

## 7-7 Base e and Natural Logarithms

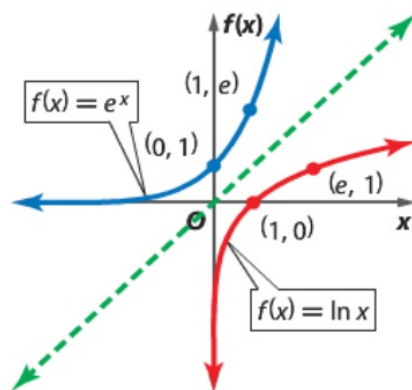
**1 Base e and Natural Logarithms** Like  $\pi$  and  $\sqrt{2}$ , the number  $e$  is an irrational number. The value of  $e$  is 2.71828... . It is referred to as the **natural base,  $e$** . An exponential function with base  $e$  is called a **natural base exponential function**.

### KeyConcept Natural Base Functions

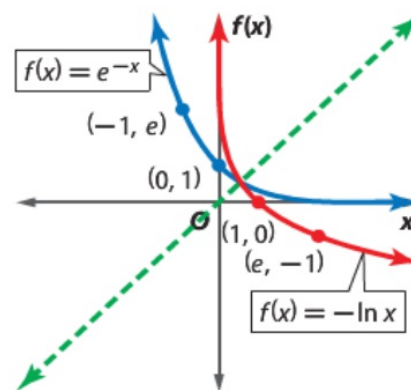
The function  $f(x) = e^x$  is used to model continuous exponential growth.

The function  $f(x) = e^{-x}$  is used to model continuous exponential decay.

The inverse of a natural base exponential function is called the **natural logarithm**. This logarithm can be written as  $\log_e x$ , but is more often abbreviated as  $\ln x$ .



Exponential Growth



Exponential Decay

Write each exponential equation in logarithmic form.

a.  $e^x = 8$

$$e^x = 8 \rightarrow \log_e 8 = x$$
$$\ln 8 = x$$

b.  $e^5 = x$

$$e^5 = x \rightarrow \log_e x = 5$$
$$\ln x = 5$$

**Example 2** Write Equivalent Expressions

Write each logarithmic equation in exponential form.

a.  $\ln x \approx 0.7741$

$$\ln x \approx 0.7741 \rightarrow \log_e x = 0.7741$$
$$x \approx e^{0.7741}$$

b.  $\ln 10 = x$

$$\ln 10 = x \rightarrow \log_e 10 = x$$
$$10 = e^x$$

**Examples 1–2** Write an equivalent exponential or logarithmic function.

1.  $e^x = 30$   $\ln 30 = x$

3.  $e^3 = x$   $\ln x = 3$

2.  $\ln x = 42$   $e^{42} = x$

4.  $\ln 18 = x$   $e^x = 18$



5

$$3 \ln 2 + 2 \ln 4$$

$$\ln 2^3 + \ln 4^2$$

$$= \ln (2^3 \cdot 4^2)$$

Example 3

Write each as a single logarithm.

5.  $3 \ln 2 + 2 \ln 4$   $7 \ln 2$  ←

7.  $3 \ln 6 + 2 \ln 9$   $\ln 17496$

$$\ln (8 \cdot 16)$$

$$\ln (128) \leftarrow$$

$$\ln 2^7$$

6

$$5 \ln 3 - 2 \ln 9$$

$$\ln 3^5 - \ln 9^2$$

$$= \ln \frac{3^5}{9^2}$$

6.  $5 \ln 3 - 2 \ln 9$   $\ln 3$

8.  $3 \ln 5 + 4 \ln x$   $\ln 125x^4$



### Example 4 Solve Base $e$ Equations

Solve  $4e^{-2x} - 5 = 3$ . Round to the nearest ten-thousandth.

a

$$4e^{-2x} - 24 = 16$$
$$+24 \quad +24$$

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$$4e^{-2x} = 40$$
$$\frac{4e^{-2x}}{4} = \frac{40}{4}$$
$$e^{-2x} = 10$$
$$\ln e^{-2x} = \ln 10$$
$$-2x = \ln 10$$
$$x = -\frac{\ln 10}{2} \approx -1.6094$$

**Example 4** Solve each equation. Round to the nearest ten-thousandth.

9.  $5e^x - 24 = 16$  **2.0794**

10.  $-3e^x + 9 = 4$  **0.5108**

11.  $3e^{-3x} + 4 = 6$  **0.1352**

12.  $2e^{-x} - 3 = 8$  **-1.7047**



**Example 5** Solve Natural Log Equations and Inequalities

$$\textcircled{13} \quad \ln 3x = 8$$
$$\frac{e^8}{3} = 3x$$

$$e^8/3$$

993.6526623

$x < 10.7183$  Use a calculator.

**Example 5** Solve each equation or inequality. Round to the nearest ten-thousandth.

13.  $\ln 3x = 8$  **993.6527**

14.  $-4 \ln 2x = -26$

15.  $\ln (x + 5)^2 < 6$

16.  $\ln (x - 2)^3 > 15$

17.  $e^x > 29$

18.  $5 + e^{-x} > 14$

## KeyConcept Continuously Compounded Interest

Calculate continuously compounded interest using the following formula:

$$A = Pe^{rt},$$

where  $A$  is the amount in the account after  $t$  years,  $P$  is the principal amount invested, and  $r$  is the annual interest rate.

### Real-World Example 6 Solve Base $e$ Inequalities

**FINANCIAL LITERACY** When Angelina was born, her grandparents deposited \$3000 into a college savings account paying 4% interest compounded continuously.

- a. Assuming there are no deposits or withdrawals from the account, what will the balance be after 10 years?

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### Real-World Example 6 Solve Base $e$ Inequalities

**FINANCIAL LITERACY** When Angelina was born, her grandparents deposited \$3000 into a college savings account paying 4% interest compounded continuously.

**b. How long will it take the balance to reach at least \$10,000?**

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### Real-World Example 6 Solve Base $e$ Inequalities

**FINANCIAL LITERACY** When Angelina was born, her grandparents deposited \$3000 into a college savings account paying 4% interest compounded continuously.

- c. If her grandparents want Angelina to have \$10,000 after 18 years, how much would they need to invest?



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#### Example 6

19. **SCIENCE** A virus is spreading through a computer network according to the formula  $v(t) = 30e^{0.1t}$ , where  $v$  is the number of computers infected and  $t$  is the time in minutes. How long will it take the virus to infect 10,000 computers?

**Examples 1–2** Write an equivalent exponential or logarithmic function. **22.**  $0.25 = e^x$  **23.**  $5.4 = e^x$

**20.**  $e^{-x} = 8$   **$\ln 8 = -x$**  **21.**  $e^{-5x} = 0.1$   **$\ln 0.1 = -5x$**  **22.**  $\ln 0.25 = x$  **23.**  $\ln 5.4 = x$   
**24.**  $e^{x-3} = 2$   **$\ln 2 = x - 3$**  **25.**  $\ln(x + 4) = 36$   **$e^{36} = x + 4$**  **26.**  $e^{-2} = x^6$   **$-2 = 6 \ln x$**  **27.**  $\ln e^x = 7$   
 **$e^7 = e^x$**

**Example 3** Write each as a single logarithm.

**28.**  $\ln 125 - 2 \ln 5$   **$\ln 5$**  **29.**  $3 \ln 10 + 2 \ln 100$   **$7 \ln 10$**  **30.**  $4 \ln \frac{1}{3} - 6 \ln \frac{1}{9}$   **$-8 \ln \frac{1}{3}$**   
**31.**  $7 \ln \frac{1}{2} + 5 \ln 2$   **$-2 \ln 2$**  **32.**  $8 \ln x - 4 \ln 5$   **$\ln \frac{x^8}{625}$**  **33.**  $3 \ln x^2 + 4 \ln 3$   **$\ln 81x^6$**

**Example 4** Solve each equation. Round to the nearest ten-thousandth.

**34.**  $6e^x - 3 = 35$   **$1.8458$**  **35.**  $4e^x + 2 = 180$   **$3.7955$**  **36.**  $3e^{2x} - 5 = -4$   **$-0.5493$**   
**37.**  $-2e^{3x} + 19 = 3$   **$0.6931$**  **38.**  $6e^{4x} + 7 = 4$  **no solution** **39.**  $-4e^{-x} + 9 = 2$   **$-0.5596$**

**Examples 5–6** **40.** **CCSS SENSE-MAKING** The value of a certain car depreciates according to  $v(t) = 18500e^{-0.186t}$ , where  $t$  is the number of years after the car is purchased new.

- a. What will the car be worth in 18 months?  **$\$13,996$**   
 b. When will the car be worth half of its original value? **about 3.73 yr**  
 c. When will the car be worth less than \$1000? **about 15.69 yr**

**45.**  $\{x \mid x < -239.8802 \text{ or } x > 239.8802\}$

Solve each inequality. Round to the nearest ten-thousandth. **46.**  $\{x \mid 6 < x \leq 26.0855\}$

**41.**  $e^x \leq 8.7$   **$\{x \mid x \leq 2.1633\}$**  **42.**  $e^x \geq 42.1$   **$\{x \mid x \geq 3.7400\}$**  **43.**  $\ln(3x + 4)^3 > 10$   **$\{x \mid x > 8.0105\}$**   
**44.**  $4 \ln x^2 < 72$  **45.**  $\ln(8x^4) > 24$  **46.**  $-2[\ln(x - 6)^{-1}] \leq 6$   
 **$\{x \mid -8103.0839 < x < 8103.0839, x \neq 0\}$**