7-4 Solving Logarithmic Equations and Inequalities

1. $\log_8 x = \frac{4}{3}$

$8^{4/3} = x$

$2^4 = x$

2. $16^{3/4} = x$

$2^3 = x$

$8 = x$

Example 1 **Solve each equation.**

1. $\log_8 x = \frac{4}{3}$ **16**

2. $\log_{16} x = \frac{3}{4}$ **8**

KeyConcept Property of Equality for Logarithmic Functions

Symbols If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.

Example If $\log_5 x = \log_5 8$, then $x = 8$. If $x = 8$, then $\log_5 x = \log_5 8$.

Standardized Test Example 2 Solve a Logarithmic Equation

Solve $\log_2 (x^2 - 4) = \log_2 3x$.

A -2 **B** -1 **C** 2 **D** 4

Read the Test Item

You need to find x for the logarithmic equation.

Solve the Test Item

$\log_2 (x^2 - 4) = \log_2 3x$	Original equation
$x^2 - 4 = 3x$	Property of Equality for Logarithmic Functions
$x^2 - 3x - 4 = 0$	Subtract $3x$ from each side.
$(x - 4)(x + 1) = 0$	Factor.
$x - 4 = 0 \quad \text{or} \quad x + 1 = 0$	Zero Product Property

KeyConcept Property of Equality for Logarithmic Functions

Symbols	If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.
Example	If $\log_5 x = \log_5 8$, then $x = 8$. If $x = 8$, then $\log_5 x = \log_5 8$.

Example 2 3. **MULTIPLE CHOICE** Solve $\log_5 (x^2 - 10) = \log_5 3x$.

A 10 **B** 2 **C** 5 **D** 2, 5

$x^2 - 10 = 3x$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, -2$

$\log_5 3x = \log_5 -6 \leftarrow -x$

can't use!

KeyConcept Property of Inequality for Logarithmic Functions

If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$.

If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.

This property also holds true for $\leq$ and $\geq$.

$x > 0$

Example 3 Solve a Logarithmic Inequality

Solve $\log_3 x > 4$.

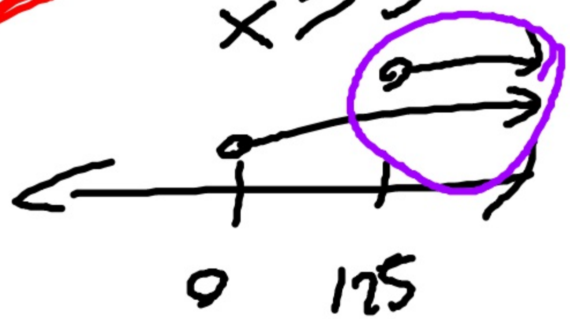
$\log_3 x > 4$	Original inequality
$x > 3^4$	Property of Inequality for Logarithmic Functions
$x > 81$	Simplify.

Example 3 **Solve each inequality.**

4. $\log_5 x > 3$ $\{x \mid x > 125\}$

$x > 0$

$x > 5^3$ $\quad x > 125$



5. $\log_8 x \leq -2$ $\left\{x \mid 0 < x \leq \frac{1}{64}\right\}$

$x \leq 8^{-2}$

$x \leq \frac{1}{8^2}$

$x \leq \frac{1}{64}$

KeyConcept Property of Inequality for Logarithmic Functions

Symbols If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.

Example If $\log_6 x > \log_6 35$, then $x > 35$

This property also holds true for $\leq$ and $\geq$.

Example 4 Solve Inequalities with Logarithms on Each Side

Solve $\log_4 (x + 3) > \log_4 (2x + 1)$.

$\log_4 (x + 3) > \log_4 (2x + 1)$ Original inequality

$x + 3 > 2x + 1$ Property of Inequality for Logarithmic Functions

$2 > x$ Subtract $x + 1$ from each side.

Exclude all values of x for which $x + 3 \leq 0$ or $2x + 1 \leq 0$. So, $x > -3$, $x > -\frac{1}{2}$, and $x < 2$. The solution set is $\left\{x \mid -\frac{1}{2} < x < 2\right\}$ or $\left(-\frac{1}{2}, 2\right)$.

6. $\log_4 (2x + 5) \leq \log_4 (4x - 3)$ $\{x \mid x \geq 4\}$

7. $\log_8 (2x) > \log_8 (6x - 8)$ $\left\{x \middle| 2 > x > \frac{4}{3}\right\}$

Examples 1–2 **CCSS STRUCTURE** **Solve each equation.**

8. $\log_{81} x = \frac{3}{4}$ **27**

9. $\log_{25} x = \frac{5}{2}$ **3125**

10. $\log_8 \frac{1}{2} = x$ $-\frac{1}{3}$

11. $\log_6 \frac{1}{36} = x$ **−2**

12. $\log_x 32 = \frac{5}{2}$ **4**

13. $\log_x 27 = \frac{3}{2}$ **9**

14. $\log_3 (3x + 8) = \log_3 (x^2 + x)$ **−2 or 4**

15. $\log_{12} (x^2 - 7) = \log_{12} (x + 5)$ **4 or −3**

16. $\log_6 (x^2 - 6x) = \log_6 (-8)$ **no solution**

17. $\log_9 (x^2 - 4x) = \log_9 (3x - 10)$ **5**

18. $\log_4 (2x^2 + 1) = \log_4 (10x - 7)$ **1 or 4**

19. $\log_7 (x^2 - 4) = \log_7 (-x + 2)$ **−3**

SCIENCE **The equation for wind speed w, in miles per hour, near the center of a tornado is $w = 93 \log_{10} d + 65$, where d is the distance in miles that the tornado travels.**

20. Write this equation in exponential form. $d = 10^{\frac{w - 65}{93}}$

21. In May of 1999, a tornado devastated Oklahoma City with the fastest wind speed ever recorded. If the tornado traveled 525 miles, estimate the wind speed near the center of the tornado. **318 mph**

Solve each inequality.

30. $\left\{x \,\middle|\, \frac{6}{7} < x < 5\right\}$

Examples 3–4

22. $\log_6 x < -3$ $\left\{x \,\middle|\, 0 < x < \frac{1}{216}\right\}$

23. $\log_4 x \geq 4$ $\{x \mid x \geq 256\}$

24. $\log_3 x \geq -4$ $\left\{x \,\middle|\, x \geq \frac{1}{81}\right\}$

25. $\log_2 x \leq -2$ $\left\{x \,\middle|\, 0 < x \leq \frac{1}{4}\right\}$

26. $\log_5 x > 2$ $\{x \mid x > 25\}$

27. $\log_7 x < -1$ $\left\{x \,\middle|\, 0 < x < \frac{1}{7}\right\}$

28. $\log_2 (4x - 6) > \log_2 (2x + 8)$ $\{x \mid x > 7\}$

29. $\log_7 (x + 2) \geq \log_7 (6x - 3)$ $\left\{x \,\middle|\, \frac{1}{2} < x \leq 1\right\}$

30. $\log_3 (7x - 6) < \log_3 (4x + 9)$

31. $\log_5 (12x + 5) \leq \log_5 (8x + 9)$ $\left\{x \,\middle|\, -\frac{5}{12} < x \leq 1\right\}$

32. $\log_{11} (3x - 24) \geq \log_{11} (-5x - 8)$ $\{x \mid x > 2\}$

33. $\log_9 (9x + 4) \leq \log_9 (11x - 12)$ $\{x \mid x \geq 8\}$

11. $\log_6 \frac{1}{36} = x$ **−2**

$$6^x = \frac{1}{36}$$

$$6^x = \frac{1}{6^2}$$

$$6^{x} = 6^{-2}$$

$$x = -2$$

13. $\log_x 27 = \frac{3}{2}$ **9**

$$\left(x^{3/2}\right)^{2/3} = (27)^{2/3}$$

$$x = 3^2 = 9$$