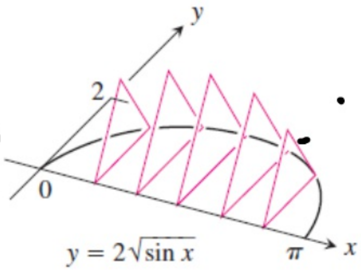
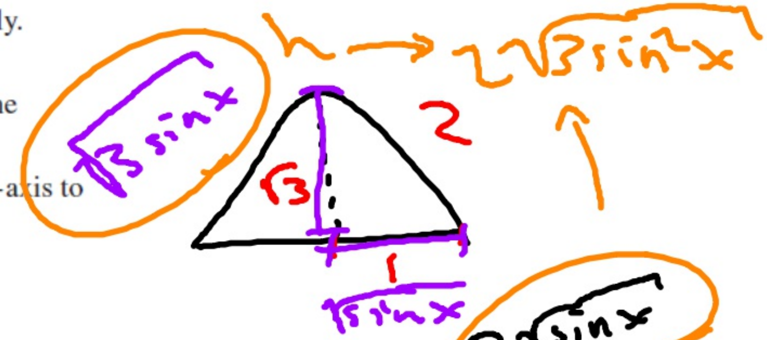
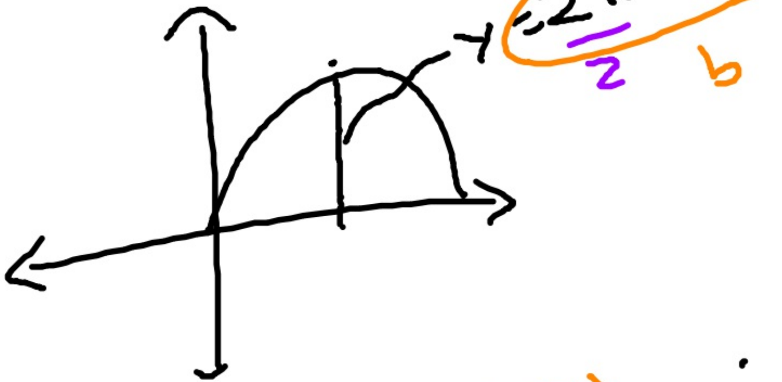
In Exercises 39–42, find the volume of the solid analytically.

39. The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are

(**a**) equilateral triangles with bases running from the x-axis to the curve as shown in the figure. $2\sqrt{3}$





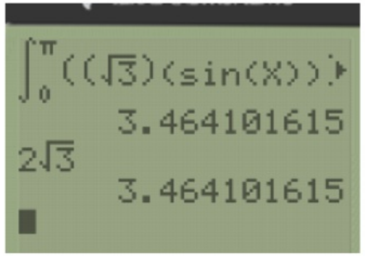


Create a Formula
w/ one variable...

$A = \frac{1}{2}bh$

$= \frac{1}{2}\left(2\sin x(\sqrt{3})\right) = (\sin x)(\sqrt{3})$

$\int_0^\pi (\sqrt{3})(\sin x)$



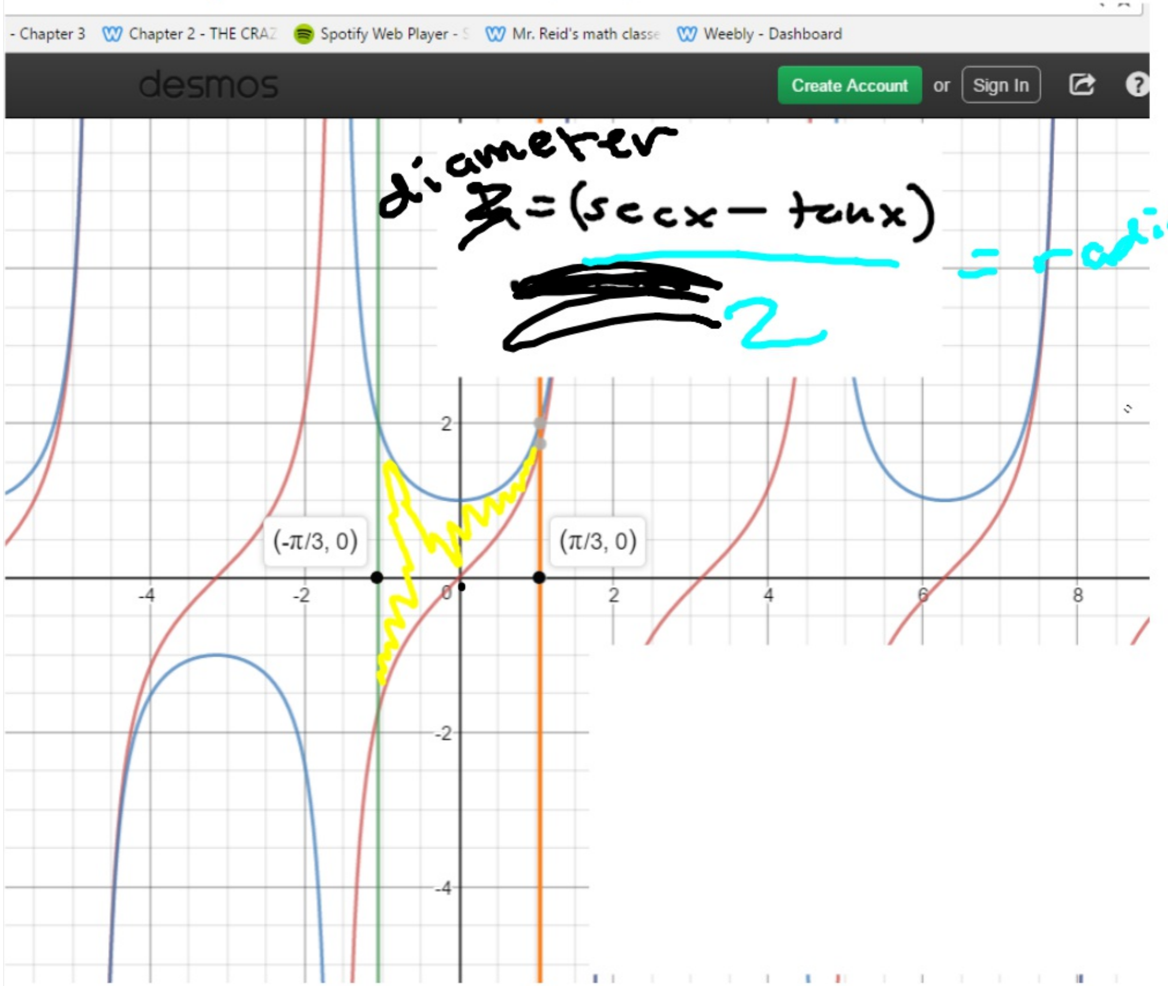
40. The solid lies between planes perpendicular to the x-axis at $x = -\pi/3$ and $x = \pi/3$. The cross sections perpendicular to the x-axis are

$A = \pi r^2$

(a) circular disks with diameters running from the curve $y = \tan x$ to the curve $y = \sec x$. $\pi\sqrt{3} - (\pi^2/6)$

(b) squares whose bases run from the curve $y = \tan x$ to the curve $y = \sec x$. $4\sqrt{3} - (2\pi/3)$

$$\int_{-\pi/3}^{\pi/3} \pi \left(\frac{\sec x - \tan x}{2}\right)^2$$



diameter

$d = \dfrac{(\sec x - \tan x)}{2}$ = radius

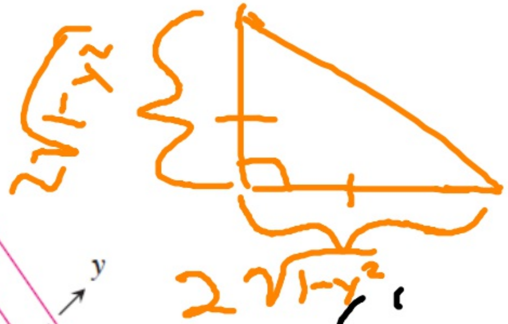
40. The solid lies between planes perpendicular to the x-axis at $x = -\pi/3$ and $x = \pi/3$. The cross sections perpendicular to the x-axis are

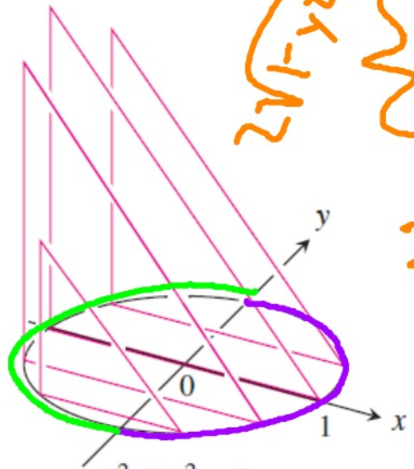
(a) circular disks with diameters running from the curve $y = \tan x$ to the curve $y = \sec x$. $\pi\sqrt{3} - (\pi^2/6)$

(b) squares whose bases run from the curve $y = \tan x$ to the curve $y = \sec x$. $4\sqrt{3} - (2\pi/3)$

41. The solid lies between planes perpendicular to the y-axis at $y = 0$ and $y = 2$. The cross sections perpendicular to the y-axis are circular disks with diameters running from the y-axis to the parabola $x = \sqrt{5}y^2$. 8π

42. The base of the solid is the disk $x^2 + y^2 \le 1$. The cross sections by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk. 8/3





$x^2 + y^2 = 1$

area of a triangle

$$\int_{-1}^{1} \frac{(f(y))^2}{2}\, dy$$

$$f(y) = \left(\sqrt{1-y^2}\right) - \left(-\sqrt{1-y^2}\right)$$

$$x^2 = 1 - y^2$$

$$x = \pm\sqrt{1-y^2}$$

$$\int_{-1}^{1} \frac{\left(2\sqrt{1-y^2}\right)^2}{2}\, dy = \int_{-1}^{1} 2(1-y^2)\, dy$$