New Vocabulary

- polynomial
- binomial
- trinomial
- degree of a monomial
- degree of a polynomial
- standard form of a polynomial
- leading coefficient

1 Polynomials in Standard Form A **polynomial** is a monomial or the sum of monomials, each called a *term* of the polynomial. Some polynomials have special names. A **binomial** is the sum of *two* monomials, and a **trinomial** is the sum of *three* monomials.

Monomial	Binomial	Trinomial
$5x$	$2x^2 + 7$	$x^3 - 10x + 1$

WAIT. Remember what I said about monomials back in 7-1?

1 Multiply Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. It has only one term. In the formula to calculate the horsepower of a car, the term $w\left(\frac{v}{234}\right)^3$ is a monomial.

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An expression that involves division by a variable, like $\frac{ab}{c}$, is not a monomial.

abc^{-1}

This will come up in today's work. Be careful!

The **degree of a monomial** is the sum of the exponents of all its variables. A nonzero constant term has degree 0, and zero has no degree.

The **degree of a polynomial** is the greatest degree of any term in the polynomial. You can find the degree of a polynomial by finding the degree of each term. Polynomials are named based on their degree.

Degree	Name
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic
6 or more	6th degree, 7th degree, and so on

1B

$-3y^2+2y-1$

$4y + (-5xz)$

PT

Example 1 Identify Polynomials

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

Expression	Is it a polynomial?	Degree	Monomial, binomial, or trinomial?
a. $4y - 5xz$	Yes; $4y - 5xz$ is the sum of $4y$ and $-5xz$.	2	binomial
b. -6.5	Yes; -6.5 is a real number.	0	monomial
c. $7a^{-3} + 9b$	No; $7a^{-3} = \frac{7}{a^3}$, which is not a monomial.	—	———
d. $6x^3 + 4x + x + 3$	Yes; $6x^3 + 4x + x + 3 = 6x^3 + 5x + 3$, the sum of three monomials.	3	trinomial

main concern;

is the exponent a non-negative integer?

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1A. x **yes; 1; monomial**

1B. $-3y^2 - 2y + 4y - 1$ **yes; 2; trinomial**

1C. $5rx + 7tuv$ **yes; 3; binomial**

1D. $10x^{-4} - 8x^a$

1D. No; $10x^{-4} = \frac{10}{x^4}$, which is not a monomial, and $8x^a$ has a variable exponent.

Example 1 Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial, binomial,* or *trinomial.*

1. $7ab + 6b^2 - 2a^3$ yes; 3; trinomial
2. $2y - 5 + 3y^2$
3. $3x^2$ yes; 2; monomial
4. $\frac{4m}{3p}$ No; a monomial cannot have a variable in the denominator.
5. $5m^2p^3 + 6$ yes; 5; binomial
6. $5q^{-4} + 6q$ No; $5q^{-4} = \frac{5}{q^4}$, and a monomial cannot have a variable in the denominator.

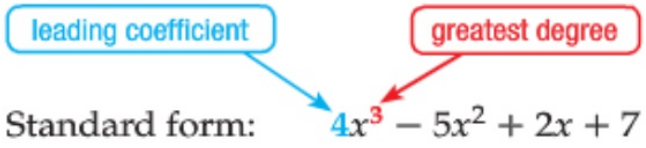
Practice and Problem Solving

Extra Practice is on page R8.

Example 1 Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial, binomial,* or *trinomial.*

20. $\frac{5y^3}{x^2} + 4x$ No; a monomial cannot have a variable in the denominator.
21. 21 yes; 0; monomial
22. $c^4 - 2c^2 + 1$ yes; 4; trinomial
23. $d + 3d^c$ No; the exponent is a variable.
24. $a - a^2$ yes; 2; binomial
25. $5n^3 + nq^3$ yes; 4; binomial

The terms of a polynomial can be written in any order. However, polynomials in one variable are usually written in standard form. The **standard form of a polynomial** has the terms in order from greatest to least degree. In this form, the coefficient of the first term is called the **leading coefficient**.



Example 2 Standard Form of a Polynomial

Write each polynomial in standard form. Identify the leading coefficient.

a. $3x^2 + 4x^5 - 7x$

Find the degree of each term.

Degree: 2 5 1

Polynomial: $3x^2 + 4x^5 - 7x$

The greatest degree is 5. Therefore, the polynomial can be rewritten as $4x^5 + 3x^2 - 7x$, with a leading coefficient of 4.

b. $5y - 9 - 2y^4 - 6y^3$

Find the degree of each term.

Degree: 1 0 4 3

Polynomial: $5y - 9 - 2y^4 - 6y^3$

The greatest degree is 4. Therefore, the polynomial can be rewritten as $-2y^4 - 6y^3 + 5y - 9$, with a leading coefficient of -2.

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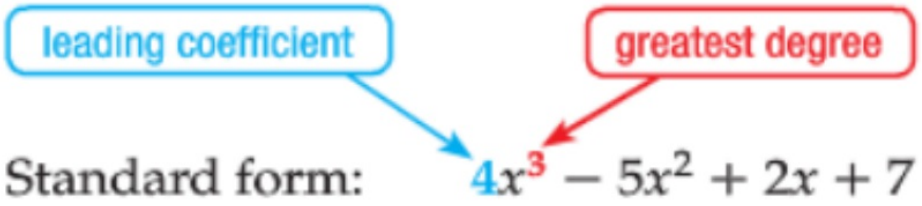
2A. $8 - 2x^2 + 4x^4 - 3x$

2B. $y + 5y^3 - 2y^2 - 7y^6 + 10$



Standard form: $4x^3 - 5x^2 + 2x + 7$

Example 2 **Write each polynomial in standard form. Identify the leading coefficient.**

7. $2x^5 - 12 + 3x$ $2x^5 + 3x - 12; 2$

8. $-4d^4 + 1 - d^2$

9. $4z - 2z^2 - 5z^4$ $-5z^4 - 2z^2 + 4z; -5$

10. $2a + 4a^3 - 5a^2 - 1$

$-5z^4 - 2z^2 + 4z$

Example 2 **Write each polynomial in standard form. Identify the leading coefficient.**

26. $5x^2 - 2 + 3x$ $5x^2 + 3x - 2; 5$

27. $8y + 7y^3$ $7y^3 + 8y; 7$

28. $4 - 3c - 5c^2$ $-5c^2 - 3c + 4; -5$

29. $-y^3 + 3y - 3y^2 + 2$ $-y^3 - 3y^2 + 3y + 2; -1$

30. $11t + 2t^2 - 3 + t^5$ $t^5 + 2t^2 + 11t - 3; 1$

31. $2 + r - r^3$ $-r^3 + r + 2; -1$

32. $\frac{1}{2}x - 3x^4 + 7$ $-3x^4 + \frac{1}{2}x + 7; -3$

33. $-9b^2 + 10$ $+ 10b; -1$

2 Add and Subtract Polynomials

Adding polynomials involves adding like terms. You can group like terms by using a horizontal or vertical format.

Example 3 Add Polynomials

Find each sum.

a. $(2x^2 + 5x - 7) + (3 - 4x^2 + 6x)$

Horizontal Method

Group and combine like terms.

$$(2x^2 + 5x - 7) + (3 - 4x^2 + 6x)$$
$$= [2x^2 + (-4x^2)] + [5x + 6x] + [-7 + 3]$$ Group like terms.
$$= -2x^2 + 11x - 4$$ Combine like terms.

b. $(3y + y^3 - 5) + (4y^2 - 4y + 2y^3 + 8)$

Vertical Method

Align like terms in columns and combine.

$$\begin{array}{r} y^3 + 0y^2 + 3y - 5 \\ (+)\ 2y^3 + 4y^2 - 4y + 8 \\ \hline 3y^3 + 4y^2 - y + 3 \end{array}$$

Insert a placeholder to help align the terms.

Align and combine like terms.

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3A. $(5x^2 - 3x + 4) + (6x - 3x^2 - 3)$ $2x^2 + 3x + 1$

3B. $(y^4 - 3y + 7) + (2y^3 + 2y - 2y^4 - 11)$

Example 4 Subtract Polynomials

Find each difference.

a. $(3 - 2x + 2x^2) - (4x - 5 + 3x^2)$

Horizontal Method

Subtract $4x - 5 + 3x^2$ by adding its additive inverse.

$(3 - 2x + 2x^2) - (4x - 5 + 3x^2)$

$= (3 - 2x + 2x^2) + (-4x + 5 - 3x^2)$ The additive inverse of $4x - 5 + 3x^2$ is $-4x + 5 - 3x^2$.

$= [2x^2 + (-3x^2)] + [(-2x) + (-4x)] + [3 + 5]$ Group like terms.

$= -x^2 - 6x + 8$ Combine like terms.

b. $(7p + 4p^3 - 8) - (3p^2 + 2 - 9p)$

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 4p^3 + 0p^2 + 7p - 8 \\ (-) & 3p^2 - 9p + 2 \\ \hline \end{array} \quad \text{Add the opposite.} \quad \begin{array}{rr} & 4p^3 + 0p^2 + 7p - 8 \\ (+) & -3p^2 + 9p - 2 \\ \hline & 4p^3 - 3p^2 + 16p - 10 \end{array}$$

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4A. $(4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)$

4B. $(8y - 10 + 5y^2) - (7 - y^3 + 12y)$

You're basically doing the same thing as the last example, just make sure to *distribute the negative!*

Examples 3–4 Find each sum or difference. 13. $-a^2 + 6a - 3$ 15. $-8z^3 - 3z^2 - 2z + 13$

11. $(6x^3 - 4) + (-2x^3 + 9)$ $4x^3 + 5$

12. $(g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)$

13. $(4 + 2a^2 - 2a) - (3a^2 - 8a + 7)$

14. $(8y - 4y^2) + (3y - 9y^2)$

15. $(-4z^3 - 2z + 8) - (4z^3 + 3z^2 - 5)$

16. $(-3d^2 - 8 + 2d) + (4d - 12 + d^2)$

17. $(y + 5) + (2y + 4y^2 - 2)$ $4y^2 + 3y + 3$

18. $(3n^3 - 5n + n^2) - (-8n^2 + 3n^3)$

$-2 - (-8)$

$-2 + 8$

Examples 3–4 Find each sum or difference.

34. $(2c^2 + 6c + 4) + (5c^2 - 7)$ $7c^2 + 6c - 3$

35. $(2x + 3x^2) - (7 - 8x^2)$ $11x^2 + 2x - 7$

36. $(3c^3 - c + 11) - (c^2 + 2c + 8)$ $3c^3 - c^2 - 3c + 3$

37. $(z^2 + z) + (z^2 - 11)$ $2z^2 + z - 11$

38. $(2x - 2y + 1) - (3y + 4x)$ $-2x - 5y + 1$

39. $(4a - 5b^2 + 3) + (6 - 2a + 3b^2)$ $-2b^2 + 2a + 9$

40. $(x^2y - 3x^2 + y) + (3y - 2x^2y)$ $-x^2y - 3x^2 + 4y$

41. $(-8xy + 3x^2 - 5y) + (4x^2 - 2y + 6xy)$ $7x^2 - 2xy - 7y$

42. $(5n - 2p^2 + 2np) - (4p^2 + 4n)$ $-6p^2 + 2np + n$

43. $(4rxt - 8r^2x + x^2) - (6rx^2 + 5rxt - 2x^2)$ $3x^2 - rxt - 8r^2x - 6rx^2$

Example 5

19. CCSS **SENSE-MAKING** The total number of students T who traveled for spring break consists of two groups: students who flew to their destinations F and students who drove to their destination D. The number (in thousands) of students who flew and the total number of students who flew or drove can be modeled by the following equations, where n is the number of years since 1995.

$$T = 14n + 21 \qquad F = 8n + 7$$

a. Write an equation that models the number of students who drove to their destination for this time period.

b. Predict the number of students who will drive to their destination in 2012.

c. How many students will drive or fly to their destination in 2015?