8-2 Adding and Subtracting Rational Expressions

1 LCM of Polynomials Just as with rational numbers in fractional form, to add or subtract two rational expressions that have unlike denominators, you must first find the least common denominator (LCD). The LCD is the least common multiple (LCM) of the denominators.

To find the LCM of two or more numbers or polynomials, factor them. The LCM contains each factor the greatest number of times it appears as a factor.

Numbers

$\frac{5}{6} + \frac{4}{9}$

LCM of 6 and 9

$6 = 2 \cdot 3$

$9 = 3 \cdot 3$

LCM $= 2 \cdot 3 \cdot 3$ or 18

Polynomials

$$\frac{3}{x^2 - 3x + 2} + \frac{5}{2x^2 - 2}$$

LCM of $x^2 - 3x + 2$ and $2x^2 - 2$

$x^2 - 3x + 2 = (x - 1)(x - 2)$

$2x^2 - 2 = 2 \cdot (x - 1)(x + 1)$

LCM $= 2(x - 1)(x - 2)(x + 1)$

$16x$ → $2 \cdot 2 \cdot 2 \cdot 2 \cdot x$

$8x^2y^3$ → $2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y$

$5x^3y$ → $2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$

b. $3y(y-3)(y-5)$ LCM $= 3y(y-3)(y-5)$

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 Find the LCM of each set of polynomials.

1. $16x, 8x^2y^3, 5x^3y$ $80x^3y^3$
2. $7a^2, 9ab^3, 21abc^4$ $63a^2b^3c^4$
3. $3y^2 - 9y, y^2 - 8y + 15$ $3y(y-3)(y-5)$
4. $x^3 - 6x^2 - 16x, x^2 - 4$

KeyConcept

Adding Rational Expressions	
Words	To add rational expressions, find the least common denominator (LCD). Rewrite each expression with the LCD. Then add.
Symbols	For all $\frac{a}{b}$ and $\frac{c}{d}$, with $b \neq 0$ and $d \neq 0$, $\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$.
Subtracting Rational Expressions	
Words	To subtract rational expressions, find the least common denominator (LCD). Rewrite each expression with the LCD. Then subtract.
Symbols	For all $\frac{a}{b}$ and $\frac{c}{d}$, with $b \neq 0$ and $d \neq 0$, $\frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{bc}{bd} = \frac{ad - bc}{bd}$.

Example 2 Monomial Denominators

Simplify $\frac{3y}{2x^3} + \frac{5z}{8xy^2}$.

Examples 2–3 Simplify each expression.

5. $\frac{12y}{5x} + \frac{5x}{4y^3}$

6. $\frac{5}{6ab} + \frac{3b^2}{14a^3}$

7. $\frac{7b}{12a} - \frac{1}{18ab^3}$

8. $\frac{y^2}{8c^2d^2} - \frac{3x}{14c^4d}$

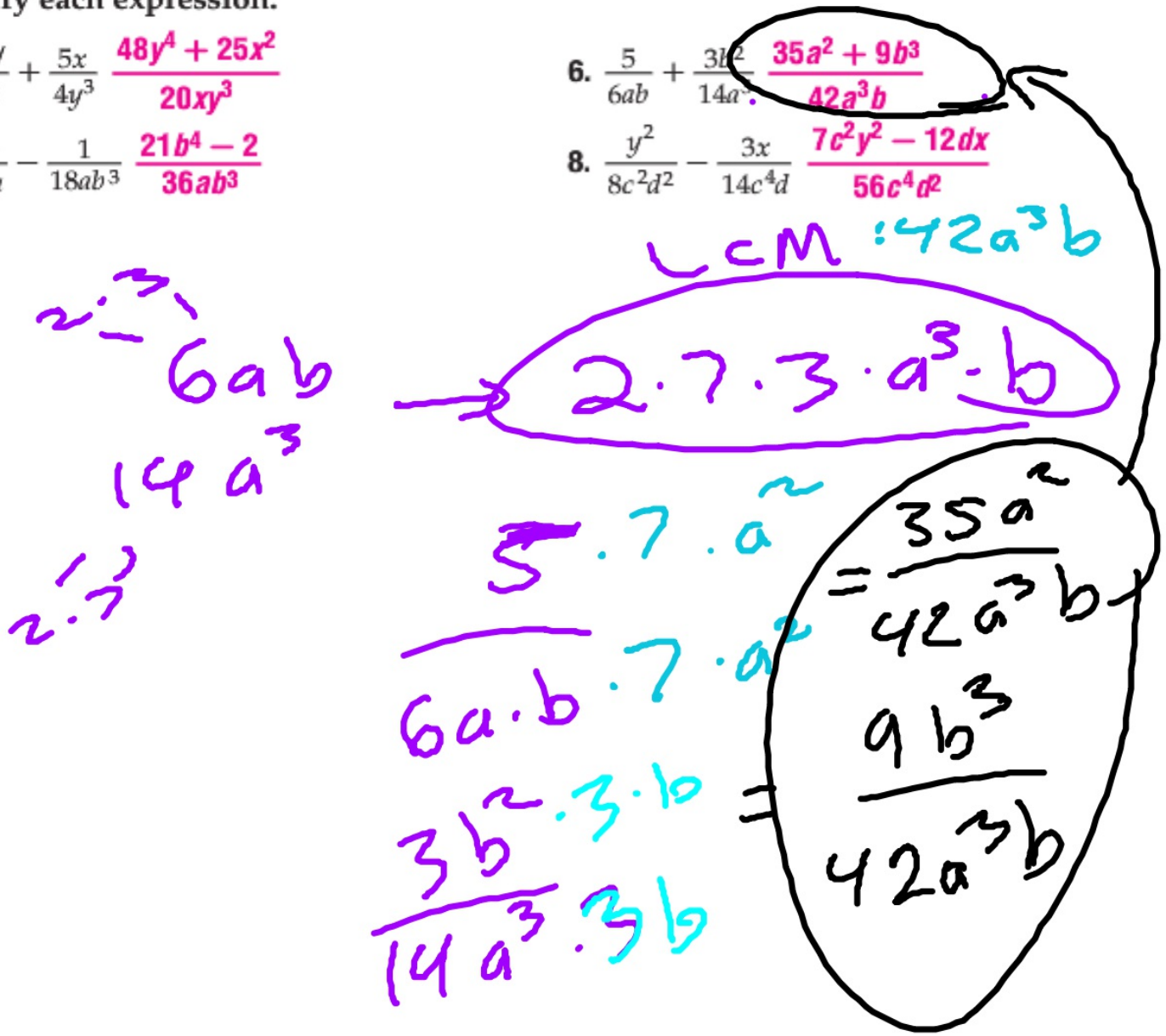
Examples 2–3 Simplify each expression.

5. $\frac{12y}{5x} + \frac{5x}{4y^3}$ $\frac{48y^4 + 25x^2}{20xy^3}$

6. $\frac{5}{6ab} + \frac{3b^2}{14a^3}$ $\frac{35a^2 + 9b^3}{42a^3b}$

7. $\frac{7b}{12a} - \frac{1}{18ab^3}$ $\frac{21b^4 - 2}{36ab^3}$

8. $\frac{y^2}{8c^2d^2} - \frac{3x}{14c^4d}$ $\frac{7c^2y^2 - 12dx}{56c^4d^2}$



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Example 3 Polynomial Denominators

Simplify $\frac{5}{6x - 18} - \frac{x - 1}{4x^2 - 14x + 6}$.

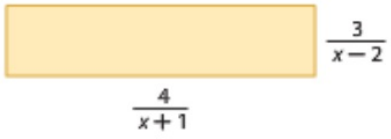
9. $\frac{4x}{x^2 + 9x + 18} + \frac{5}{x + 6}$

10. $\frac{8}{y - 3} + \frac{2y - 5}{y^2 - 12y + 27}$

11. $\frac{4}{3x + 6} - \frac{x + 1}{x^2 - 4}$

12. $\frac{3a + 2}{a^2 - 16} - \frac{7}{6a + 24}$

13. GEOMETRY Find the perimeter of the rectangle.



9. $\frac{4x}{x^2+9x+18} + \frac{5}{x+6}$ **$\frac{9x+15}{(x+3)(x+6)}$**

10. $\frac{8}{y-3} + \frac{2y-5}{y^2-12y+27}$ **$\frac{10y-77}{(y-3)(y-9)}$**

11. $\frac{4}{3x+6} - \frac{x+1}{x^2-4}$ **$\frac{x-11}{3(x+2)(x-2)}$**

12. $\frac{3a+2}{a^2-16} - \frac{7}{6a+24}$ **$\frac{11a+40}{6(a+4)(a-4)}$**

13. **GEOMETRY** Find the perimeter of the rectangle.

$\frac{14x-10}{(x+1)(x-2)}$

(9) $\frac{4x}{(x+3)(x+6)} + \frac{5}{(x+6)} \cdot \frac{(x+3)}{(x+3)}$

$\frac{4x+5x+15}{(x+3)(x+6)}$

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Example 4 Complex Fractions with Different LCDs

Simplify $\dfrac{1 + \frac{1}{x}}{1 - \frac{x}{y}}$.

Example 5 Complex Fractions with Same LCDs

Simplify $\dfrac{1 + \frac{1}{x}}{1 - \frac{x}{y}}$.

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Examples 4–5 Simplify each expression.

14. $\dfrac{4 + \frac{2}{x}}{3 - \frac{2}{x}}$

15. $\dfrac{6 + \frac{4}{y}}{2 + \frac{6}{y}}$

16. $\dfrac{\frac{3}{x} + \frac{2}{y}}{1 + \frac{4}{y}}$

17. $\dfrac{\frac{2}{b} + \frac{5}{a}}{\frac{3}{a} - \frac{8}{b}}$

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Practice and Problem Solving

Extra Practice is on page R8.

Example 1 Find the LCM of each set of polynomials.

18. $24cd, 40a^2c^3d^4, 15abd^3$ **$120a^2bc^3d^4$**

19. $4x^2y^3, 18xy^4, 10xz^2$ **$180x^2y^4z^2$**

20. $x^2 - 9x + 20, x^2 + x - 30$ **$(x-4)(x-5)(x+6)$**

21. $6x^2 + 21x - 12, 4x^2 + 22x + 24$ **$6(x+4)(2x-1)(2x+3)$**

Examples 2–3 CCSS **PERSEVERANCE** Simplify each expression.

22. $\frac{5a}{24cf^4} + \frac{a}{36bc^4f^3}$ **$\frac{15abc^3 + 2af}{72bc^4f^4}$**

23. $\frac{4b}{15x^3y^2} - \frac{3b}{35x^2y^4z}$ **$\frac{28by^2z - 9bx}{105x^3y^4z}$**

24. $\frac{5b}{6a} + \frac{3b}{10a^2} + \frac{2}{ab^2}$ **$\frac{25ab^3 + 9b^3 + 60a}{30a^2b^2}$**

25. $\frac{4}{3x} + \frac{8}{x^3} + \frac{2}{5xy}$ **$\frac{20x^2y + 120y + 6x^2}{15x^3y}$**

26. $\frac{8}{3y} + \frac{2}{9} - \frac{3}{10y^2}$ **$\frac{240y + 20y^2 - 27}{90y^2}$**

27. $\frac{1}{16a} + \frac{5}{12b} - \frac{9}{10b^3}$ **$\frac{15b^3 + 100ab^2 - 216a}{240ab^3}$**

28. $\frac{8}{x^2 - 6x - 16} + \frac{9}{x^2 - 3x - 40}$

29. $\frac{6}{y^2 - 2y - 35} + \frac{4}{y^2 + 9y + 20}$ **$\frac{10y - 4}{(y-7)(y+5)(y+4)}$**

30. $\frac{12}{3y^2 - 10y - 8} - \frac{3}{y^2 - 6y + 8}$

31. $\frac{6}{2x^2 + 11x - 6} - \frac{8}{x^2 + 3x - 18}$ **$\frac{-10x - 10}{(2x-1)(x+6)(x-3)}$**

32. $\frac{2x}{4x^2 + 9x + 2} + \frac{3}{2x^2 - 8x - 24}$

33. $\frac{4x}{3x^2 + 3x - 18} - \frac{2x}{2x^2 + 11x + 15}$ **$\frac{2x^2 + 32x}{3(x-2)(x+3)(2x+5)}$**

34. **BIOLOGY** After a person eats something, the pH or acid level A of his or her mouth can be determined by the formula $A = \frac{20.4t}{t^2 + 36} + 6.5$, where t is the number of minutes that have elapsed since the food was eaten.

a. Simplify the equation. **$A = \frac{6.5t^2 + 20.4t + 234}{t^2 + 36}$**

b. What would the acid level be after 30 minutes? **≈ 7.2**

28. **$\frac{17x + 58}{(x-8)(x+2)(x+5)}$**

30. **$\frac{3y - 30}{(3y+2)(y-4)(y-2)}$**

32. **$\frac{4x^2 - 12x + 3}{2(x-6)(4x+1)(x+2)}$**