"I can't plug in zero..." domain!!

8-3 Graphing Reciprocal Functions

Open to page 545!

$n \neq 0$

"n" "c"

Why?

The East High School Chorale wants to raise $5000 to fund a trip to a national competition in Tampa. They have decided to sell candy bars. They will make a $1 profit on each candy bar they sell, so they need to sell 5000 candy bars.

If *c* represents the number of candy bars each student has to sell and *n* represents the number of students, then $c = \frac{5000}{n}$.

"I can't calculate out zero..."

range...

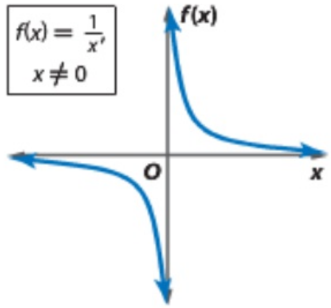
$\frac{5000}{1} = 5000$

$\frac{5000}{25} = 200$

$u(x)$ is a linear function and $u(x) \neq 0$.

KeyConcept Parent Function of Reciprocal Functions

Parent function:	$f(x) = \frac{1}{x}$
Type of graph:	**hyperbola**
Domain and range:	all nonzero real numbers
Asymptotes:	$x = 0$ and $f(x) = 0$
Intercepts:	none
Not defined:	$x = 0$



The domain of a reciprocal function is limited to values for which the function is defined.

Functions:	$f(x) = \frac{-3}{x+2}$	$g(x) = \frac{4}{x-5}$	$h(x) = \frac{3}{x}$
Not defined at:	$x = -2$	$x = 5$	$x = 0$

Example 1 Limitations on Domain

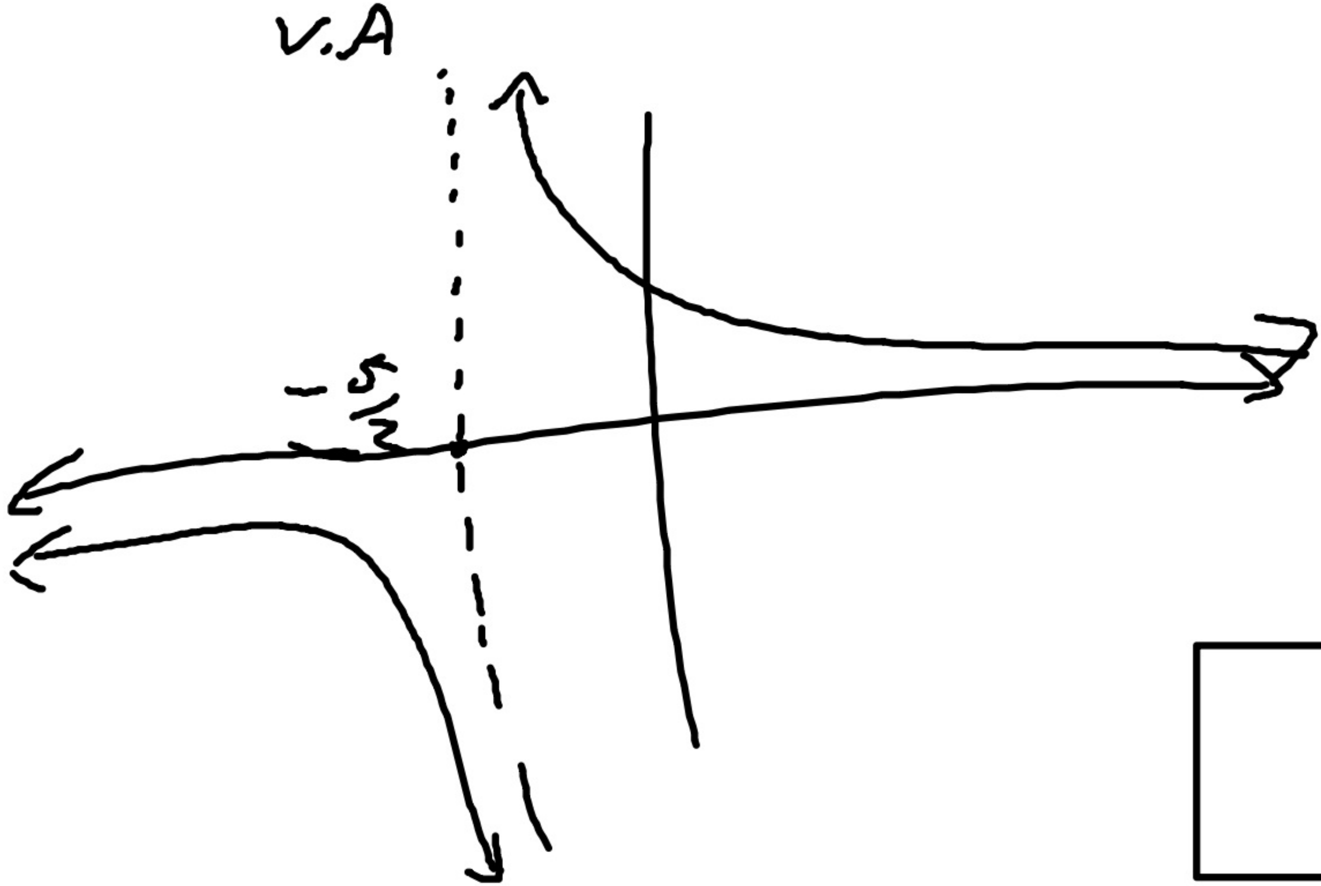
Determine the value of x for which $f(x) = \dfrac{3}{2x+5}$ is not defined.

Find the value for which the denominator of the expression equals 0.

$$\frac{3}{2x+5} \quad \rightarrow \quad 2x + 5 = 0$$

$$x = -\frac{5}{2}$$

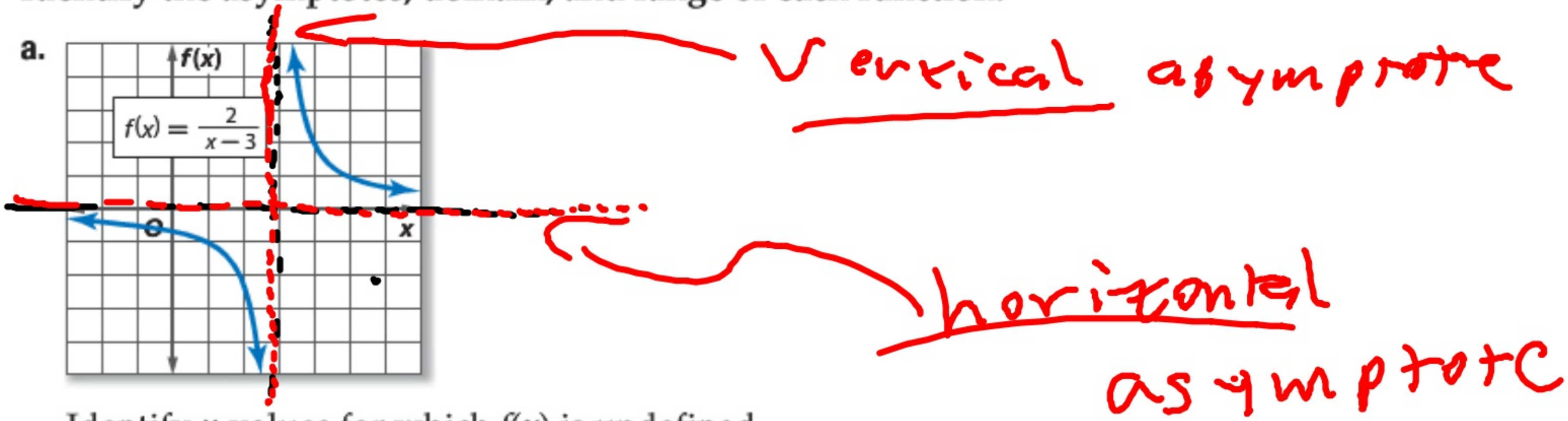
The function is undefined for $x = -\frac{5}{2}$.



Example 2 Determine Properties of Reciprocal Functions

Identify the asymptotes, domain, and range of each function.

a.



Identify x-values for which $f(x)$ is undefined.

$$x - 3 = 0$$
$$x = 3$$

$f(x)$ is not defined when $x = 3$. So there is an asymptote at $x = 3$.

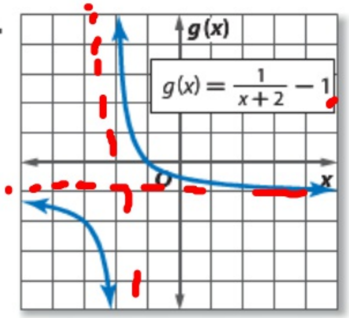
From $x = 3$, as x-values decrease, $f(x)$-values approach 0, and as x-values increase, $f(x)$-values approach 0. So there is an asymptote at $f(x) = 0$.

The domain is all real numbers not equal to 3 and the range is all real numbers not equal to 0.

Example 2 Determine Properties of Reciprocal Functions

Identify the asymptotes, domain, and range of each function.

b.



Identify x-values for which $g(x)$ is undefined.

$x + 2 = 0$

$x = -2$

$g(x)$ is not defined when $x = -2$. So there is an asymptote at $x = -2$.

From $x = -2$, as x-values decrease, $g(x)$-values approach -1, and as x-values increase, $g(x)$-values approach -1. So there is an asymptote at $g(x) = -1$.

The domain is all

not equal to -1.

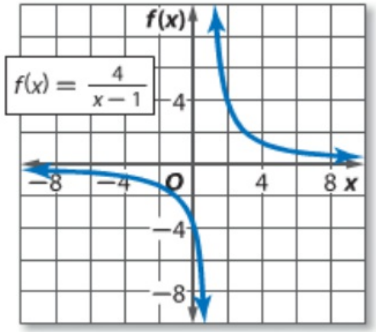
$x = 3$, $f(x) = -2$; $x = -1$, $g(x) = 5$;

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

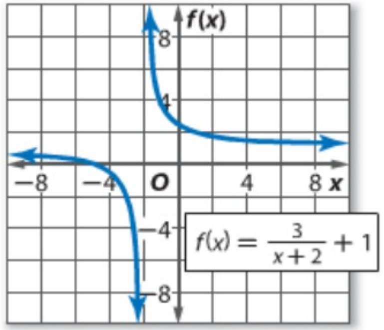
Examples 1–2 Identify the asymptotes, domain, and range of each function.

1.



$x = 1$, $f(x) = 0$;
$D = \{x \mid x \neq 1\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

2.



$x = -2$, $f(x) = 1$;
$D = \{x \mid x \neq -2\}$;
$R = \{f(x) \mid f(x) \neq 1\}$

KeyConcept Transformations of Reciprocal Functions

$$f(x) = \frac{a}{x-h} + k$$

h – Horizontal Translation	*k* – Vertical Translation
h units right if *h* is positive	*k* units up if *k* is positive
$|h|$ units left if *h* is negative	$|k|$ units down if *k* is negative
The *vertical* asymptote is at $x = h$.	The *horizontal* asymptote is at $f(x) = k$.

a – Orientation and Shape

If $a < 0$, the graph is reflected across the *x*-axis.

If $|a| > 1$, the graph is stretched vertically. If $0 < |a| < 1$, the graph is compressed vertically.

Example 3 Graph Transformations

Graph each function. State the domain and range.

$x \neq 4 \quad y \neq 2$

a. $f(x) = \frac{2}{x-4} + 2$

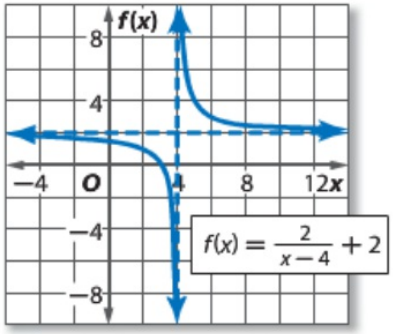
This represents a transformation of the graph of $f(x) = \frac{1}{x}$.

$a = 2$: The graph is stretched vertically.

$h = 4$: The graph is translated 4 units right. There is an asymptote at $x = 4$.

$k = 2$: The graph is translated 2 units up. There is an asymptote at $f(x) = 2$.

Domain: $\{x \mid x \neq 4\}$ Range: $\{f(x) \mid f(x) \neq 2\}$



KeyConcept Transformations of Reciprocal Functions

$$f(x) = \frac{a}{x - h} + k$$

h – Horizontal Translation	*k* – Vertical Translation
h units right if *h* is positive	*k* units up if *k* is positive
$\lvert h \rvert$ units left if *h* is negative	$\lvert k \rvert$ units down if *k* is negative
The *vertical* asymptote is at $x = h$.	The *horizontal* asymptote is at $f(x) = k$.

a – Orientation and Shape	
If $a < 0$, the graph is reflected across the *x*-axis.	If $\lvert a \rvert > 1$, the graph is stretched vertically. If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

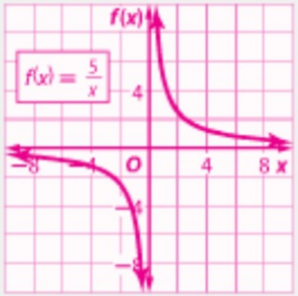
Example 3 Graph Transformations

Graph each function. State the domain and range.

b. $f(x) = \frac{-3}{x + 1} - 4$

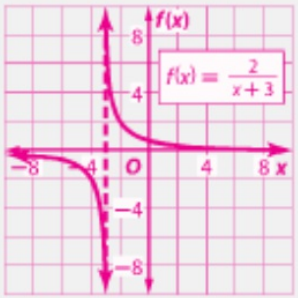
Additional Answers

3.



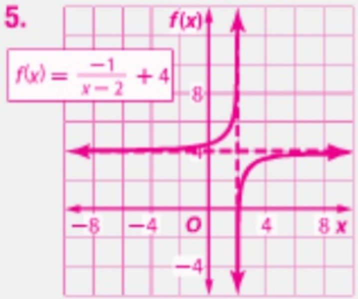
$D = \{x \mid x \neq 0\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

4.



$D = \{x \mid x \neq -3\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

5.



$D = \{x \mid x \neq 2\}$;
$R = \{f(x) \mid f(x) \neq 4\}$

KeyConcept Transformations of Reciprocal Functions

$$f(x) = \frac{a}{x-h} + k$$

h – Horizontal Translation	*k* – Vertical Translation
h units right if *h* is positive	*k* units up if *k* is positive
$\lvert h \rvert$ units left if *h* is negative	$\lvert k \rvert$ units down if *k* is negative
The *vertical* asymptote is at $x = h$.	The *horizontal* asymptote is at $f(x) = k$.

a – Orientation and Shape

If $a < 0$, the graph is reflected across the *x*-axis.	If $\lvert a \rvert > 1$, the graph is stretched vertically. If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

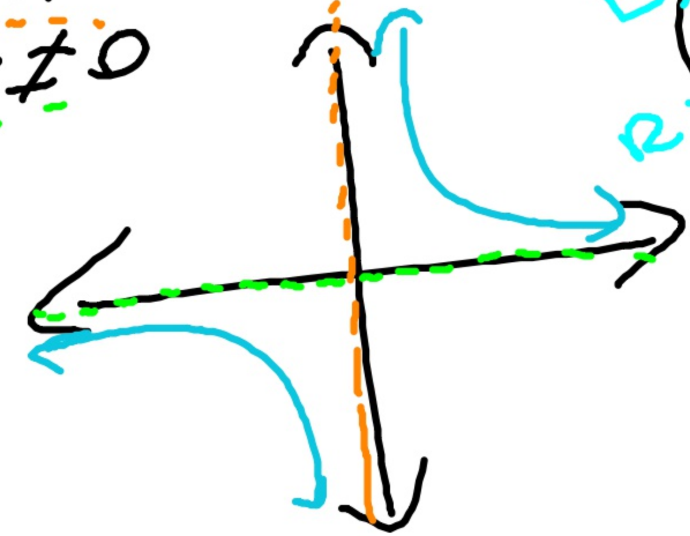
Graph each function. State the domain and range. **3–5. See margin.**

3. $f(x) = \frac{5}{x}$

4. $f(x) = \frac{2}{x+3}$

5. $f(x) = \frac{-1}{x-2} + 4$

(3) D: $x \neq 0$
R: $y \neq 0$



D: $x \neq 2$
R: $y \neq 4$

Real-World Example 4 Write Equations

TRAVEL An airline has a daily nonstop flight between Los Angeles, California, and Sydney, Australia. A one-way trip is about 7500 miles.

a. Write an equation to represent the travel time from Los Angeles to Sydney as a function of flight speed. Then graph the equation.

b. Explain any limitations to the range or domain in this situation.

Example 4

6. **CCSS SENSE-MAKING** A group of friends plans to get their youth group leader a gift certificate for a day at a spa. The certificate costs $150.

   a. If c represents the cost for each friend and f represents the number of friends, write an equation to represent the cost to each friend as a function of how many friends give. [redacted]

   b. Graph the function. **See margin.** [redacted]

   c. Explain any limitations to the range or domain in this situation.

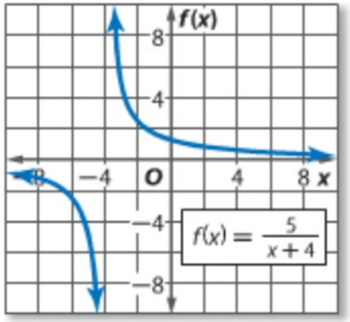
[redacted]

Practice and Problem Solving

Extra Practice is on page

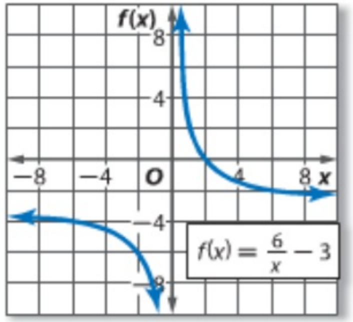
Examples 1–2 Identify the asymptotes, domain, and range of each function.

7.



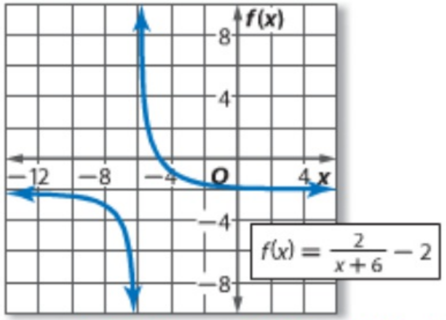
$x = -4$, $f(x) = 0$; $D = \{x \mid x \neq -4\}$; $R = \{f(x) \mid f(x) \neq 0\}$

8.



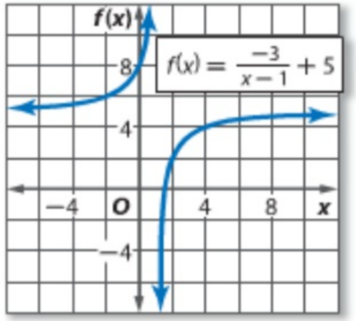
$x = 0$, $f(x) = -3$; $D = \{x \mid x \neq 0\}$; $R = \{f(x) \mid f(x) \neq -3\}$

9.



$x = -6$, $f(x) = -2$; $D = \{x \mid x \neq -6\}$; $R = \{f(x) \mid f(x) \neq -2\}$

10.



$x = 1$, $f(x) = 5$; $D = \{x \mid x \neq 1\}$; $R = \{f(x) \mid f(x) \neq 5\}$

$R = \{f(x) \mid f(x) \neq -2\}$

Example 3 **Graph each function. State the domain and range.** **11–22. See Chapter 8 Answer Appendix.**

11. $f(x) = \frac{3}{x}$

12. $f(x) = \frac{-4}{x+2}$

13. $f(x) = \frac{2}{x-6}$

14. $f(x) = \frac{6}{x} - 5$

15 $f(x) = \frac{2}{x} + 3$

16. $f(x) = \frac{8}{x}$

17. $f(x) = \frac{-2}{x-5}$

18. $f(x) = \frac{3}{x-7} - 8$

19. $f(x) = \frac{9}{x+3} + 6$

20. $f(x) = \frac{8}{x+3}$

21. $f(x) = \frac{-6}{x+4} - 2$

22. $f(x) = \frac{-5}{x-2} + 2$

Example 4 **23. CYCLING** Marina's New Year's resolution is to ride her bike 5000 miles.

a. If m represents the mileage Marina rides each day and d represents the number of days, write an equation to represent the mileage each day as a function of the number of days that she rides. $m = \frac{5000}{d}$

b. Graph the function. **See Chapter 8 Answer Appendix.**

c. If she rides her bike every day of the year, how many miles should she ride each day to meet her goal? **13.7 mi**

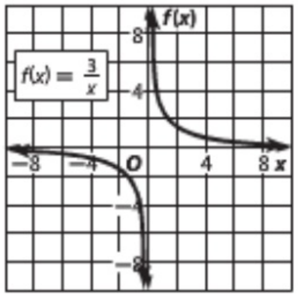
24. CCSS MODELING Parker has 200 grams of an unknown liquid. Knowing the density will help him discover what type of liquid this is.

a. Density of a liquid is found by dividing the mass by the volume. Write an equation to represent the density of this unknown as a function of volume. $d = \frac{200}{v}$

b. Graph the function. **See Chapter 8 Answer Appendix.**

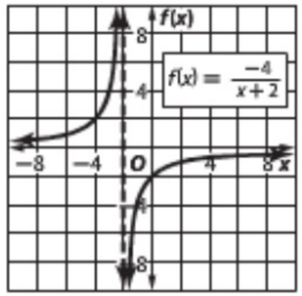
c. From the graph, identify the asymptotes, domain, and range of the function.
$v = 0$, $d = 0$; $D = \{v \mid v \neq 0\}$; $R = \{d \mid d \neq 0\}$

11.



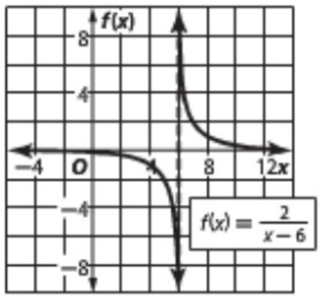
$D = \{x \mid x \neq 0\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

12.



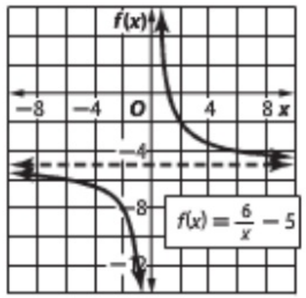
$D = \{x \mid x \neq -2\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

13.



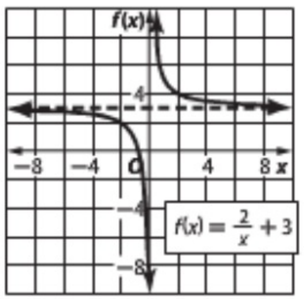
$D = \{x \mid x \neq 6\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

14.



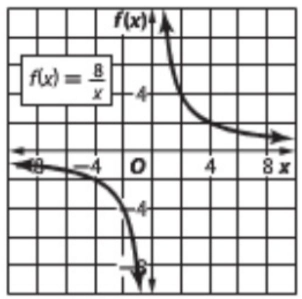
$D = \{x \mid x \neq 0\}$;
$R = \{f(x) \mid f(x) \neq -5\}$

15.



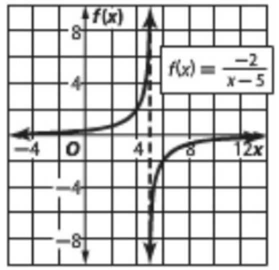
$D = \{x \mid x \neq 0\}$;
$R = \{f(x) \mid f(x) \neq 3\}$

16.



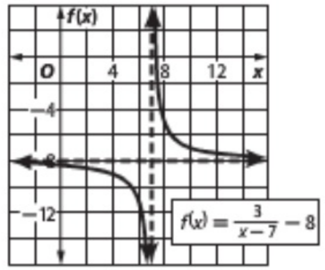
$D = \{x \mid x \neq 0\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

17.



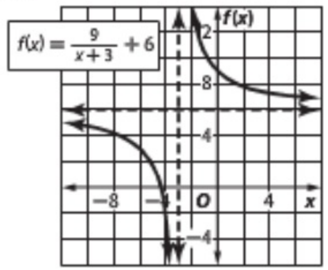
$D = \{x \mid x \neq 5\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

18.



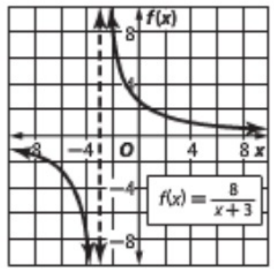
$D = \{x \mid x \neq 7\}$;
$R = \{f(x) \mid f(x) \neq -8\}$

19.



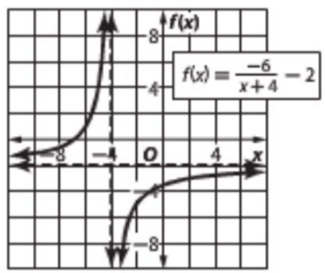
$D = \{x \mid x \neq -3\}$;
$R = \{f(x) \mid f(x) \neq 6\}$

20.



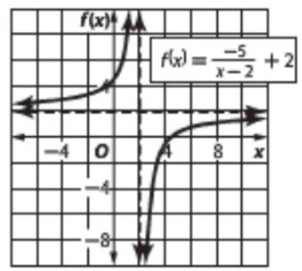
$D = \{x \mid x \neq -3\}$;
$R = \{f(x) \mid f(x) \neq 0\}$

21.



$D = \{x \mid x \neq -4\}$; $R = \{f(x) \mid f(x) \neq -2\}$

22.



$D = \{x \mid x \neq 2\}$; $R = \{f(x) \mid f(x) \neq 2\}$