7-6 Common Logarithms

1 Common Logarithms You have seen that the base 10 logarithm function, $y = \log_{10} x$, is used in many applications. Base 10 logarithms are called **common logarithms**. Common logarithms are usually written without the subscript 10.

$$\log_{10} x = \log x, x > 0$$

Most scientific calculators have a LOG key for evaluating common logarithms.

PT

Example 1 Find Common Logarithms

Use a calculator to evaluate each expression to the nearest ten-thousandth.

a. log 5

KEYSTROKES: LOG 5 ENTER .6989700043

$\log 5 \approx 0.6990$

b. log 0.3

KEYSTROKES: LOG 0.3 ENTER −.5228787453

$\log 0.3 \approx -0.5229$

Example 1 **Use a calculator to evaluate each expression to the nearest ten-thousandth.**

1. $\log 5$ **0.6990** **2.** $\log 21$ **1.3222** **3.** $\log 0.4$ **−0.3979** **4.** $\log 0.7$ **−0.1549**

The common logarithms of numbers that differ by integral powers of ten are closely related. Remember that a logarithm is an exponent. For example, in the equation $y = \log x$, y is the power to which 10 is raised to obtain the value of x.

$$\begin{array}{rccccl} \log x = y & \rightarrow & \text{means} & \rightarrow & 10^y = x \\ \log 1 = 0 & & \text{since} & & 10^0 = 1 \\ \log 10 = 1 & & \text{since} & & 10^1 = 10 \\ \log 10^m = m & & \text{since} & & 10^m = 10^m \end{array}$$

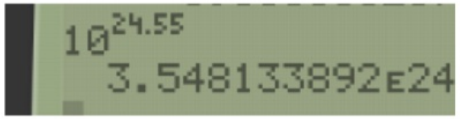
Real-World Example 2 Solve Logarithmic Equations

ROCK CONCERT **The loudness L, in decibels, of a sound is $L = 10 \log \frac{I}{m}$, where I is the intensity of the sound and m is the minimum intensity of sound detectable by the human ear. Residents living several miles from a concert venue can hear the music at an intensity of 66.6 decibels. How many times the minimum intensity of sound detectable by the human ear was this sound, if m is defined to be 1?**

$\log E = 11.8 + 1.5(8.5)$

$\log_{10} E = 24.55$

$E = 10^{24.55}$



Example 2

5. **SCIENCE** The amount of energy E in ergs that an earthquake releases is related to its Richter scale magnitude M by the equation $\log E = 11.8 + 1.5M$. Use the equation to find the amount of energy released by the 1960 Chilean earthquake, which measured 8.5 on the Richter scale. **3.55×10^{24} ergs**

Example 3 Solve Exponential Equations Using Logarithms

Solve $4^x = 19$. Round to the nearest ten-thousandth.

$$\log 6^x = \log 40 \rightarrow x = \frac{\log 40}{\log 6}$$

$$\frac{x \cdot \log 6}{\log 6} = \frac{\log 40}{\log 6}$$

Example 3 Solve each equation. Round to the nearest ten-thousandth.

6. $6^x = 40$ **2.0588**

7. $2.1^{a+2} = 8.25$ **0.8442**

8. $7^{x^2} = 20.42$ **±1.2451**

9 $11^{b-3} = 5^b$ **9.1237**

$$\log 2.1^{a+2} = \log 8.25$$

$$\frac{(a+2)\log 2.1}{\log 2.1} = \frac{\log 8.25}{\log 2.1}$$

$$a = \frac{\log 8.25}{\log 2.1} - 2$$

Example 4 Solve Exponential Inequalities Using Logarithms

Solve $3^{5y} < 7^{y-2}$. Round to the nearest ten-thousandth.

$3^{5y} < 7^{y-2}$	Original inequality
$\log 3^{5y} < \log 7^{y-2}$	Property of Inequality for Logarithmic Functions
$5y \log 3 < (y-2) \log 7$	Power Property of Logarithms
$5y \log 3 < y \log 7 - 2 \log 7$	Distributive Property
$5y \log 3 - y \log 7 < -2 \log 7$	Subtract $y \log 7$ from each side.
$y(5 \log 3 - \log 7) < -2 \log 7$	Distributive Property
$y < \frac{-2 \log 7}{5 \log 3 - \log 7}$	Divide each side by $5 \log 3 - \log 7$.
$\{y \mid y < -1.0972\}$	Use a calculator.

Example 4 **Solve each inequality. Round to the nearest ten-thousandth.**

10. $5^{4n} > 33$ $\{n \mid n > 0.5431\}$

11. $6^{p-1} \leq 4^{p}$ $\{p \mid p \leq 4.4190\}$

$\log 5^{4n} > \log 33$

$\frac{\cancel{4}n \cancel{\log 5}}{\cancel{4} \cancel{\log 5}} > \frac{\log 33}{4 \log 5}$

$n > \frac{\log 33}{4 \log 5}$

Example 5 Change of Base Formula

Express $\log_3 20$ in terms of common logarithms. Then round to the nearest ten-thousandth.

Additional Answers

12. $\frac{\log 7}{\log 3} \approx 1.7712$

13. $\frac{\log 23}{\log 4} \approx 2.2618$

14. $\frac{\log 13}{\log 9} \approx 1.1674$

15. $\frac{\log 5}{\log 2} \approx 2.3219$

Example 5 **Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.** **12–15. See margin.**

12. $\log_3 7$ **13.** $\log_4 23$ **14.** $\log_9 13$ **15.** $\log_2 5$

12 $\log_3 7 = x$

$3^x = 7$

$\log 3^x = \log 7$

$\frac{x \cdot \log 3}{\log 3} = \frac{\log 7}{\log 3}$

$x = \frac{\log 7}{\log 3}$

Practice and Problem Solving

Extra Practice is on

Example 1 **Use a calculator to evaluate each expression to the nearest ten-thousandth.**

16. $\log 3$ **0.4771**
17. $\log 11$ **1.0414**
18. $\log 3.2$ **0.5051**
19. $\log 8.2$ **0.9138**
20. $\log 0.9$ **−0.0458**
21. $\log 0.04$ **−1.3979**

Example 2

22. CCSS **SENSE-MAKING** Loretta had a new muffler installed on her car. The noise level of the engine dropped from 85 decibels to 73 decibels.

a. How many times the minimum intensity of sound detectable by the human ear was the car with the old muffler, if m is defined to be 1? **about 316,227,766 times**

b. How many times the minimum intensity of sound detectable by the human ear is the car with the new muffler? Find the percent of decrease of the intensity of the sound with the new muffler. **about 19,952,623 times; about 93.7%**

Example 3 **Solve each equation. Round to the nearest ten-thousandth.**

23. $8^x = 40$ **1.7740**
24. $5^x = 55$ **2.4899**
25. $2.9^{a-4} = 8.1$ **5.9647**
26. $9^{b-1} = 7^b$ **8.7429**
27. $13^{x^2} = 33.3$ **±1.1691**
28. $15^{x^2} = 110$ **±1.3175**

Example 4 **Solve each inequality. Round to the nearest ten-thousandth.**

29. $6^{3n} > 36$ $\{n \mid n > 0.6667\}$
30. $2^{4x} \leq 20$ $\{x \mid x \leq 1.0805\}$
31. $3^{y-1} \leq 4^y$ $\{y \mid y \geq -3.8188\}$
32. $5^{p-2} \geq 2^p$ $\{p \mid p \geq 3.5129\}$

Example 5 **Express each logarithm in terms of common logarithms. Then approximate its value to the nearest ten-thousandth.**

33. $\log_7 18$ $\frac{\log 18}{\log 7} \approx 1.4854$
34. $\log_5 31$ $\frac{\log 31}{\log 5} \approx 2.1337$
35. $\log_2 16$ $\frac{\log 16}{\log 2} = 4$
36. $\log_4 9$ $\frac{\log 9}{\log 4} \approx 1.5850$
37. $\log_3 11$ $\frac{\log 11}{\log 3} \approx 2.1827$
38. $\log_6 33$ $\frac{\log 33}{\log 6} \approx 1.9514$