$2(2x-1)$

7-2 Solving Exponential Equations and Inequalities

KeyConcept Property of Equality for Exponential Functions

Words	Let $b > 0$ and $b \neq 1$. Then $b^x = b^y$ if and only if $x = y$.
Example	If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$.

Example 1 Solve Exponential Equations

Solve each equation.

a. $2^x = 8^3$

$2^x = 8^3$	Original equation
$2^x = (2^3)^3$	Rewrite 8 as 2^3.
$2^x = 2^9$	Power of a Power
$x = 9$	Property of Equality for Exponential Functions

b. $9^{2x-1} = 3^{6x}$

$9^{2x-1} = 3^{6x}$	Original equation
$(3^2)^{2x-1} = 3^{6x}$	Rewrite 9 as 3^2.
$3^{4x-2} = 3^{6x}$	Power of a Power
$4x - 2 = 6x$	Property of Equality for Exponential Functions
$-2 = 2x$	Subtract $4x$ from each side.
$-1 = x$	Divide each side by 2.

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Example 1 Solve each equation.

1. $3^{5x} = 27^{2x-4}$ 12
2. $16^{2y-3} = 4^{y+1}$ $\frac{7}{3}$
3. $2^{6x} = 32^{x-2}$ −10
4. $49^{x+5} = 7^{8x-6}$ $\frac{8}{3}$

(1)

$$3^{5x} = (3^3)^{2x-4}$$

$$3^{5x} = 3^{6x-12}$$

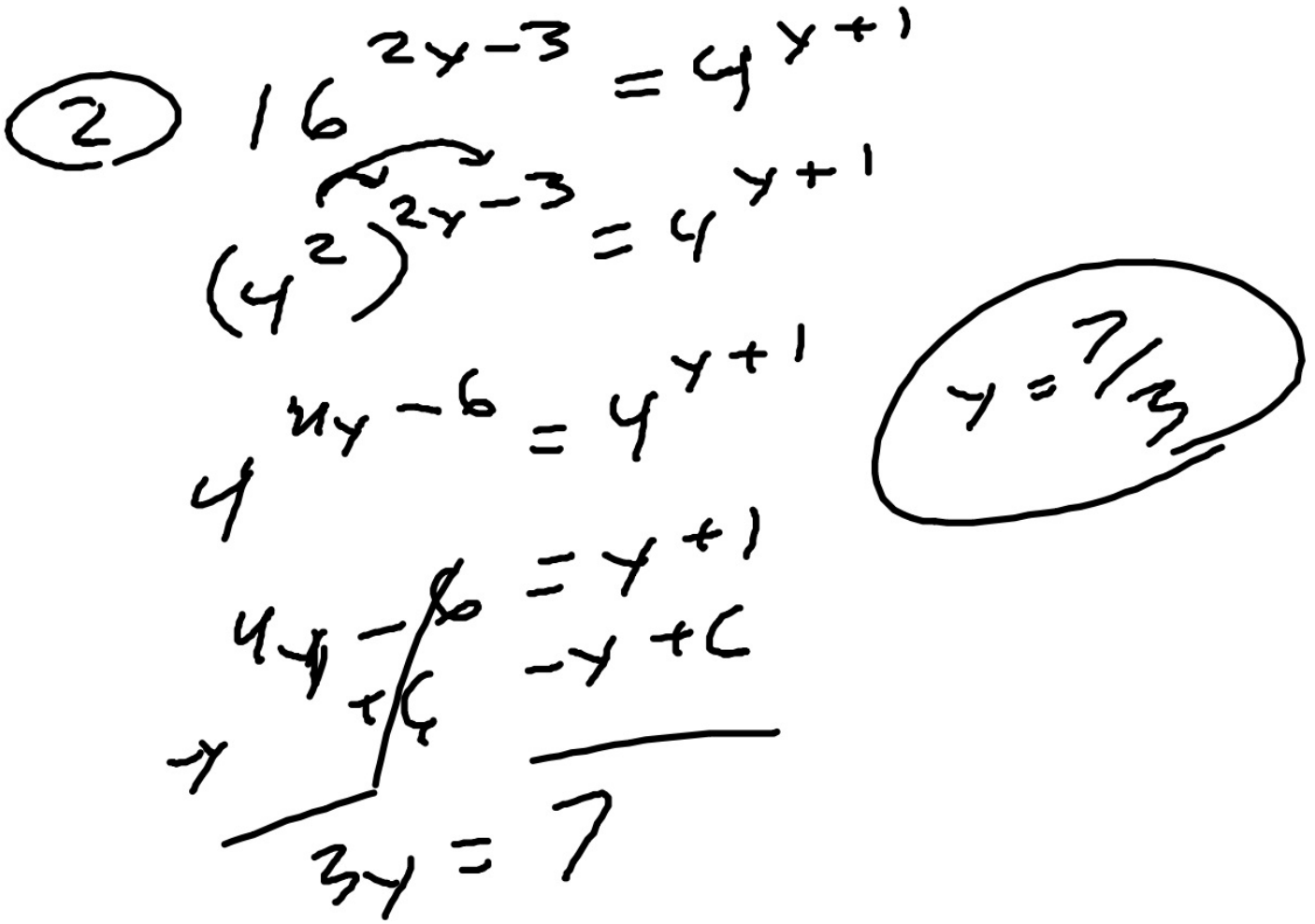
$$5x = 6x - 12$$

$$-6x \quad -6x$$

$$-x = -12$$

Example 1 Solve each equation.

1. $3^{5x} = 27^{2x-4}$ **12**
2. $16^{2y-3} = 4^{y+1}$ $\frac{7}{3}$ **$\frac{8}{3}$**
3. $2^{6x} = 32^{x-2}$ **−10**
4. $49^{x+5} = 7^{8x-6}$



Real-World Example 2 Write an Exponential Function

SCIENCE **Kristin starts an experiment with 7500 bacteria cells. After 4 hours, there are 23,000 cells.**

a. Write an exponential function that could be used to model the number of bacteria after x hours if the number of bacteria changes at the same rate.

At the beginning of the experiment, the time is 0 hours and there are 7500 bacteria cells. Thus, the y-intercept, and the value of a, is 7500.

When $x = 4$, the number of bacteria cells is 23,000. Substitute these values into an exponential function to determine the value of b.

$y = ab^x$	Exponential function
$23{,}000 = 7500 \cdot b^4$	Replace x with 4, y with 23,000, and a with 7500.
$3.067 \approx b^4$	Divide each side by 7500.
$\sqrt[4]{3.067} \approx b$	Take the 4th root of each side.
$1.323 \approx b$	Use a calculator.

An equation that models the number of bacteria is $y \approx 7500(1.323)^x$.

b. How many bacteria cells can be expected in the sample after 12 hours?

$y \approx 7500(1.323)^x$	Modeling equation
$\approx 7500(1.323)^{12}$	Replace x with 12.
$\approx 215{,}665$	Use a calculator.

Example 2

5a. $c = 2^{\frac{t}{15}}$

5b. 16 cells

5. **SCIENCE** Mitosis is a process in which one cell divides into two. The *Escherichia coli* is one of the fastest growing bacteria. It can reproduce itself in 15 minutes.

   a. Write an exponential function to represent the number of cells c after t minutes.

   b. If you begin with one *Escherichia coli* cell, how many cells will there be in one hour?

Exponential functions are used in situations involving compound interest. **Compound interest** is interest paid on the principal of an investment and any previously earned interest.

KeyConcept Compound Interest

You can calculate compound interest using the following formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

where A is the amount in the account after t years, P is the principal amount invested, r is the annual interest rate, and n is the number of compounding periods each year.

Example 3 Compound Interest

An investment account pays 4.2% annual interest compounded monthly. If $2500 is invested in this account, what will be the balance after 15 years?

GuidedPractice

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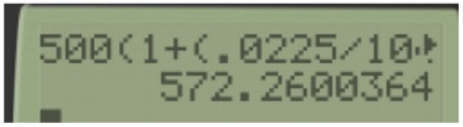
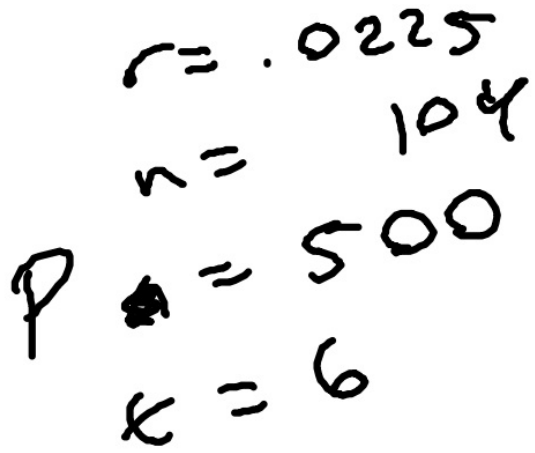
Example 3

6. A certificate of deposit (CD) pays 2.25% annual interest compounded biweekly. If you deposit \$500 into this CD, what will the balance be after 6 years? **\$572.23**





KeyConcept Property of Inequality for Exponential Functions

Words	Let $b > 1$. Then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.
Example	If $2^x > 2^6$, then $x > 6$. If $x > 6$, then $2^x > 2^6$.

This property also holds true for $\leq$ and $\geq$.

Example 4 Solve Exponential Inequalities

Solve $16^{2x-3} < 8$.

Example 4 **Solve each inequality.**

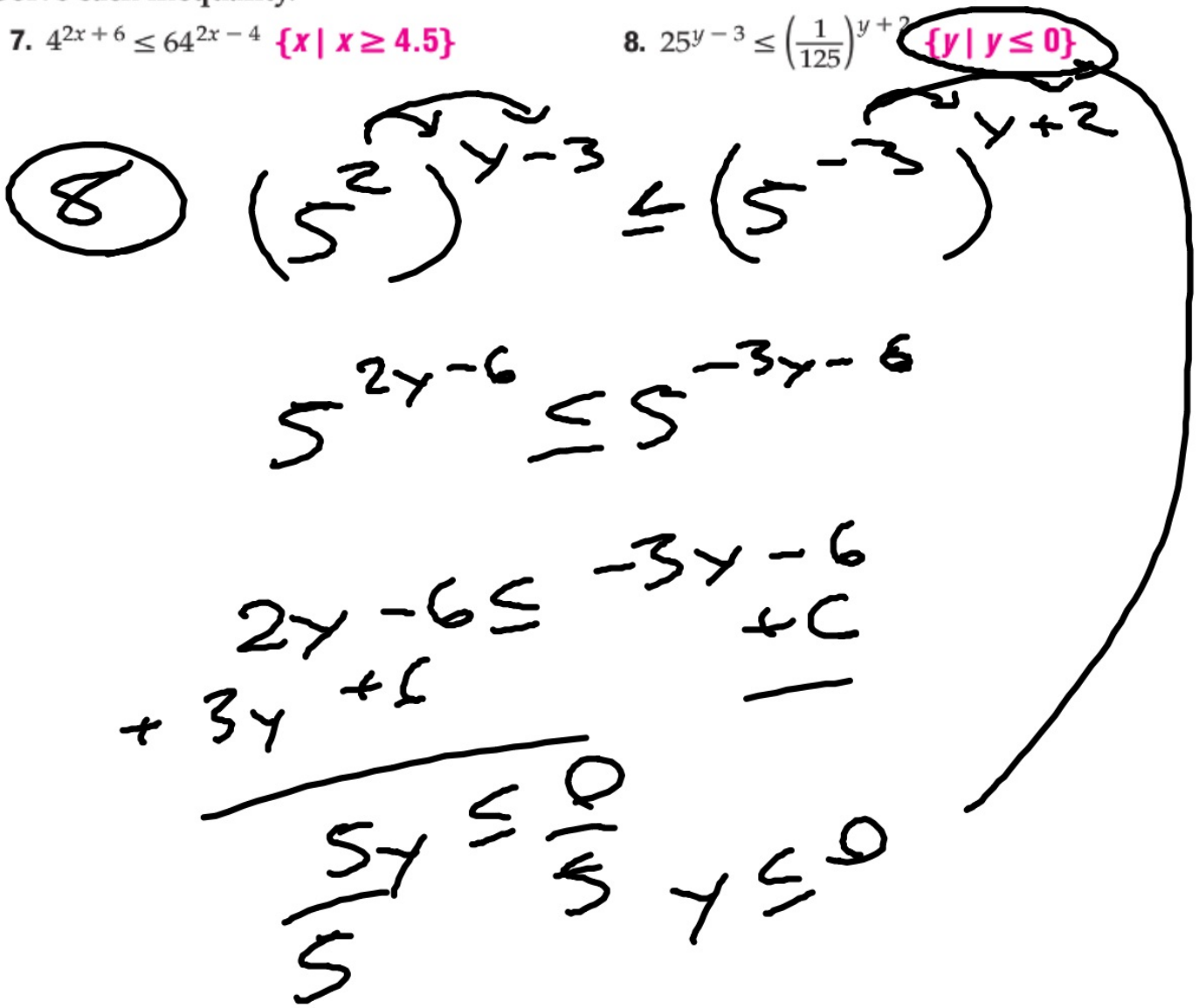
7. $4^{2x+6} \leq 64^{2x-4}$

8. $25^{y-3} \leq \left(\frac{1}{125}\right)^{y+2}$

Example 4 Solve each inequality.

7. $4^{2x+6} \leq 64^{2x-4}$ {x | x ≥ 4.5}

8. $25^{y-3} \leq \left(\frac{1}{125}\right)^{y+2}$ {y | y ≤ 0}



Example 1 **Solve each equation.**

9. $8^{4x+2} = 64$ **0**

10. $5^{x-6} = 125$ **9**

11 $81^{a+2} = 3^{3a+1}$ **−7**

12. $256^{b+2} = 4^{2-2b}$ **−1**

13. $9^{3c+1} = 27^{3c-1}$ $\frac{5}{3}$

14. $8^{2y+4} = 16^{y+1}$ **−4**

Example 2

15. CCSS **MODELING** In 2009, My-Lien received $10,000 from her grandmother. Her parents invested all of the money, and by 2021, the amount will have grown to $16,960.

a. Write an exponential function that could be used to model the money y. Write the function in terms of x, the number of years since 2009. $y = 10{,}000(1.045)^x$

b. Assume that the amount of money continues to grow at the same rate. What would be the balance in the account in 2031? **about $26,336.52**

Write an exponential function for the graph that passes through the given points.

16. (0, 6.4) and (3, 100) $y = 6.4(2.5)^x$

17. (0, 256) and (4, 81) $y = 256(0.75)^x$

18. (0, 128) and (5, 371,293) $y = 128(4.926)^x$

19. (0, 144), and (4, 21,609) $y = 144(3.5)^x$

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$y = a \cdot b^x$

$b^0 = 1$

(16) $6.4 = a \cdot b^0$

$6.4 = a$

$100 = a \cdot b^3$

$\frac{100}{6.4} = \frac{6.4\,b^3}{6.4}$

$\sqrt[3]{15.625} = \sqrt[3]{b^3}$

15.625^(1/3) = 2.5

$2.5 = b$

Example 3

20. Find the balance of an account after 7 years if \$700 is deposited into an account paying 4.3% interest compounded monthly. **\$945.34**

21. Determine how much is in a retirement account after 20 years if \$5000 was invested at 6.05% interest compounded weekly. **\$16,755.63**

22. A savings account offers 0.7% interest compounded bimonthly. If \$110 is deposited in this account, what will the balance be after 15 years? **\$122.17**

23. A college savings account pays 13.2% annual interest compounded semiannually. What is the balance of an account after 12 years if \$21,000 was initially deposited? **\$97,362.61**

Example 4 **Solve each inequality.**

24. $625 \geq 5^{a+8}$ $\{a \mid a \leq -4\}$

25. $10^{5b+2} > 1000$ $\left\{b \mid b > \frac{1}{5}\right\}$

26. $\left(\frac{1}{64}\right)^{c-2} < 32^{2c}$ $\left\{c \mid c > \frac{3}{4}\right\}$

27. $\left(\frac{1}{27}\right)^{2d-2} \leq 81^{d+4}$ $\{d \mid d \geq -1\}$

28. $\left(\frac{1}{9}\right)^{3t+5} \geq \left(\frac{1}{243}\right)^{t-6}$ $\{t \mid t \leq -40\}$

29. $\left(\frac{1}{36}\right)^{w+2} < \left(\frac{1}{216}\right)^{4w}$ $\left\{w \mid w < \frac{2}{5}\right\}$