What you'll learn about

- Linear Motion Revisited
- General Strategy
- Consumption Over Time
- Net Change from Data
- Work

7.1 Integral As Net Change

We actually did this before, back at 5.2...

...remember these?

Answers:

29. $\int_8^{11} 87\,dt = 261$ miles

30. $\int_0^{60} 25\,dt = 1500$ gallons

31. $\int_6^{7.5} 300\,dt = 450$ calories

In Exercises 29–32, express the desired quantity as a definite integral and evaluate the integral using Theorem 2.

29. Find the distance traveled by a train moving at 87 mph from 8:00 A.M. to 11:00 A.M.

30. Find the output from a pump producing 25 gallons per minute during the first hour of its operation.

31. Find the calories burned by a walker burning 300 calories per hour between 6:00 P.M. and 7:30 P.M.

These were easy, mostly because the integrand was a constant. What if it changed over the course of time?

EXAMPLE 1 Interpreting a Velocity Function

Figure 7.1 shows the velocity

$$\frac{ds}{dt} = v(t) = t^2 - \frac{8}{(t+1)^2} \quad \frac{\text{cm}}{\text{sec}}$$

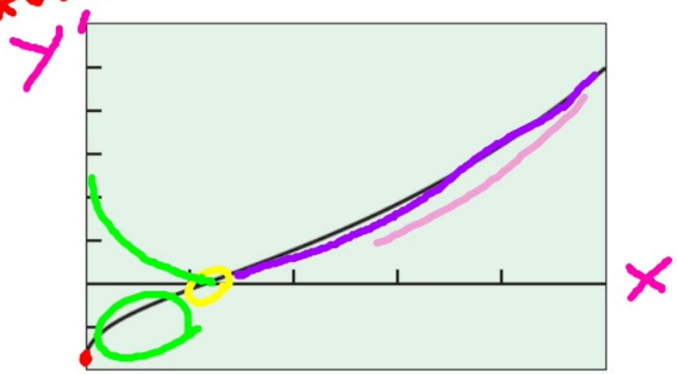
of a particle moving along a horizontal s-axis for $0 \le t \le 5$. Describe the motion.

SOLUTION

Solve Graphically The graph of v (Figure 7.1) starts with $v(0) = -8$, which we interpret as saying that the particle has an initial velocity of 8 cm/sec to the left. It slows to a halt at about $t = 1.25$ sec, after which it moves to the right ($v > 0$) with increasing speed, reaching a velocity of $v(5) \approx 24.8$ cm/sec at the end. ***Now try Exercise 1(a).***

← negative direction

SO. It'll be good for us to be savvy with our graphing calculator....



[0, 5] by [–10, 30]

Figure 7.1 The velocity function in Example 1.

EXAMPLE 2 Finding Position from Displacement

Suppose the initial position of the particle in Example 1 is $s(0) = 9$. What is the particle's position at **(a)** $t = 1$ sec? **(b)** $t = 5$ sec?

$$\begin{aligned}
\text{Displacement} &= \int_0^1 v(t)\,dt \\
&= \int_0^1 \left(t^2 - \frac{8}{(t+1)^2}\right) dt \\
&= \left[\frac{t^3}{3} + \frac{8}{t+1}\right]_0^1 \\
&= \frac{1}{3} + \frac{8}{2} - 8 = -\frac{11}{3}.
\end{aligned}$$

During the first second of motion, the particle moves $11/3$ cm to the left. It starts at $s(0) = 9$, so its position at $t = 1$ is

$$\text{New position} = \text{initial position} + \text{displacement} = 9 - \frac{11}{3} = \frac{16}{3}.$$

EXAMPLE 2 Finding Position from Displacement

Suppose the initial position of the particle in Example 1 is $s(0) = 9$. What is the particle's position at **(a)** $t = 1$ sec? **(b)** $t = 5$ sec?

(b) If we model the displacement from $t = 0$ to $t = 5$ in the same way, we arrive at

$$\text{Displacement} = \int_0^5 v(t)\,dt = \left[\frac{t^3}{3} + \frac{8}{t+1}\right]_0^5 = 35.$$

The motion has the net effect of displacing the particle 35 cm to the right of its starting point. The particle's final position is

$$\begin{aligned}\text{Final position} &= \text{initial position} + \text{displacement}\\ &= s(0) + 35 = 9 + 35 = 44.\end{aligned}$$

EXAMPLE 3 Calculating Total Distance Traveled

Find the *total distance traveled* by the particle in Example 1.

SOLUTION

Solve Analytically We partition the time interval as in Example 2 but record every position shift as *positive* by taking absolute values. The Riemann sum approximating total distance traveled is

$$\sum |v(t_k)|\, \Delta t,$$

and we are led to the integral

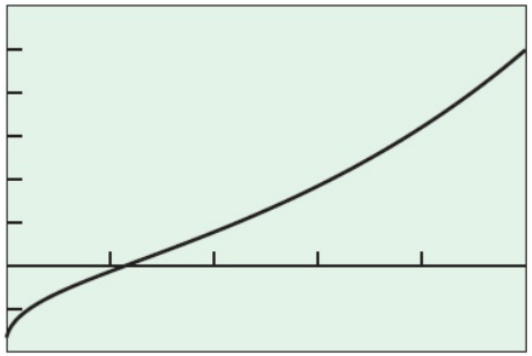
$$\text{Total distance traveled} = \int_0^5 |v(t)|\, dt = \int_0^5 \left| t^2 - \frac{8}{(t+1)^2} \right| dt.$$

Evaluate Numerically We have

$$\text{NINT}\left(\left| t^2 - \frac{8}{(t+1)^2} \right|, t, 0, 5 \right) \approx 42.59.$$

Now try Exercise 1(c).

Why do we take the absolute value?



[0, 5] by [–10, 30]

in

Compare the motion of the particle to the graph of v(t)...

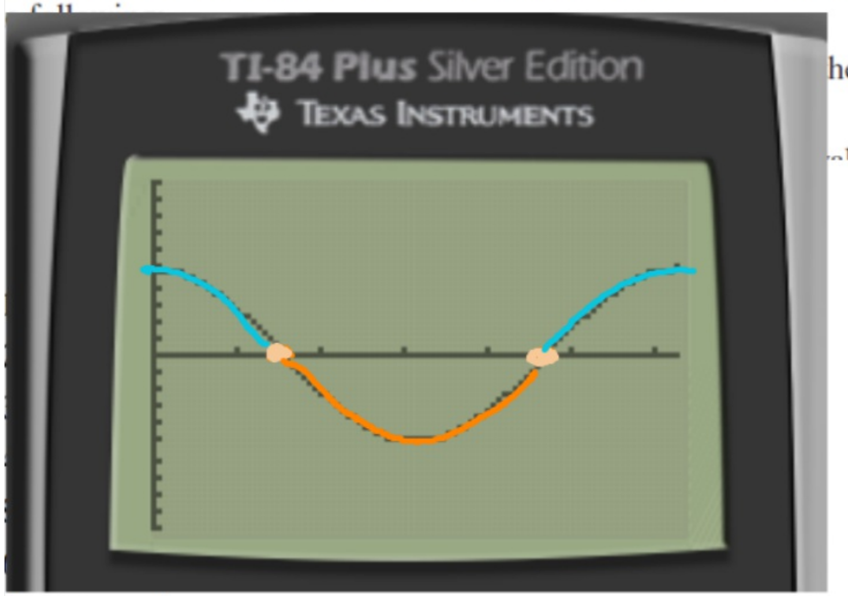
$$\mathbf{1.}\ v(t) = 5\cos t, \quad 0 \le t \le 2\pi$$

$s(0) = 3$

Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a
rticle moving along the x-axis. Use analytic methods to do each of

b) $\int_0^{2\pi} 5\cos t\, dt + 3$

c) $\int_0^{2\pi} |5\cos t|\, dt$



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al. If

7.

1. (a) Right: $0 \le t < \pi/2$, $3\pi/2 < t \le 2\pi$

Left: $\pi/2 < t < 3\pi/2$

Stopped: $t = \pi/2, 3\pi/2$

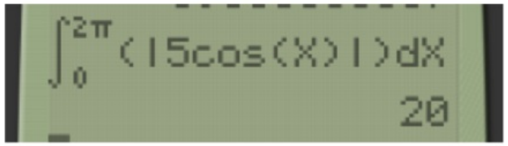
(b) 0; 3 (c) 20

2. (a) Right: $0 < t < \pi/3$

Left: $\pi/3 < t \le \pi/2$

Stopped: $t = 0, \pi/3$

(b) 2; 5 (c) 6



In Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to do each of the following:

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval. If $s(0) = 3$, what is the particle's final position?

(c) Find the total distance traveled by the particle.

1. $v(t) = 5 \cos t, \quad 0 \le t \le 2\pi$ See page 389.

2. $v(t) = 6 \sin 3t, \quad 0 \le t \le \pi/2$ See page 389.

3. $v(t) = 49 - 9.8t, \quad 0 \le t \le 10$ See page 389.

4. $v(t) = 6t^2 - 18t + 12, \quad 0 \le t \le 2$ See page 389.

5. $v(t) = 5 \sin^2 t \cos t, \quad 0 \le t \le 2\pi$ See page 389.

6. $v(t) = \sqrt{4 - t}, \quad 0 \le t \le 4$ See page 389.

7. $v(t) = e^{\sin t} \cos t, \quad 0 \le t \le 2\pi$ See page 389.

8. $v(t) = \dfrac{t}{1 + t^2}, \quad 0 \le t \le 3$ See page 389.

1. (a) Right: $0 \le t < \pi/2$, $3\pi/2 < t \le 2\pi$
Left: $\pi/2 < t < 3\pi/2$
Stopped: $t = \pi/2, 3\pi/2$

(b) 0; 3 **(c)** 20

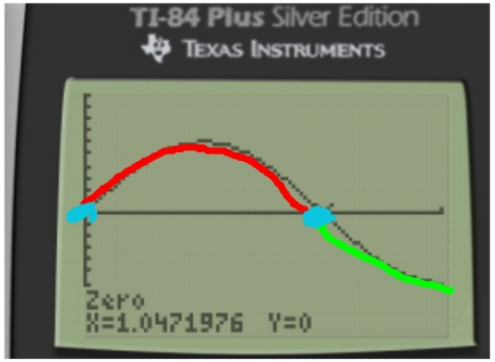
2. (a) Right: $0 < t < \pi/3$
Left: $\pi/3 < t \le \pi/2$
Stopped: $t = 0, \pi/3$
(b) 2; 5 **(c)** 6

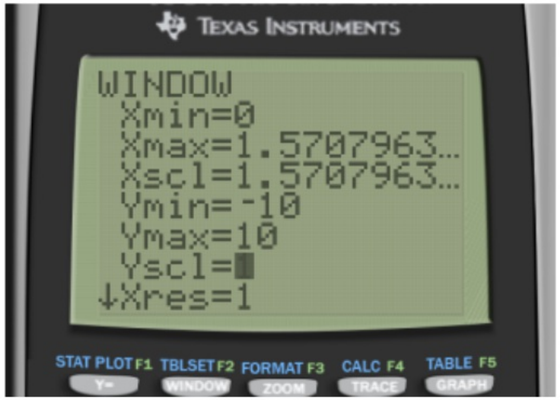
3. (a) Right: $0 \le t < 5$
Left: $5 < t \le 10$
Stopped: $t = 5$
(b) 0; 3 **(c)** 245

4. (a) Right: $0 \le t < 1$
Left: $1 < t < 2$
Stopped: $t = 1, 2$
(b) 4; 7 **(c)** 6

5. (a) Right: $0 < t < \pi/2$, $3\pi/2 < t < 2\pi$
Left: $\pi/2 < t < \pi$, $\pi < t < 3\pi/2$
Stopped: $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$
(b) 0 ; 3 **(c)** 20/3

2. $v(t) = 6 \sin 3t, \quad 0 \le t \le \pi/2$ See page 389.



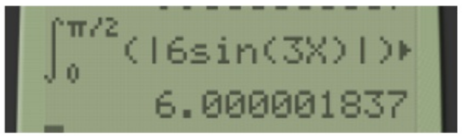


2. (a) Right: $0 < t < \pi/3$
Left: $\pi/3 < t \le \pi/2$
Stopped: $t = 0, \pi/3$
(b) 2; 5 (c) 6

② b) $\int_0^{\pi/2} 6\sin 3t \, dt + 3$

$\Rightarrow 2 + 3 = 5$

c)



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6. **(a)** Right: $0 \le t < 4$
Left: never
Stopped: $t = 4$
(b) 16/3; 25/3 **(c)** 16/3

7. **(a)** Right: $0 \le t < \pi/2$, $3\pi/2 < t \le 2\pi$
Left: $\pi/2 < t < 3\pi/2$
Stopped: $t = \pi/2, 3\pi/2$
(b) 0; 3 **(c)** $2e - (2/e) \approx 4.7$

8. **(a)** Right: $0 < t \le 3$
Left: never
Stopped: $t = 0$
(b) $(\ln 10)/2 \approx 1.15$; 4.15 **(c)** $(\ln 10)/2 \approx 1.15$

(11) $a(t) = 32 \text{ ft/sec}^2$ $\quad v(0) = 90$

a) $v(t) = -32t \text{ ft/sec} + 90$ initial velocity

$v(3) = -32(3) + 90$

$= -96 + 90 = -6$

11. ***Projectile*** Recall that the acceleration due to Earth's gravity is 32 ft/sec^2. From ground level, a projectile is fired straight upward with velocity 90 feet per second.

(a) What is its velocity after 3 seconds? -6 ft/sec

(b) When does it hit the ground? 5.625 sec

(c) When it hits the ground, what is the net distance it has traveled? 0

(d) When it hits the ground, what is the total distance it has traveled? 253.125 feet

$s(t) = -16t^2 + 90t + C$ initial displacement

$C = 0$

$s(t) = -16t^2 + 90t = 0$

$t(-16t + 90) = 0$

$$h(t) = y(t) = y_0 + v_0\,t + \frac{1}{2}\,a\,t^2\,.$$

from college...

$-16t + 90 = 0$

$t = 90/16$

90/16
5.625

d) $s(t) = \int_0^{5.625} |v(t)|\,dt$

$\int_0^{5.625} (|-32X+90|)d$
253.125

9. An automobile accelerates from rest at $1 + 3\sqrt{t}$ mph/sec for 9 seconds.

$v(0) = 0$

(a) What is its velocity after 9 seconds? 63 mph

(b) How far does it travel in those 9 seconds? 344.52 feet

a

velocity → $v(t) = \int a(t)\,dt$ ← acceleration

$3t^{1/2}$

$= \int (1 + 3\sqrt{t})\,dt$

$v(t) = t + 2t^{3/2}$

$v(t) = t + 2t^{3/2}$

$v(9) = 9 + 2(9)^{3/2}$

$9 + 2(27)$

$9 + 54 = 63$

b) $v(t) = t + 2t^{3/2}$

total distance $= \int_0^9 t + 2t^{3/2}\,dt$

$= \frac{1}{2}t^2 + \frac{4}{5}t^{5/2} \Big|_0^9$

$= \frac{1}{2}(81) + \frac{4}{5}(3)^5$

1 mile = 5280 ft

1 hr = 3600 sec

$\left(\frac{5280}{3600}\right)(234.9) = 344.52$

```
5280/3600
        1.466666667
Ans*234.9
             344.52
```

10. A particle travels with velocity

$$v(t) = (t - 2)\sin t \ \ \text{m/sec}$$

for $0 \le t \le 4$ sec.

(a) What is the particle's displacement? ≈ -1.44952 meters

(b) What is the total distance traveled? ≈ 1.91411 meters

absolute value!

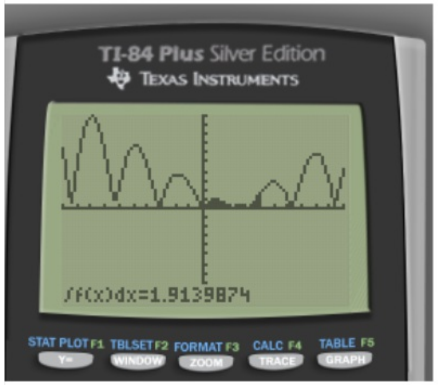
$$s(t) = \int v(t)$$

$$= \int_0^4 (t-2)\sin t \, dt$$

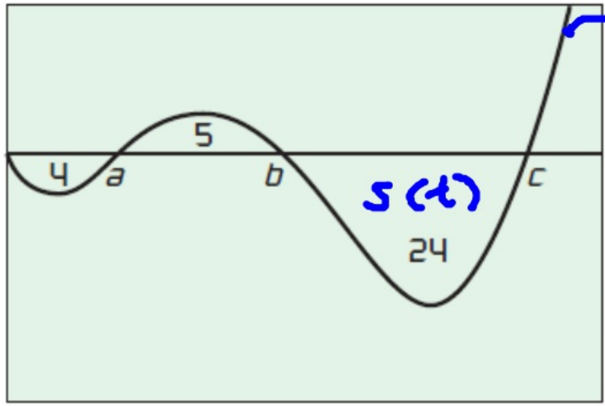
$$= -1.44952$$

$$\int |v(t)|$$

$$\int_0^4 |(t-2)\sin t| \, dt$$



In Exercises 12–16, a particle moves along the x-axis (units in cm). Its initial position at $t = 0$ sec is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the *areas* of the enclosed regions.



$v(t)$

12 $-4+5-24$

13 $4+5+24$

12. What is the particle's displacement between $t = 0$ and $t = c$? -23 cm

13. What is the total distance traveled by the particle in the same time period? 33 cm

a: 11 b: 16 c: -8

14. Give the positions of the particle at times a, b, and c.

15. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$? $t = a$

16. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$? $t = c$

$a(t) =$ slope of $v(t)$ (positive)

In Exercises 17–20, the graph of the velocity of a particle moving on the x-axis is given. The particle starts at $x = 2$ when $t = 0$.

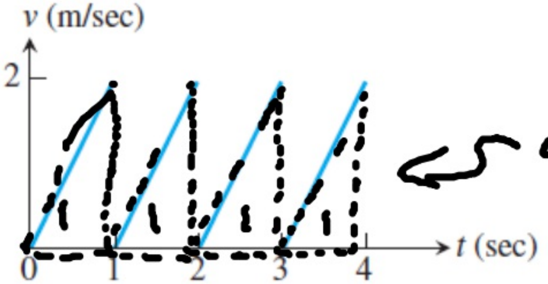
$s(0) = 2$

(**a**) Find where the particle is at the end of the trip.

(**b**) Find the total distance traveled by the particle.

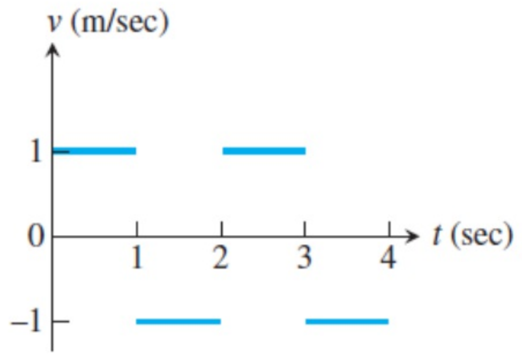
17. (a) 6 (b) 4 meters

starts @ 2,
ends @ $2+4=6$

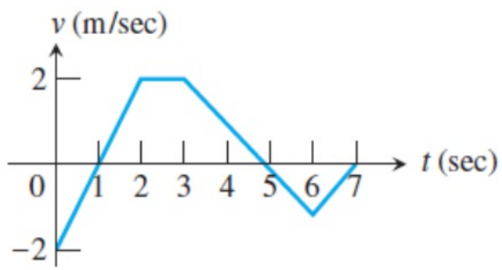


area: 4

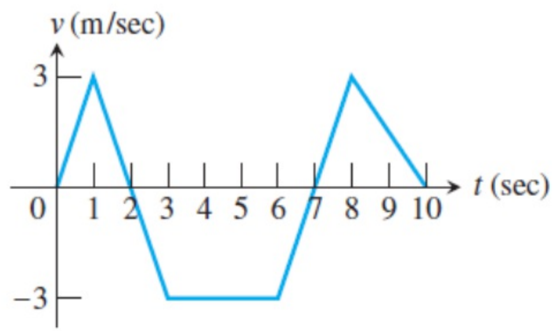
18. (a) 2 (b) 4 meters



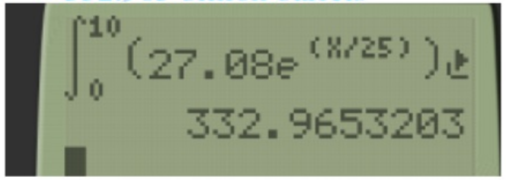
19. (a) 5 (b) 7 meters



20. (a) −2.5 (b) 19.5 meters



21. ***U.S. Oil Consumption*** The rate of consumption of oil in the United States during the 1980s (in billions of barrels per year) is modeled by the function $C = 27.08 \cdot e^{t/25}$, where t is the number of years after January 1, 1980. Find the total consumption of oil in the United States from January 1, 1980 to January 1, 1990. ≈332.965 billion barrels



...."what is the rate at 10 yrs?"

let $t = 10$ ...

$C = 27.08 \cdot e^{t/25}$

rate

total $= \int_0^{10} 27.08 e^{t/25}$

22. ***Home Electricity Use*** The rate at which your home consumes electricity is measured in kilowatts. If your home consumes electricity at the rate of 1 kilowatt for 1 hour, you will be charged for 1 "kilowatt-hour" of electricity. Suppose that the average consumption rate for a certain home is modeled by the function $C(t) = 3.9 - 2.4 \sin(\pi t/12)$, where $C(t)$ is measured in kilowatts and t is the number of hours past midnight. Find the average daily consumption for this home, measured in kilowatt-hours. 93.6 kilowatt-hours

$t = 24$

$$\int_0^{24} 3.9 - 2.4 \sin(\pi t/12)$$

$= 93.6$:)

23. *Population Density* Population density measures the number of people per square mile inhabiting a given living area. Washerton's population density, which decreases as you move away from the city center, can be approximated by the function $10{,}000(2 - r)$ at a distance r miles from the city center.

pop. density $= 10000(2-r)$

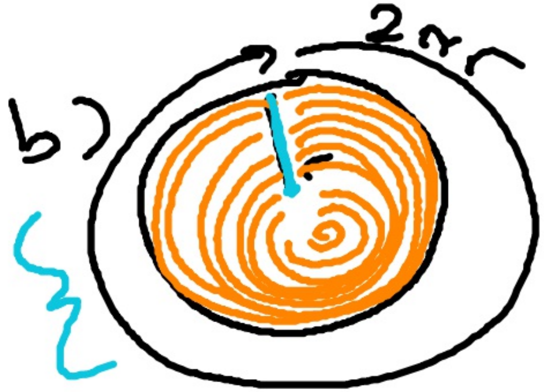
(a) If the population density approaches zero at the edge of the city, what is the city's radius? 2 miles

a) $0 = 10000(2-r)$

$r = 2$

(b) A thin ring around the center of the city has thickness Δr and radius r. If you straighten it out, it suggests a rectangular strip. Approximately what is its area? $2\pi r\Delta r$

b)



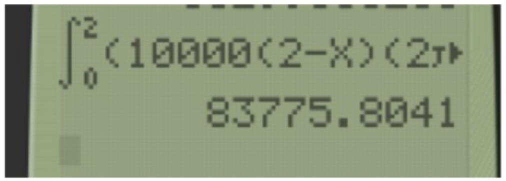
(c) **Writing to Learn** Explain why the population of the ring in part (b) is approximately

$$10{,}000(2 - r)(2\pi r)\Delta r. \quad \Delta r \sim dr$$

Population = Population density × Area

(d) Estimate the total population of Washerton by setting up and evaluating a definite integral. ≈83,776

$$\int_0^2 10{,}000(2-r)(2\pi r)\,dr$$

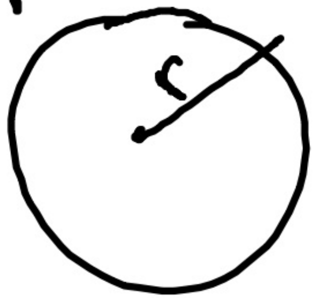


24. *Oil Flow* Oil flows through a cylindrical pipe of radius 3 inches, but friction from the pipe slows the flow toward the outer edge. The speed at which the oil flows at a distance r inches from the center is $8(10 - r^2)$ inches per second.

Thick = Δr

(a) In a plane cross section of the pipe, a thin ring with thickness Δr at a distance r inches from the center approximates a rectangular strip when you straighten it out. What is the area of the strip (and hence the approximate area of the ring)? $2\pi r \Delta r$

a) $2\pi r \Delta r$

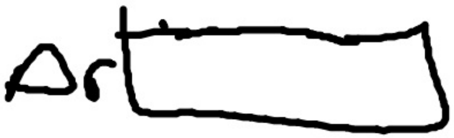


$2\pi r$

(b) Explain why we know that oil passes through this ring at approximately $8(10 - r^2)(2\pi r)\,\Delta r$ cubic inches per second.

(c) Set up and evaluate a definite integral that will give the rate (in cubic inches per second) at which oil is flowing through the pipe. 396π in^3/sec or ≈ 1244.07 in^3/sec

24. (b) $8(10 - r^2)$ in/sec $\cdot$ $(2\pi r)\Delta r$ in^2 = flow in in^3/sec



c) $\int_0^3 8(10-r^2)(2\pi r)\,dr$ ✓