomial that is a real number. The monomial $3x$ is an example of a *linear expression* since the exponent of x is 1. The monomial $2x^2$ is a *nonlinear expression* since the exponent is a positive number other than 1.

EXAMPLE 1 **Identify Monomials**

Determine whether each expression is a monomial. Explain your reasoning.

A. $17 - c$ **Answer:** No; the expression involves subtraction, so it has more than one term.

B. $8f^2g$ **Answer:** Yes; the expression is the product of a number and two variables.

C. $\dfrac{3}{4}$ **Answer:** Yes; the expression is a constant.

D. $\dfrac{5}{t}$ **Answer:** No; the expression involves division by a variable.

EXAMPLE 1 ✔ **Check Your Progress**

Which expression is a monomial?

A. x^5

B. $3p - 1$

C. $\dfrac{9x}{y}$

D. $\dfrac{c}{d}$

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$$b \cdot b \cdot b = b^{\textcircled{3}} \leftarrow \text{You have 3 factors of "b"}$$

KeyConcept Product of Powers

Words	To multiply two powers that have the same base, add their exponents.
Symbols	For any real number a and any integers m and p, $a^m \cdot a^p = a^{m+p}$.
Examples	$b^3 \cdot b^5 = b^{3+5}$ or b^8 $g^4 \cdot g^6 = g^{4+6}$ or g^{10}

$$\underbrace{b \cdot b \cdot b}_{b^3} \; \underbrace{b \cdot b \cdot b \cdot b \cdot b}_{b^5} = b^8$$

$$1b^3 + 1b^3 =$$
$$2b^3$$

$$b^3 \cdot b^3 = b^6$$

EXAMPLE 2 **Product of Powers**

A. Simplify $(r^4)(-12r^7)$**.**

$(r^4)(-12r^7) = [1 \bullet (-12)](r^4)(r^7)$ Group the coefficients and the variables.

$= [1 \bullet (-12)](r^{4+7})$ Product of Powers

$= -12r^{11}$ Simplify.

EXAMPLE 2 **Product of Powers**

B. Simplify $(6cd^5)(5c^5d^2)$.

$(6cd^5)(5c^5d^2) = (6 \cdot 5)(c \cdot c^5)(d^5 \cdot d^2)$ Group the coefficients and the variables.

$= (6 \cdot 5)(c^{1+5})(d^{5+2})$ Product of Powers

$= 30c^6d^7$ Simplify.

Answer: $30c^6d^7$

KeyConcept Power of a Power

Words   To find the power of a power, multiply the exponents.

Symbols   For any real number a and any integers m and p, $(a^m)^p = a^{m \cdot p}$.

Examples $(b^3)^5 = b^{3 \cdot 5}$ or b^{15} $(g^6)^7 = g^{6 \cdot 7}$ or g^{42}

EXAMPLE 3 Power of a Power

Simplify $[(2^3)^3]^2$.

$[(2^3)^3]^2 = (2^{3 \cdot 3})^2$ Power of a Power

$= (2^9)^2$ Simplify.

$= 2^{9 \cdot 2}$ Power of a Power

$= 2^{18}$ or $262,144$ Simplify.

Answer: 2^{18} or $262,144$

KeyConcept Power of a Product

Words	To find the power of a product, find the power of each factor and multiply.
Symbols	For any real numbers a and b and any integer m, $(ab)^m = a^m b^m$.
Example	$\left(-2xy^3\right)^5 = (-2)^5 x^5 y^{15}$ or $-32x^5 y^{15}$

EXAMPLE 4 **Power of a Product**

GEOMETRY Find the volume of a cube with side length 5*xyz*.

EXAMPLE 5 Simplify Expressions

Simplify $[(8g^3h^4)^2]^2(2gh^5)^4$.

Example 1 Determine whether each expression is a monomial. Write *yes* or *no*. Explain your reasoning. **1–6. See margin.**

1. 15

2. $2 - 3a$

3. $\frac{5c\pi}{d}$

4. $-15g^2$

5. $\frac{r}{2}$

6. $7b + 9$

Examples 2–3 Simplify each expression.

7. $k(k^3)$ k^4

8. $m^4(m^2)$ m^6

9 $2q^2(9q^4)$ $18q^6$

10. $(5u^4v)(7u^4v^3)$ $35u^8v^4$

11. $[(3^2)^2]^2$ 3^8 or 6561

12. $(xy^4)^6$ x^6y^{24}

13. $(4a^4b^9c)^2$ $16a^8b^{18}c^2$
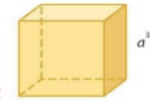
14. $(-2f^2g^3h^2)^3$ $-8f^6g^9h^6$

15. $(-3p^5t^6)^4$ $81p^{20}t^{24}$

Example 4

16. **GEOMETRY** The formula for the surface area of a cube is $SA = 6s^2$, where SA is the surface area and s is the length of any side.

a. Express the surface area of the cube as a monomial. $6a^6b^2$

b. What is the surface area of the cube if $a = 3$ and $b = 4$? $69{,}984$ units^2

a^3b

Example 5 Simplify each expression.

17. $(5x^2y)^2(2xy^3z)^3(4xyz)$ $800x^8y^{12}z^4$

18. $(-3d^2f^3g)^2[(-3d^2f)^3]^2$ $6561d^{16}f^{12}g^2$

19. $(-2g^3h)(-3gj^4)^2(-ghj)^2$ $-18g^7h^3j^{10}$

20. $(-7ab^4c)^3[(2a^2c)^2]^3$ $-21{,}952a^{15}b^{12}c^9$

$(-2g^3h)(9g^2j^8)(g^2h^3j^2)$

⑰ $(25x^4y^2)(8x^3y^9z^3)(4xyz)$

$= 800x^8y^{12}z^4$

Example 1   Determine whether each expression is a monomial. Write *yes* or *no*.
Explain your reasoning. **21–26. See margin.**

21. 122 **22.** $3a^4$ **23.** $2c + 2$

24. $\dfrac{-2g}{4h}$ **25.** $\dfrac{5k}{10}$ **26.** $6m + 3n$

Examples 2–3 Simplify each expression.

(27) $\left(q^2\right)\left(2q^4\right)$ $2q^6$ **28.** $\left(-2u^2\right)\left(6u^6\right)$ $-12u^8$ **29.** $\left(9w^2x^8\right)\left(w^6x^4\right)$ $9w^8x^{12}$

30. $\left(y^6z^9\right)\left(6y^4z^2\right)$ $6y^{10}z^{11}$ **31.** $\left(b^8c^6d^5\right)\left(7b^6c^2d\right)$ $7b^{14}c^8d^6$ **32.** $\left(14fg^2h^2\right)\left(-3f^4g^2h^2\right)$ $-42f^5g^4h^4$

33. $\left(j^5k^7\right)^4$ $j^{20}k^{28}$ **34.** $\left(n^3p\right)^4$ $n^{12}p^4$ **35.** $\left[\left(2^2\right)^2\right]^2$ 2^8 or 256

36. $\left[\left(3^2\right)^2\right]^4$ 3^{16} or $43{,}046{,}721$ **37.** $\left[\left(4r^2t\right)^3\right]^2$ $4096r^{12}t^6$ **38.** $\left[\left(-2xy^2\right)^3\right]^2$ $64x^6y^{12}$

Example 4   **GEOMETRY** Express the area of each triangle as a monomial.

39. $20c^5d^5$ **40.** $3g^3h^6$
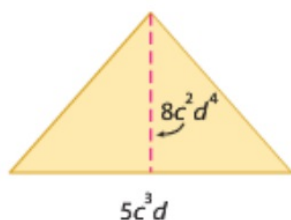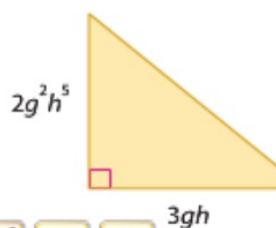
$8c^2d^4$

$5c^3d$

$2g^2h^5$

$3gh$

$$\frac{1}{2}\left(5c^3d\right)\left(8c^2d^4\right)$$

$$\left(A = \frac{1}{2}bh\right)$$

Example 5 Simplify each expression.

41. $(2a^3)^4(a^3)^3$ $\mathbf{16a^{21}}$

42. $(c^3)^2(-3c^5)^2$ $\mathbf{9c^{16}}$

43. $(2gh^4)^3\left[(-2g^4h)^3\right]^2$ $\mathbf{512g^{27}h^{18}}$

44. $(5k^2m)^3\left[(4km^4)^2\right]^2$ $\mathbf{32{,}000k^{10}m^{19}}$

45. $(p^5r^2)^4(-7p^3r^4)^2(6pr^3)$ $\mathbf{294p^{27}r^{19}}$

46. $(5x^2y)^2(2xy^3z)^3(4xyz)$ $\mathbf{800x^8y^{12}z^4}$

47. $(5a^2b^3c^4)(6a^3b^4c^2)$ $\mathbf{30a^5b^7c^6}$

48. $(10xy^5z^3)(3x^4y^6z^3)$ $\mathbf{30x^5y^{11}z^6}$

49. $(0.5x^3)^2$ $\mathbf{0.25x^6}$

50. $(0.4h^5)^3$ $\mathbf{0.064h^{15}}$

51. $\left(-\dfrac{3}{4}c\right)^3$ $\mathbf{-\dfrac{27}{64}c^3}$

52. $\left(\dfrac{4}{5}a^2\right)^2$ $\mathbf{\dfrac{16}{25}a^4}$

53. $(8y^3)(-3x^2y^2)\left(\dfrac{3}{8}xy^4\right)$ $\mathbf{-9x^3y^9}$

54. $\left(\dfrac{4}{7}m\right)^2(49m)(17p)\left(\dfrac{1}{34}p^5\right)$ $\mathbf{8m^3p^6}$

55. $(-3r^3w^4)^3(2rw)^2(-3r^2)^3(4rw^2)^3(2r^2w^3)^4$ $\mathbf{2{,}985{,}984r^{28}w^{32}}$