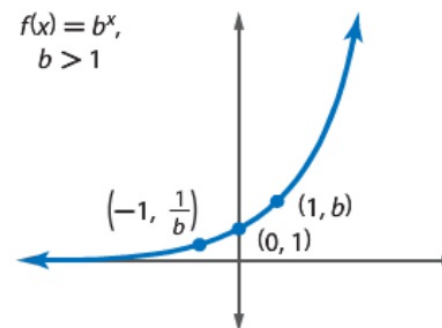


7-1 Graphing Exponential Functions

Key Concept Parent Function of Exponential Growth Functions

Parent Functions:	$f(x) = b^x, b > 1$
Type of graph:	continuous, one-to-one, and increasing
Domain:	all real numbers
Range:	all positive real numbers
Asymptote:	x -axis
Intercept:	$(0, 1)$

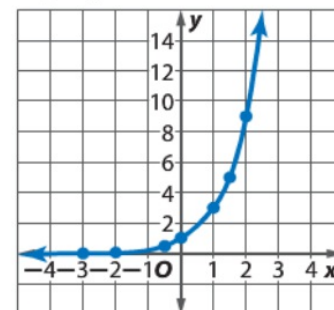


Example 1 Graph Exponential Growth Functions

Graph $y = 3^x$. State the domain and range.

Make a table of values. Then plot the points and sketch the graph.

x	-3	-2	$-\frac{1}{2}$	0
$y = 3^x$	$3^{-3} = \frac{1}{27}$	$3^{-2} = \frac{1}{9}$	$3^{-\frac{1}{2}} = \frac{\sqrt{3}}{3}$	$3^0 = 1$
x	1	$\frac{3}{2}$	2	
$y = 3^x$	$3^1 = 3$	$3^{\frac{3}{2}} = \sqrt{27}$	$3^2 = 9$	



The domain is all real numbers, and the range is all positive real numbers.

KeyConcept Transformations of Exponential Functions

$$f(x) = ab^{x-h} + k$$

h – Horizontal Translation

h units right if h is positive
 $|h|$ units left if h is negative

k – Vertical Translation

k units up if k is positive
 $|k|$ units down if k is negative

a – Orientation and Shape

If $a < 0$, the graph is reflected in the x -axis.

If $|a| > 1$, the graph is stretched vertically.
If $0 < |a| < 1$, the graph is compressed vertically.

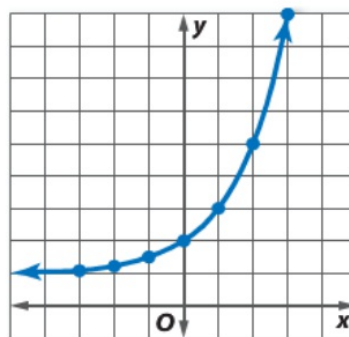
Example 2 Graph Transformations

Graph each function. State the domain and range.

a. $y = 2^x + 1$

The equation represents a translation of the graph of $y = 2^x$ one unit up.

x	$y = 2^x + 1$
-3	$2^{-3} + 1 = 1.125$
-2	$2^{-2} + 1 = 1.25$
-1	$2^{-1} + 1 = 1.5$
0	$2^0 + 1 = 2$
1	$2^1 + 1 = 3$
2	$2^2 + 1 = 5$
3	$2^3 + 1 = 9$



Domain = {all real numbers}; Range = $\{y \mid y > 1\}$

 **KeyConcept** Transformations of Exponential Functions

$$f(x) = ab^{x-h} + k$$

h – Horizontal Translation

h units right if h is positive
 $|h|$ units left if h is negative

k – Vertical Translation

k units up if k is positive
 $|k|$ units down if k is negative

a – Orientation and Shape

If $a < 0$, the graph is reflected in the x -axis.

If $|a| > 1$, the graph is stretched vertically.
If $0 < |a| < 1$, the graph is compressed vertically.

b. $y = -\frac{1}{2} \cdot 5^{x-2}$



KeyConcept Transformations of Exponential Functions

$$f(x) = ab^{x-h} + k$$

h – Horizontal Translation

h units right if h is positive
 $|h|$ units left if h is negative

k – Vertical Translation

k units up if k is positive
 $|k|$ units down if k is negative

a – Orientation and Shape

If $a < 0$, the graph is reflected in the x -axis.

If $|a| > 1$, the graph is stretched vertically.
 If $0 < |a| < 1$, the graph is compressed vertically.

Examples 1–2 Graph each function. State the domain and range. **1–6.** See Chapter 7 Answer Appendix.

1. $f(x) = 2^x$

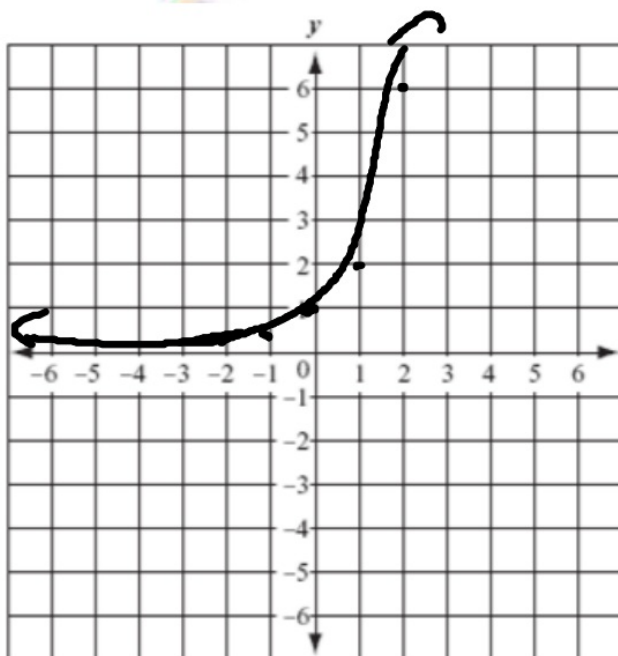
2. $f(x) = 5^x$

3. $f(x) = 3^{x-2} + 4$

4. $f(x) = 2^{x+1} + 3$

5. $f(x) = 0.25(4)^x - 6$

6. $f(x) = 3(2)^x + 8$



① $f(x) = 2^x$

$$\begin{array}{r} x & y \\ \hline -2 & 1/4 \\ -1 & 1/2 \\ 0 & 1 \\ 1 & 2 \\ 2 & 4 \end{array} = 1$$

Examples 1-2 Graph each function. State the domain and range. **1-6. See Chapter 7 Answer Appendix.**

1. $f(x) = 2^x$

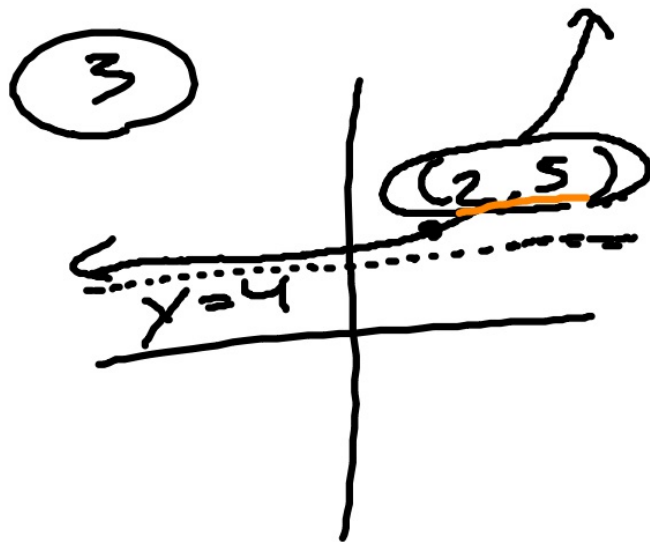
2. $f(x) = 5^x$

3 $f(x) = 3^{x-2} + 4$

4. $f(x) = 2^{x+1} + 3$

5. $f(x) = 0.25(4)^x - 6$

6. $f(x) = 3(2)^x + 8$



x	y
0	$3^{-2} + 4 = 4\frac{1}{9}$
1	$3^{-1} + 4 = 4\frac{1}{3}$
2	$3^0 + 4 = 5$
3	$3^1 + 4 = 7$
4	$3^2 + 4 = 13$

← "center"

Examples 1-2 Graph each function. State the domain and range. **1-6. See Chapter 7 Answer Appendix.**

1. $f(x) = 2^x$

2. $f(x) = 5^x$

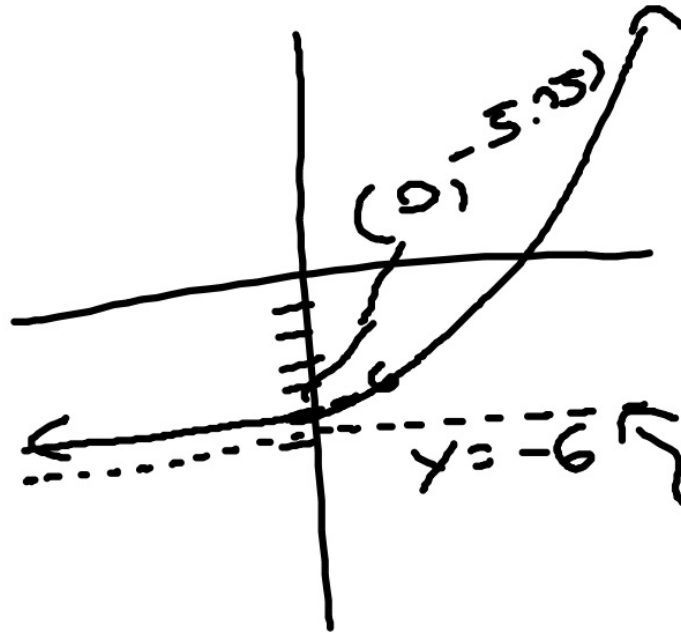
3. $f(x) = 3^{x-2} + 4$

4. $f(x) = 2^{x+1} + 3$

5. $f(x) = 0.25(4)^x - 6$

6. $f(x) = 3(2)^x + 8$

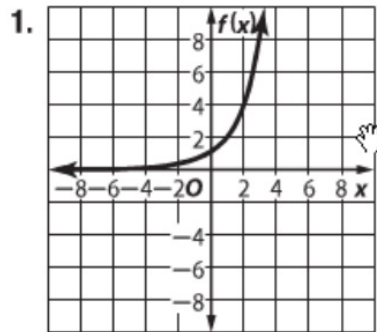
5



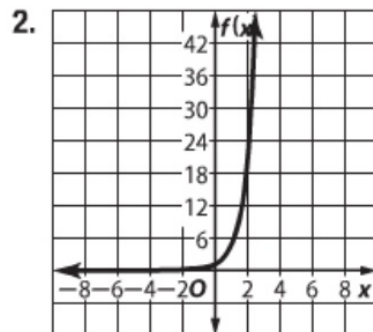
x	y
-2	$.25(4^{-2}) - 6 = -5.98$
-1	$.25(4^{-1}) - 6 = -5.93$
0	$.25(4^0) - 6 = -5.75$
1	$.25(4^1) - 6 = -5$
2	$.25(4^2) - 6 = -2$

Asymptote

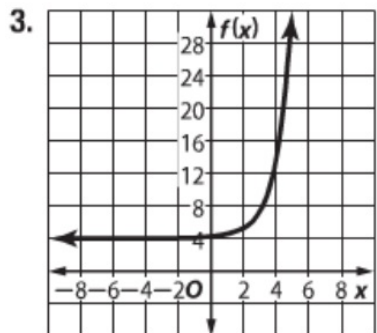
Lesson 7-1



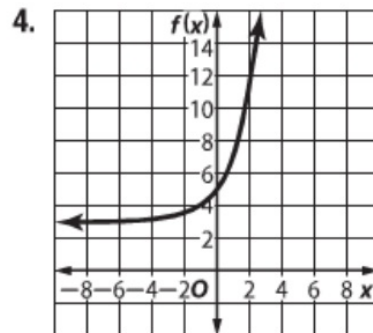
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 0\}$



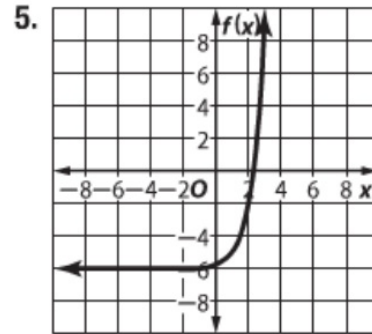
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 0\}$



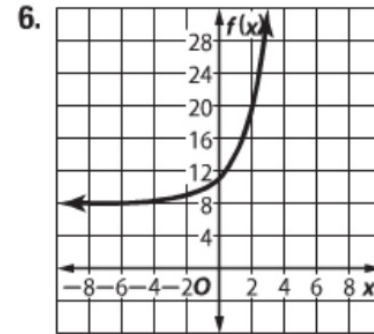
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 4\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 3\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > -6\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 8\}$

Handwritten notes:

- $\leftarrow -3$ (with an arrow pointing left)
- $1e^{fx}$ (circled)
- $x + 3$
- 2
- $2^x + 3$
- \leftarrow (with an arrow pointing left)
- 2 (circled)
- 3 (circled)
- 5 (circled)
- 2 (circled)

This is why investments with compound interest make more money.

10, 250] scl: 25 by [0, 400,000,000] scl: 40,000,000

$$a = 1$$

$$r = .25$$

Example 3

7. **CCSS SENSE-MAKING** A virus spreads through a network of computers such that each minute, 25% more computers are infected. If the virus began at only one computer, graph the function for the first hour of the spread of the virus. **See margin.**

$$y = a(1+r)^t$$

$$y = (1.25)^t$$



KeyConcept Parent Function of Exponential Decay Functions



Parent Functions: $f(x) = b^x, 0 < b < 1$

Model

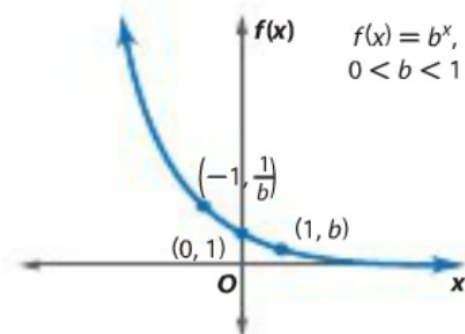
Type of graph: continuous, one-to-one, and decreasing

Domain: all real numbers

Range: positive real numbers

Asymptote: x -axis

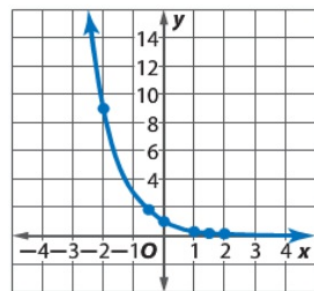
Intercept: $(0, 1)$



Graph each function. State the domain and range.

a. $y = \left(\frac{1}{3}\right)^x$

x	$y = \left(\frac{1}{3}\right)^x$
-3	$\left(\frac{1}{3}\right)^{-3} = 27$
-2	$\left(\frac{1}{3}\right)^{-2} = 9$
$-\frac{1}{2}$	$\left(\frac{1}{3}\right)^{-\frac{1}{2}} = \sqrt{3}$
0	$\left(\frac{1}{3}\right)^0 = 1$
1	$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$
$\frac{3}{2}$	$\left(\frac{1}{3}\right)^{\frac{3}{2}} = \sqrt{\frac{1}{27}}$
2	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$



The domain is all real numbers, and the range is all positive real numbers.

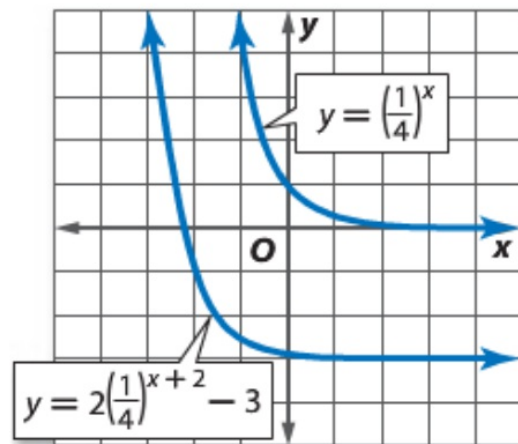
b. $y = 2\left(\frac{1}{4}\right)^{x+2} - 3$

The equation represents a transformation of the graph of $y = \left(\frac{1}{4}\right)^x$.

Examine each parameter.

- $a = 2$: The graph is stretched vertically.
- $h = -2$: The graph is translated 2 units left.
- $k = -3$: The graph is translated 3 units down.

5 · 3²



The domain is all real numbers, and the range is all real numbers greater than

Example 4

Graph each function. State the domain and range. **8-11. See Chapter 7 Answer Appendix.**

8. $f(x) = 2\left(\frac{2}{3}\right)^{x-3} - 4$

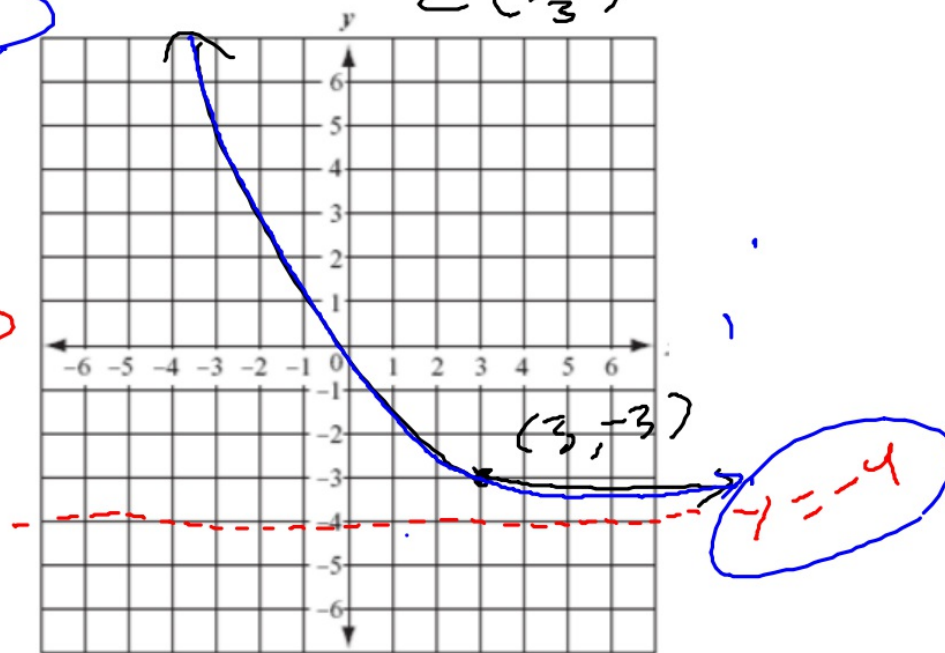
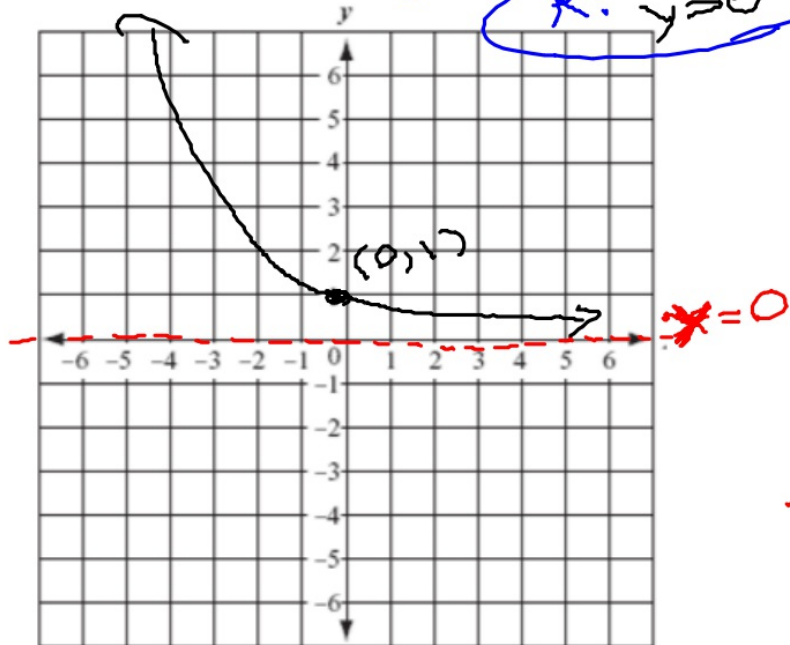
9. $f(x) = -\frac{1}{2}\left(\frac{3}{4}\right)^{x+1} + 5$

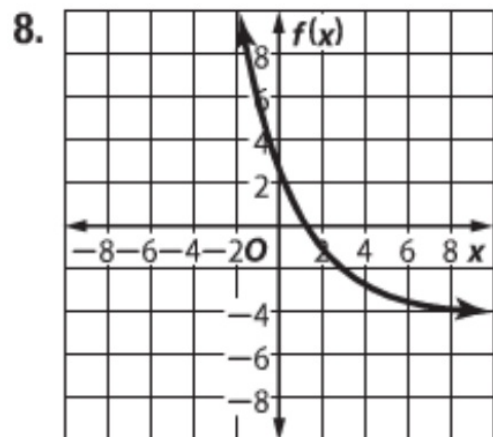
10. $f(x) = -\frac{1}{3}\left(\frac{4}{5}\right)^{x-4} + 3$

11. $f(x) = \frac{1}{8}\left(\frac{1}{4}\right)^{x+6} + 7$

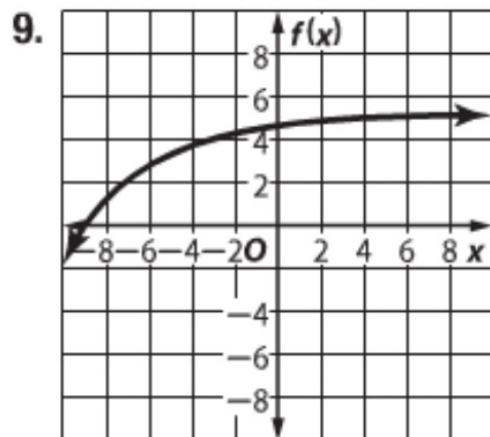
$f(x) = 2\left(\frac{2}{3}\right)^{x-3} - 4$
D: All real #'s
R: $y > -4$

$f(x) = -\frac{1}{2}\left(\frac{3}{4}\right)^{x+1} + 5$
D: All real #'s
R: $y < 5$

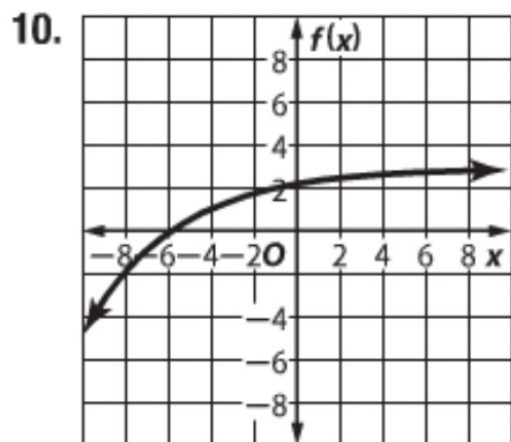




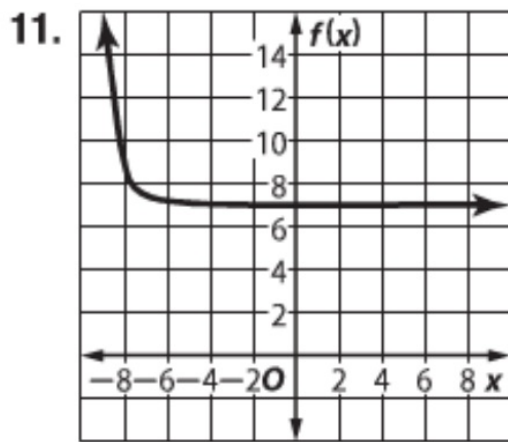
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > -4\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) < 5\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) < 3\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 7\}$

Real-World Example 5 Graph Exponential Decay Functions

TEA A cup of green tea contains 35 milligrams of caffeine. The average teen can eliminate approximately 12.5% of the caffeine from their system per hour.

- a. Draw a graph to represent the amount of caffeine remaining after drinking a cup of green tea.



- b. Estimate the amount of caffeine in a teenager's body 3 hours after drinking a cup of green tea.



Example 5

12. **FINANCIAL LITERACY** A new SUV depreciates in value each year by a factor of 15%. Draw a graph of the SUV's value for the first 20 years after the initial purchase.
See margin.



Examples 1–2 Graph each function. State the domain and range. **13–18. See Chapter 7 Answer Appendix.**

13. $f(x) = 2(3)^x$

14. $f(x) = -2(4)^x$

15. $f(x) = 4^{x+1} - 5$

16. $f(x) = 3^{2x} + 1$

17. $f(x) = -0.4(3)^{x+2} + 4$

18. $f(x) = 1.5(2)^x + 6$

Example 3

19 SCIENCE The population of a colony of beetles grows 30% each week for 10 weeks. If the initial population is 65 beetles, graph the function that represents the situation.

See Chapter 7 Answer Appendix.

Example 4

Graph each function. State the domain and range. **20–25. See Chapter 7 Answer Appendix.**

20. $f(x) = -4\left(\frac{3}{5}\right)^{x+4} + 3$

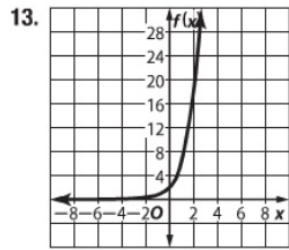
21. $f(x) = 3\left(\frac{2}{5}\right)^{x-3} - 6$

22. $f(x) = \frac{1}{2}\left(\frac{1}{5}\right)^{x+5} + 8$

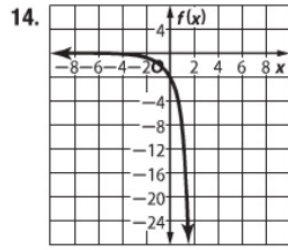
23. $f(x) = \frac{3}{4}\left(\frac{2}{3}\right)^{x+4} - 2$

24. $f(x) = -\frac{1}{2}\left(\frac{3}{8}\right)^{x+2} + 9$

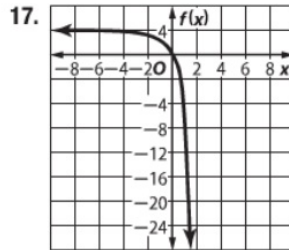
25. $f(x) = -\frac{5}{4}\left(\frac{4}{5}\right)^{x+4} + 2$



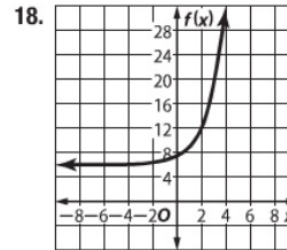
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 0\}$



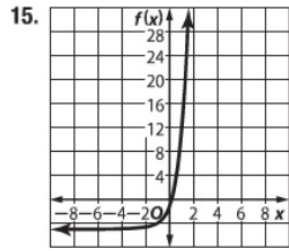
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) < 0\}$



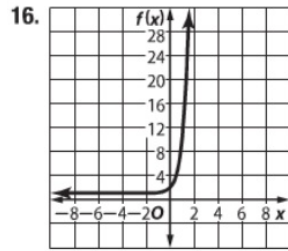
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) < 4\}$



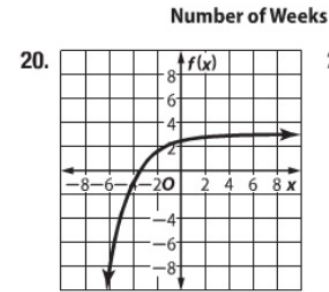
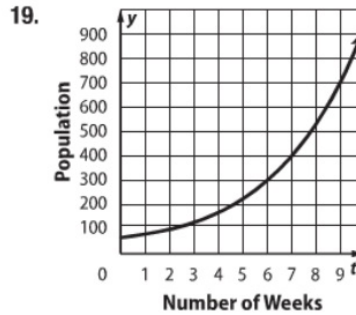
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 6\}$



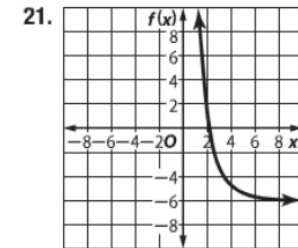
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > -5\}$



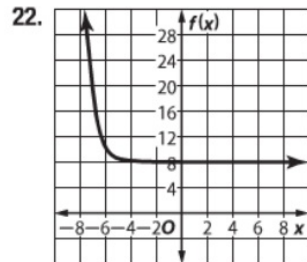
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 1\}$



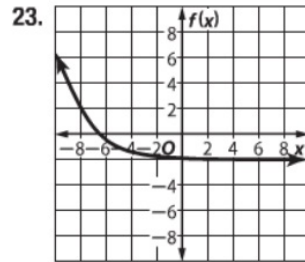
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) < 3\}$



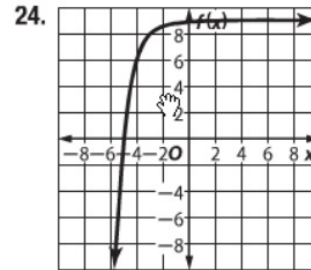
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > -6\}$



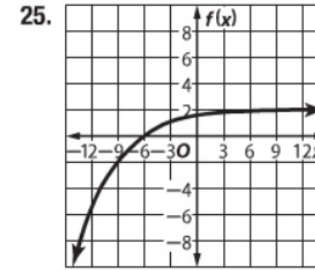
$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > 8\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) > -2\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) < 9\}$



$D = \{\text{all real numbers}\};$
 $R = \{f(x) \mid f(x) < 2\}$

