

LESSON 7-3 Rational Exponents

5-Minute Check

Over Lesson 7-2

Standardized Test Practice

- 6** Simplify $\left(\frac{2^3xy^3}{8x^2y}\right)^2$. Assume that the denominator does not equal zero.

A. $\frac{y^3}{2x^2}$

B. $\frac{y^2}{x}$

C. $\frac{y^4}{x}$

D. $\frac{y^4}{x^2}$

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- 4 Simplify $\frac{(3x^2)^2 y^3}{24x^{-2}y}$. Assume that the denominator does not equal zero.

A. $\frac{x^6 y^2}{8}$

B. $\frac{3x^6 y^2}{8}$

C. $\frac{3x^4 y^3}{8y}$

D. $\frac{3x^4 y^5}{8}$


$$\begin{aligned} & \frac{3^2 x^4 y^3}{24 x^{-2} y} = \frac{9 x^4 y^3}{8 \cancel{2} x^{-2} y} \\ & = \frac{3 x^{4 - (-2)} y^{3 - 1}}{8} \\ & = \frac{3 x^6 y^2}{8} \end{aligned}$$

 5-Minute Check

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4 Simplify $\frac{(3x^2)^2 y^3}{24x^{-2}y}$. Assume that the denominator does not equal zero.

A. $\frac{x^6 y^2}{8}$

 B. $\frac{3x^6 y^2}{8}$

C. $\frac{3x^4 y^3}{8y}$

D. $\frac{3x^4 y^5}{8}$

LESSON 7-3 Rational Exponents

$$(x^a)^b = x^{ab}$$

$$(b^2)^{1/2} = (9)^{1/2} \quad b = 9^{1/2}$$

 Key Concept $b^{1/2}$

Words For any nonnegative real number b , $b^{1/2} = \sqrt{b}$.

Examples $16^{1/2} = \sqrt{16}$ or 4 $38^{1/2} = \sqrt{38}$

EXAMPLE 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

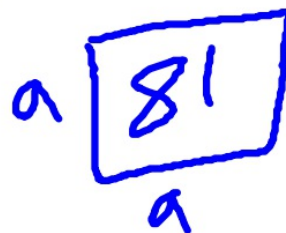
A. $81^{\frac{1}{2}}$

$= \sqrt{81}$ ~~9~~

$\frac{81}{9 \cdot 9}$

square root...

Q: What is the length of the side of a square?



EXAMPLE 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

B. $\sqrt{38}$

$$\sqrt{38} = 38^{\frac{1}{2}}$$

Definition of $b^{\frac{1}{2}}$

Answer: $38^{\frac{1}{2}}$

EXAMPLE 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

C. $12m^{\frac{1}{2}}$

$$12m^{\frac{1}{2}} = 12\sqrt{m}$$

Definition of $b^{\frac{1}{2}}$

Answer: $12\sqrt{m}$

EXAMPLE 1 Radical and Exponential Forms

Write each expression in radical form, or write each radical in exponential form.

D. $\sqrt{32w}$

$$\sqrt{32w} = (32w)^{\frac{1}{2}}$$

Definition of $b^{\frac{1}{2}}$

Answer: $(32w)^{\frac{1}{2}}$

Example 1 Write each expression in radical form, or write each radical in exponential form.

1. $12^{\frac{1}{2}}$ $\sqrt{12}$

2. $3x^{\frac{1}{2}}$ $3\sqrt{x}$

3. $\sqrt{33}$ $33^{\frac{1}{2}}$

4. $\sqrt{8n}$ $(8n)^{\frac{1}{2}}$

LESSON 7-3 Rational Exponents

Key Concept n th Root

Words For any real numbers a and b and any positive integer n , if $a^n = b$, then a is an n th root of b .

Symbols If $a^n = b$, then $\sqrt[n]{b} = a$.


Example Because $2^4 = 16$, 2 is a fourth root of 16, $\sqrt[4]{16} = 2$.

groups of 4

$$\sqrt[4]{16} = 2.$$

$$\begin{array}{cccc} & 1 & & 1 \\ & 4 & & 4 \\ \hline 2 & \cdot & 2 & \cdot & 2 & \cdot & 2 \end{array}$$

EXAMPLE 2 *n*th Roots**A.** Simplify $\sqrt[4]{256}$.

$$\sqrt[4]{256} = \sqrt[4]{4 \cdot 4 \cdot 4 \cdot 4}$$


LESSON 7-3 Rational Exponents

EXAMPLE 2 *n*th Roots

B. Simplify $\sqrt[6]{15,625}$.

⑤ $\sqrt[3]{512} = \underline{2 \cdot 2 \cdot 2}$



Examples 2-4 Simplify.

5. $\sqrt[3]{512}$ 8

6. $\sqrt[5]{243}$

LESSON 7-3 Rational Exponents

Key Concept $b^{\frac{1}{n}}$

Words For any positive real number b and any integer $n > 1$, $b^{\frac{1}{n}} = \sqrt[n]{b}$.

Example $8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2}$ or 2

EXAMPLE 3**Evaluate $b^{\frac{1}{n}}$ Expressions****A. Simplify $1331^{\frac{1}{3}}$.**

EXAMPLE 3

Evaluate $b^{\frac{1}{n}}$ ExpressionsB. Simplify $2401^{\frac{1}{4}}$.

7. $343^{\frac{1}{3}}$

8. $\left(\frac{1}{16}\right)^{\frac{1}{4}}$

LESSON 7-3 Rational Exponents

Key Concept $b^{\frac{m}{n}}$

Words For any positive real number b and any integers m and $n > 1$,

$$b^{\frac{m}{n}} = (\sqrt[n]{b})^m \text{ or } \sqrt[n]{b^m}.$$

Example $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 \text{ or } 4$

EXAMPLE 4

Evaluate $b^{\frac{m}{n}}$ ExpressionsA. Simplify $32^{\frac{2}{5}}$.

$$= (32^{\frac{1}{5}})^2$$

$$32^{\frac{2}{5}} = \left(\sqrt[5]{32}\right)^2 \quad b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m$$

$$= 2^2$$

$$2^5 = 32$$

$$= 4$$

Simplify.

EXAMPLE 4**Evaluate $b^{\frac{m}{n}}$ Expressions****B. Simplify $81^{\frac{5}{2}}$.****Answer: 59,049**

LESSON 7-3 Rational Exponents

Key Concept Power Property of Equality

Words For any real number $b > 0$ and $b \neq 1$, $b^x = b^y$ if and only if $x = y$.

Examples If $5^x = 5^3$, then $x = 3$. If $n = \frac{1}{2}$, then $4^n = 4^{\frac{1}{2}}$.

What if the bases are not the same?

→ How can I make the bases the same?

EXAMPLE 5 Solve Exponential Equations**A.** Solve $9^x = 729$.

EXAMPLE 5 Solve Exponential Equations**B.** Solve $16^{2x-1} = 8$.

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EXAMPLE 5

 Check Your Progress

Solve $93^{x+1} = 27^4$.

- A. $\frac{1}{9}$
- B. $\frac{5}{3}$**
- C. $\frac{11}{6}$
- D. 2

Example 5 Solve each equation.

13. $8^x = 4096$

14. $3^{3x+1} = 81$

15. $4^{x-3} = 32$



Check Your Understanding



Step-by-Step Solutions begin on page R13.



Example 1 Write each expression in radical form, or write each radical in exponential form.

1. $12^{\frac{1}{2}}$ $\sqrt{12}$

2. $3x^{\frac{1}{2}}$ $3\sqrt{x}$

3. $\sqrt{33}$ $33^{\frac{1}{2}}$

4. $\sqrt{8n}$ $(8n)^{\frac{1}{2}}$

Examples 2-4 Simplify.

5. $\sqrt[3]{512}$ 8

6. $\sqrt[3]{243}$ 3

7. $343^{\frac{1}{3}}$ 7

8. $(\frac{1}{16})^{\frac{1}{4}}$ $\frac{1}{2}$

9. $343^{\frac{2}{3}}$ 49

10. $81^{\frac{3}{4}}$ 27

11. $216^{\frac{4}{3}}$ 1296

12. $(\frac{1}{49})^{\frac{3}{2}}$ $\frac{1}{343}$

Example 5 Solve each equation.

13. $8^x = 4096$ 4

14. $3^{3x+1} = 81$ 1

15. $4^{x-3} = 32$ 5.5

Handwritten solutions for Example 5:

11. $(216)^{\frac{4}{3}} = (2 \cdot 3^3)^{\frac{4}{3}}$ (circled 1)
 $(2 \cdot 3^{\frac{3 \cdot \frac{4}{3}}}} = 2 \cdot 3^4 = 2 \cdot 81 = 162$ (circled 2)

14. $3^{3x+1} = 81$ (circled 1)
 $3^{3x+1} = 3^4$ (circled 1)
 $3x+1 = 4$ (circled 1)
 $3x = 3$ (circled 1)
 $x = 1$ (circled 1)

15. $4^{x-3} = 32$ (circled 2)
 $2^{2(x-3)} = 2^5$ (circled 2)
 $2(x-3) = 5$ (circled 2)
 $2x - 6 = 5$ (circled 2)
 $2x = 11$ (circled 2)
 $x = 5.5$ (circled 2)

Additional work for problem 11: $343^{\frac{2}{3}} = (343^{\frac{1}{3}})^2 = 7^2 = 49$ (circled 1)

Practice and Problem Solving

Extra Practice is on page R7.

Example 1 Write each expression in radical form, or write each radical in exponential form.

17. $15^{\frac{1}{2}}$ $\sqrt{15}$ 18. $24^{\frac{1}{2}}$ $\sqrt{24}$ 19. $4k^{\frac{1}{2}}$ $4\sqrt{k}$ 20. $(12y)^{\frac{1}{2}}$ $\sqrt{12y}$
21. $\sqrt{26}$ $26^{\frac{1}{2}}$ 22. $\sqrt{44}$ $44^{\frac{1}{2}}$ 23. $2\sqrt{ab}$ $2(ab)^{\frac{1}{2}}$ 24. $\sqrt{3xyz}$ $(3xyz)^{\frac{1}{2}}$

Examples 2–4 Simplify.

25. $\sqrt[3]{8}$ 2 26. $\sqrt[5]{1024}$ 4 27. $\sqrt[3]{216}$ 6 28. $\sqrt[4]{10,000}$ 10
29. $\sqrt[3]{0.001}$ 0.1 30. $\sqrt[4]{\frac{16}{81}}$ $\frac{2}{3}$ 31. $1331^{\frac{1}{3}}$ 11 32. $64^{\frac{1}{6}}$ 2 $64^{\frac{1}{6}}$
33. $3375^{\frac{1}{3}}$ 15 34. $512^{\frac{1}{9}}$ 2 35. $(\frac{1}{81})^{\frac{1}{4}}$ $\frac{1}{3}$ 36. $(\frac{3125}{32})^{\frac{1}{5}}$ $\frac{5}{2}$
37. $8^{\frac{2}{3}}$ 4 38. $625^{\frac{3}{4}}$ 125 39. $729^{\frac{5}{6}}$ 243 40. $256^{\frac{3}{8}}$ 8
41. $125^{\frac{4}{3}}$ 625 42. $49^{\frac{5}{2}}$ 16,807 43. $(\frac{9}{100})^{\frac{3}{2}}$ $\frac{27}{1000}$ 44. $(\frac{8}{125})^{\frac{4}{3}}$ $\frac{16}{625}$

Handwritten purple annotations:

- A circle around 35 with an arrow pointing to problem 33.
- A circle around $3 \cdot 3 \cdot 3 \cdot 3$ with an arrow pointing to problem 41.
- A circle around $81^{\frac{1}{4}}$ with an arrow pointing to problem 34.
- A circle around $\sqrt[4]{\frac{16}{81}}$ with an arrow pointing to problem 30.
- A circle around $\sqrt[5]{1024}$ with an arrow pointing to problem 26.
- A circle around $\sqrt[3]{216}$ with an arrow pointing to problem 27.
- A circle around $64^{\frac{1}{6}}$ with an arrow pointing to problem 32.
- A circle around $256^{\frac{3}{8}}$ with an arrow pointing to problem 40.
- A circle around $(\frac{8}{125})^{\frac{4}{3}}$ with an arrow pointing to problem 44.

Example 5 Solve each equation.

45. $3^x = 243$ **5**

48. $27^x = 3$ $\frac{1}{3}$

51. $2^{x-1} = 128$ **8**

54. $9^{2x+3} = 2187$ $\frac{1}{4}$

46. $12^x = 144$ **2**

49. $9^x = 27$ $\frac{3}{2}$

52. $4^{2x+1} = 1024$ **2**

55 $4^{3x} = 512$ $\frac{3}{2}$

47. $16^x = 4$ $\frac{1}{2}$

50. $32^x = 4$ $\frac{2}{5}$

53. $6^{x-4} = 1296$ **8**

56. $128^{3x} = 8$ $\frac{1}{7}$