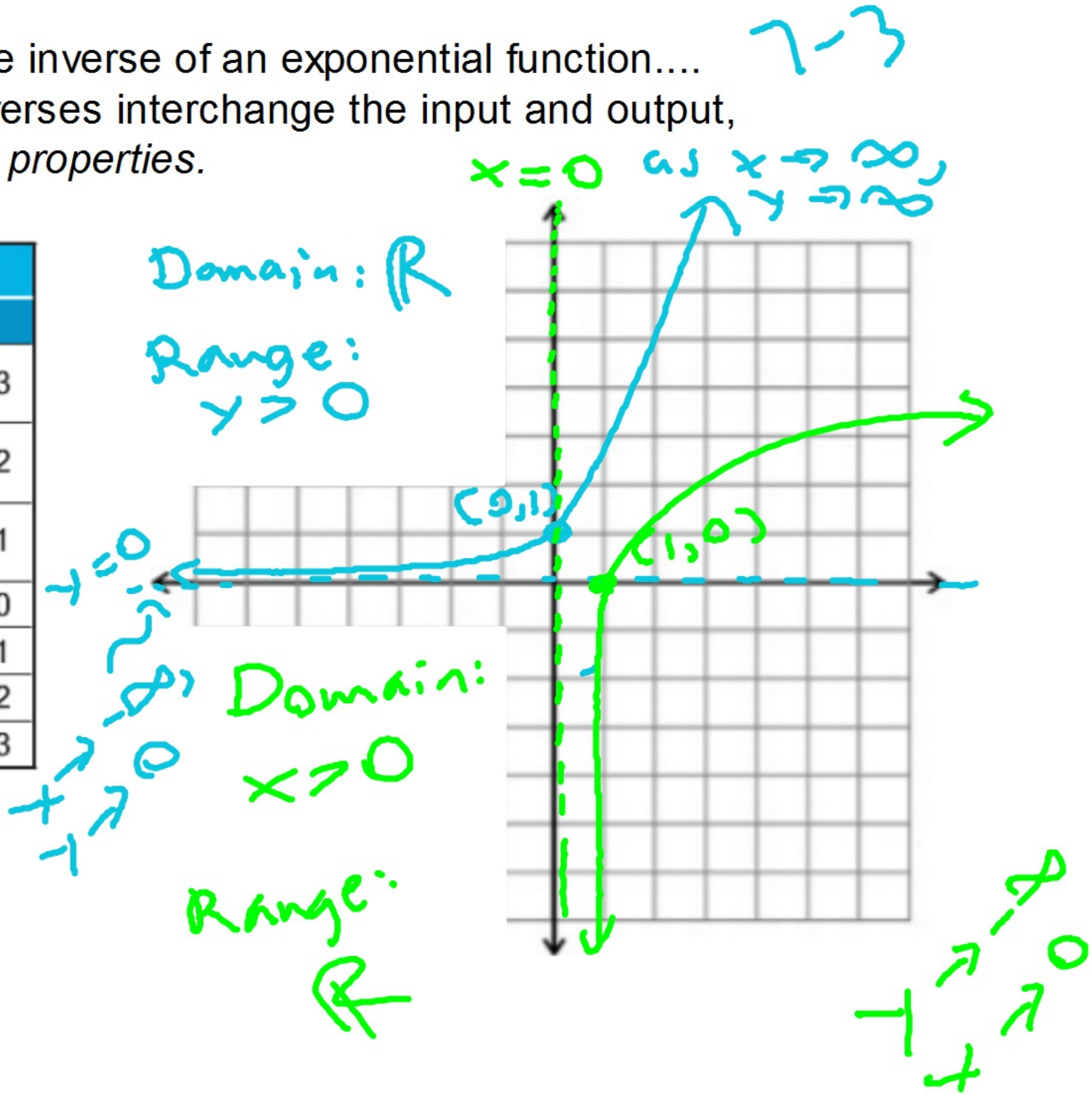
Let's graph the inverse of an exponential function....
remember, inverses interchange the input and output,
including their properties.

$y = 2^x$	
x	y
−3	$\frac{1}{8}$
−2	$\frac{1}{4}$
−1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$x = 2^y$	
x	y
$\frac{1}{8}$	−3
$\frac{1}{4}$	−2
$\frac{1}{2}$	−1
1	0
2	1
4	2
8	3

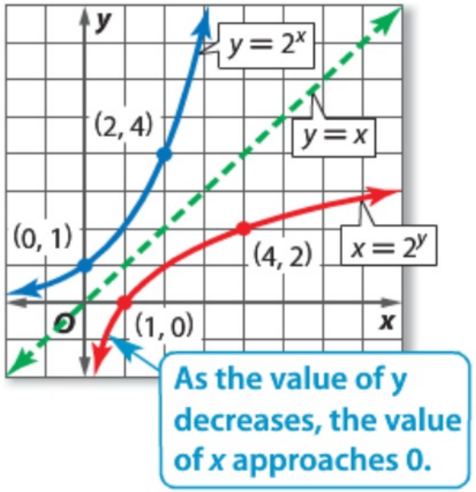


7-3 Logarithms and Logarithmic Functions

1 Logarithmic Functions and Expressions Consider the exponential function $f(x) = 2^x$ and its inverse. Recall that you can graph an inverse function by interchanging the x- and y-values in the ordered pairs of the function.

$y = 2^x$	
x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$x = 2^y$	
x	y
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



The inverse of $y = 2^x$ can be defined as $x = 2^y$. In general, the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, the variable y is called the **logarithm** of x. This is usually written as $y = \log_b x$, which is read *y equals log base b of x*.

KeyConcept Logarithm with Base b

Words	Let b and x be positive numbers, $b \neq 1$. The *logarithm of x with base b* is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.
Symbols	Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.
Example	If $\log_3 27 = y$, then $3^y = 27$.

Example 1 Logarithmic to Exponential Form

Write each equation in exponential form.

a. $\log_2 8 = 3$

$\log_2 8 = 3 \rightarrow 8 = 2^3$

b. $\log_4 \frac{1}{256} = -4$

$\log_4 \frac{1}{256} = -4 \rightarrow \frac{1}{256} = 4^{-4}$

Example 1 Write each equation in exponential form.

1. $\log_8 512 = 3$ $8^3 = 512$

2. $\log_5 625 = 4$ $5^4 = 625$

KeyConcept Logarithm with Base *b*

Words Let b and x be positive numbers, $b \neq 1$. The *logarithm of x with base b* is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.

Symbols Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that

$$\log_b x = y \text{ if and only if } b^y = x.$$

Example If $\log_3 27 = y$, then $3^y = 27$.

Example 2 Exponential to Logarithmic Form

Write each equation in logarithmic form.

a. $15^3 = 3375$ $\log_{15} 3375 = 3$

b. $4^{\frac{1}{2}} = 2$ $\log_4 2 = \frac{1}{2}$

Example 2 Write each equation in logarithmic form.

3. $11^3 = 1331$ $\log_{11} 1331 = 3$

4. $16^{\frac{3}{4}} = 8$ $\log_{16} 8 = \frac{3}{4}$

KeyConcept Logarithm with Base *b*

Words Let b and x be positive numbers, $b \neq 1$. The *logarithm of x with base b* is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.

Symbols Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that

$$\log_b x = y \text{ if and only if } b^y = x.$$

Example If $\log_3 27 = y$, then $3^y = 27$.

Example 3 Evaluate Logarithmic Expressions

Evaluate $\log_{16} 4$.

(5) $\log_{13} 169 = x$

$13^x = 169$

$13^x = 13^2$

Example 3 **Evaluate each expression.**

5. $\log_{13} 169$ **2**

6. $\log_2 \frac{1}{128}$ ☐

7. $\log_6 1$ ☐

6. $\log_2 \frac{1}{128}$ **−7**

$$\log_2 \frac{1}{128} = x$$

$$2^x = \frac{1}{128}$$

$$2^x = \frac{1}{2^7}$$

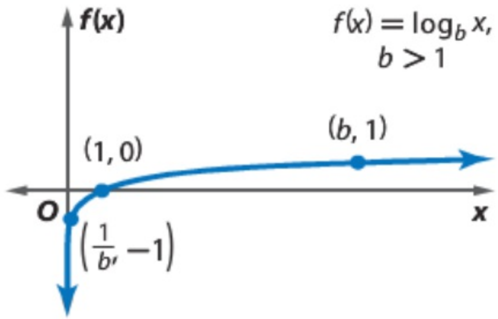
$$2^{x} = 2^{-7}$$

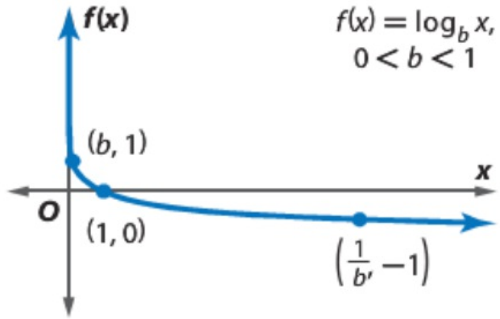
2 Graphing Logarithmic Functions

The function $y = \log_b x$, where $b \neq 1$, is called a **logarithmic function**. The graph of $f(x) = \log_b x$ represents a parent graph of the logarithmic functions.

KeyConcept Parent Function of Logarithmic Functions

Parent function:	$f(x) = \log_b x$	**Type of graph:**	continuous, one-to-one
Domain:	all positive real numbers	**Range:**	all real numbers
Asymptote:	$f(x)$-axis	**Intercept:**	(1, 0)





Example 4 Graph Logarithmic Functions

Graph each function.

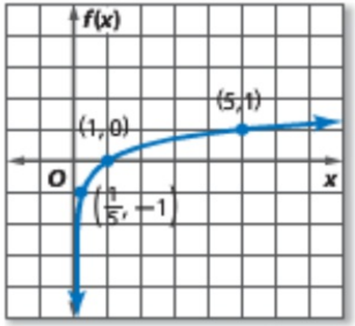
a. $f(x) = \log_5 x$

Step 1 Identify the base.

$b = 5$

Step 2 Determine points on the graph.

Because $5 > 1$, use the points $\left(\frac{1}{b}, -1\right)$, $(1, 0)$, and $(b, 1)$.



Step 3 Plot the points and sketch the graph.

$\left(\frac{1}{b}, -1\right) \rightarrow \left(\frac{1}{5}, -1\right)$

$(1, 0)$

$(b, 1) \rightarrow (5, 1)$

b. $f(x) = \log_{\frac{1}{3}} x$

The same techniques used to transform the graphs of other functions you have studied can be applied to the graphs of logarithmic functions.

KeyConcept Transformations of Logarithmic Functions

$$f(x) = a \log_b (x - h) + k$$

h – Horizontal Translation	k – Vertical Translation
h units right if h is positive $\lvert h \rvert$ units left if h is negative	k units up if k is positive $\lvert k \rvert$ units down if k is negative

a – Orientation and Shape

If $a < 0$, the graph is reflected across the x-axis.	If $\lvert a \rvert > 1$, the graph is stretched vertically. If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

Example 5 Graph Logarithmic Functions

Graph each function.

a. $f(x) = 3 \log_{10} x + 1$

This represents a transformation of the graph of $f(x) = \log_{10} x$.

- $\lvert a \rvert = 3$: The graph stretches vertically
- $h = 0$: There is no horizontal shift.
- $k = 1$: The graph is translated 1 unit up.

b. $f(x) = \frac{1}{2} \log_{\frac{1}{4}} (x - 3)$

This is a transformation of the graph of $f(x) = \log_{\frac{1}{4}} x$.

- $\lvert a \rvert = \frac{1}{2}$: The graph is compressed vertically.
- $h = 3$: The graph is translated 3 units to the right.
- $k = 0$: There is no vertical shift.

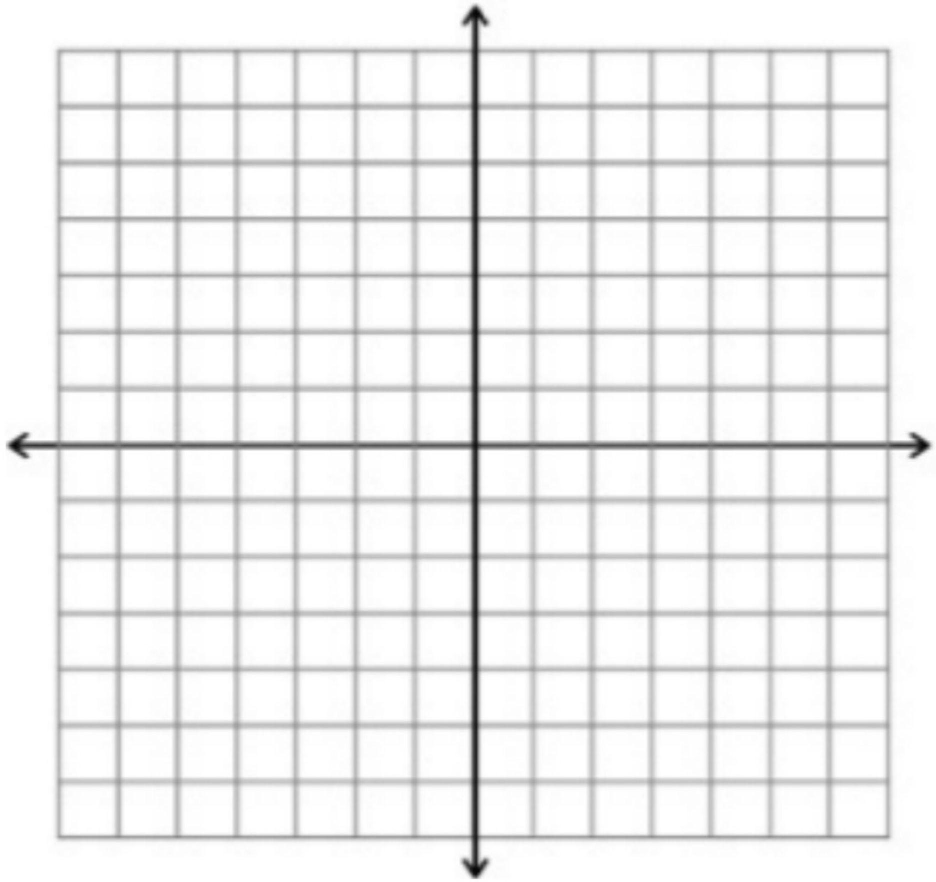
Examples 4–5 Graph each function. **8–11. See margin.**

8. $f(x) = \log_3 x$

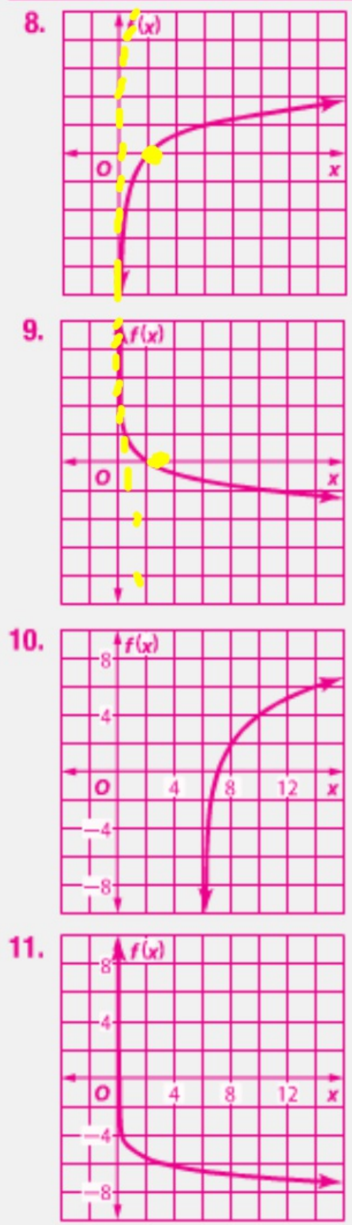
9. $f(x) = \log_{\frac{1}{6}} x$

10. $f(x) = 4 \log_4 (x - 6)$

11. $f(x) = 2 \log_{\frac{1}{10}} x - 5$



Additional Answers



Real-World Example 6 Find Inverses of Exponential Functions

EARTHQUAKES **The Richter scale measures earthquake intensity. The increase in intensity between each number is 10 times. For example, an earthquake with a rating of 7 is 10 times more intense than one measuring 6. The intensity of an earthquake can be modeled by $y = 10^{x-1}$, where x is the Richter scale rating.**

Real-World Link

The largest recorded earthquake in the United States was a magnitude 9.2 that struck Prince William Sound, Alaska, on Good Friday, March 28, 1964.

Source: United States Geological Survey

a. **Use the information at the left to find the intensity of the strongest recorded earthquake in the United States.**

b. **Write an equation of the form $y = \log_{10} x + c$ for the inverse of the function.**

Example 6

12. **SCIENCE** Use the information at the beginning of the lesson. The Palermo scale value of any object can be found using the equation $PS = \log_{10} R$, where R is the relative risk posed by the object. Write an equation in exponential form for the inverse of the function.

Example 6

49. PHOTOGRAPHY The formula $n = \log_2 \frac{1}{p}$ represents the change in the f-stop setting n to use in less light where p is the fraction of sunlight.

a. Benito's camera is set up to take pictures in direct sunlight, but it is a cloudy day. If the amount of sunlight on a cloudy day is $\frac{1}{4}$ as bright as direct sunlight, how many f-stop settings should he move to accommodate less light? **2**

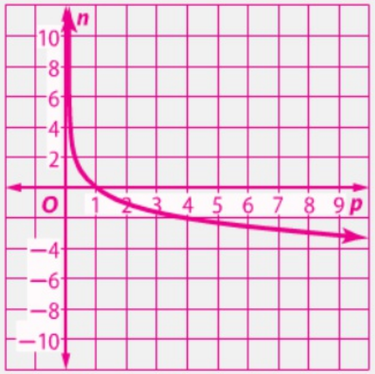
b. Graph the function.

c. Use the graph in part b to predict what fraction of daylight Benito is accommodating if he moves down 3 f-stop settings. Is he allowing more or less light into the camera?

$\log_2 p^{-1}$

Additional Answer

49b.



$n = \log_2 \frac{1}{\frac{1}{4}}$

$= \log_2 4 = x$

$2^x = 4$

$x = 2$

Practice and Problem Solving

Extra Practice is o

Example 1 Write each equation in exponential form.

13. $\log_2 16 = 4$ $2^4 = 16$
14. $\log_7 343 = 3$ $7^3 = 343$
15. $\log_9 \frac{1}{81} = -2$ $9^{-2} = \frac{1}{81}$
16. $\log_3 \frac{1}{27} = -3$ $3^{-3} = \frac{1}{27}$
17. $\log_{12} 144 = 2$ $12^2 = 144$
18. $\log_9 1 = 0$ $9^0 = 1$

Example 2 Write each equation in logarithmic form.

19. $9^{-1} = \frac{1}{9}$ $\log_9 \frac{1}{9} = -1$
20. $6^{-3} = \frac{1}{216}$ $\log_6 \frac{1}{216} = -3$
21. $2^8 = 256$ $\log_2 256 = 8$
22. $4^6 = 4096$ $\log_4 4096 = 6$
23. $27^{\frac{2}{3}} = 9$ $\log_{27} 9 = \frac{2}{3}$
24. $25^{\frac{3}{2}} = 125$ $\log_{25} 125 = \frac{3}{2}$

Example 3 Evaluate each expression.

25. $\log_3 \frac{1}{9}$ -2
26. $\log_4 \frac{1}{64}$ -3
27. $\log_8 512$ 3
28. $\log_6 216$ 3
29. $\log_{27} 3$ $\frac{1}{3}$
30. $\log_{32} 2$ $\frac{1}{5}$
31. $\log_9 3$ $\frac{1}{2}$
32. $\log_{121} 11$ $\frac{1}{2}$
33. $\log_{\frac{1}{5}} 3125$ -5
34. $\log_{\frac{1}{8}} 512$ -3
35. $\log_{\frac{1}{3}} \frac{1}{81}$ 4
36. $\log_{\frac{1}{6}} \frac{1}{216}$ 3

Examples 4–5 CCSS **PRECISION** Graph each function. **37–48. See Chapter 7 Answer Appendix.**

37. $f(x) = \log_6 x$
38. $f(x) = \log_{\frac{1}{5}} x$
39. $f(x) = 4\log_2 x + 6$
40. $f(x) = \log_{\frac{1}{9}} x$
41. $f(x) = \log_{10} x$
42. $f(x) = -3\log_{\frac{1}{12}} x + 2$
43. $f(x) = 6\log_{\frac{1}{8}} (x + 2)$
44. $f(x) = -8\log_3 (x - 4)$
45. $f(x) = \log_{\frac{1}{4}} (x + 1) - 9$
46. $f(x) = \log_5 (x - 4) - 5$
47. $f(x) = \frac{1}{6}\log_8 (x - 3) + 4$
48. $f(x) = -\frac{1}{3}\log_{\frac{1}{6}} (x + 2) - 5$

Lesson 7-3

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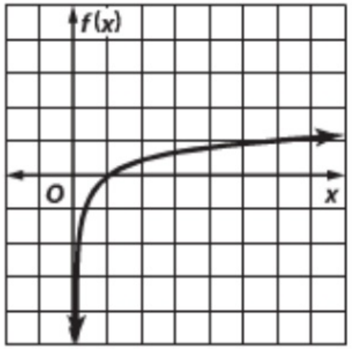

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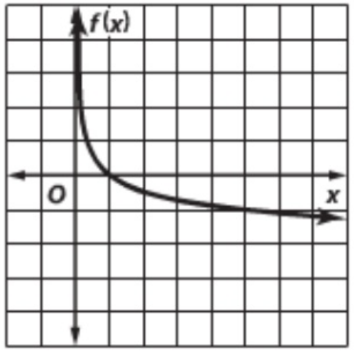

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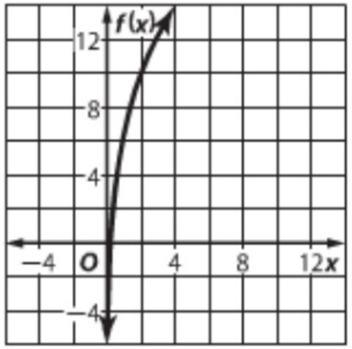

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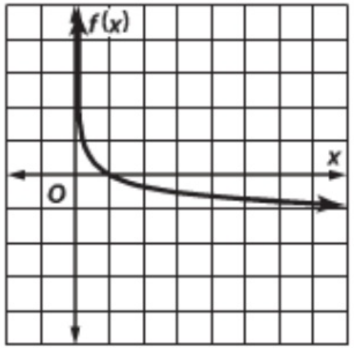

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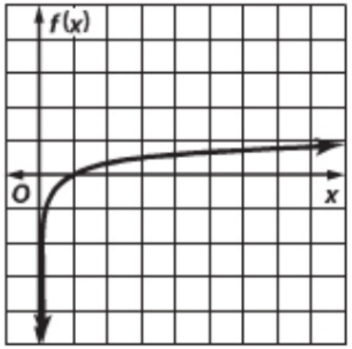

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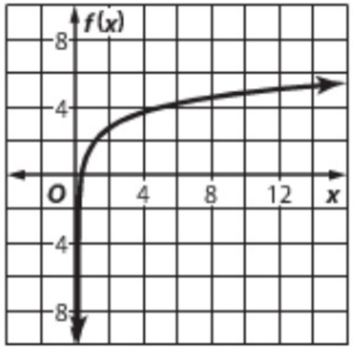

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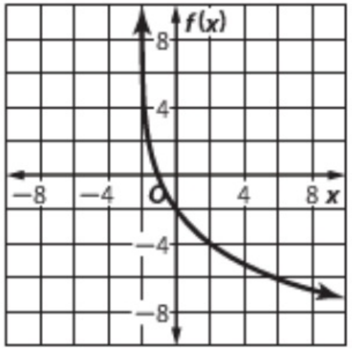

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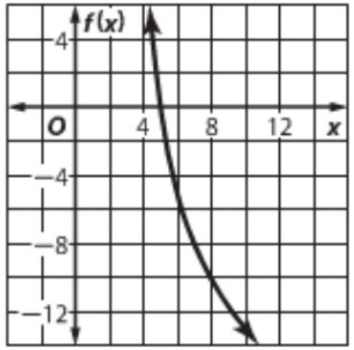

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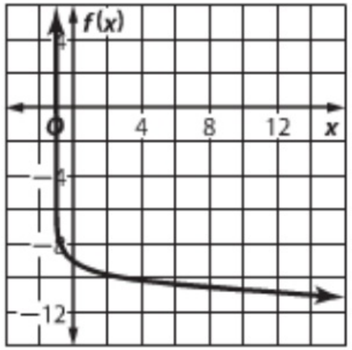

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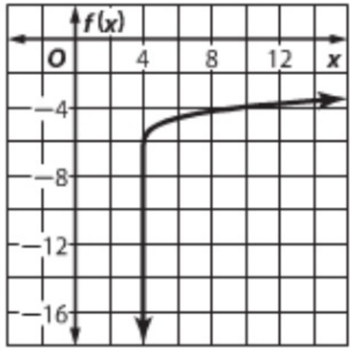

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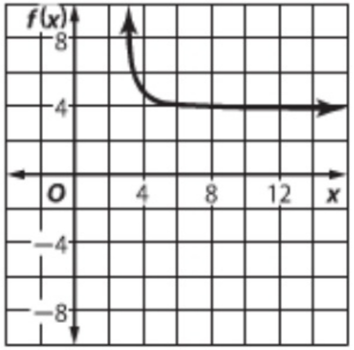

48.