

✓ 5-Minute Check

Over Lesson 8-6

3 Solve  $y^2 - 8y - 20 = 0$ .


A.  $\{-4, 3\}$

B.  $\{3, 6\}$

C.  $\{-2, 10\}$

D.  $\{1, 8\}$

Handwritten student work showing a factoring table for the equation  $y^2 - 8y - 20 = 0$ . The table has  $y^2$  in the top-left,  $-20$  in the bottom-right,  $-10$  in the bottom-left, and  $2$  in the top-right. The numbers  $-10$  and  $2$  are circled. Below the table, the factored equation  $(y - 10)(y + 2) = 0$  is written. Further down, the solutions  $y - 10 = 0$  and  $y + 2 = 0$  are shown, leading to  $y = 10$  and  $y = -2$ . The final solution set is  $y = -2, 10$ .


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# LESSON 8-7 Solving $ax^2 + bx + c = 0$

## Key Concept Factoring $ax^2 + bx + c$

**Words** To factor trinomials of the form  $ax^2 + bx + c$ , find two integers,  $m$  and  $p$ , with a sum of  $b$  and a product of  $ac$ . Then write  $ax^2 + bx + c$  as  $ax^2 + mx + px + c$ , and factor by grouping.

**Example**

$$5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6$$

$$m = -10 \text{ and } p = -3$$

$$\begin{aligned} &= 5x(x - 2) + (-3)(x - 2) \\ &= (5x - 3)(x - 2) \end{aligned}$$

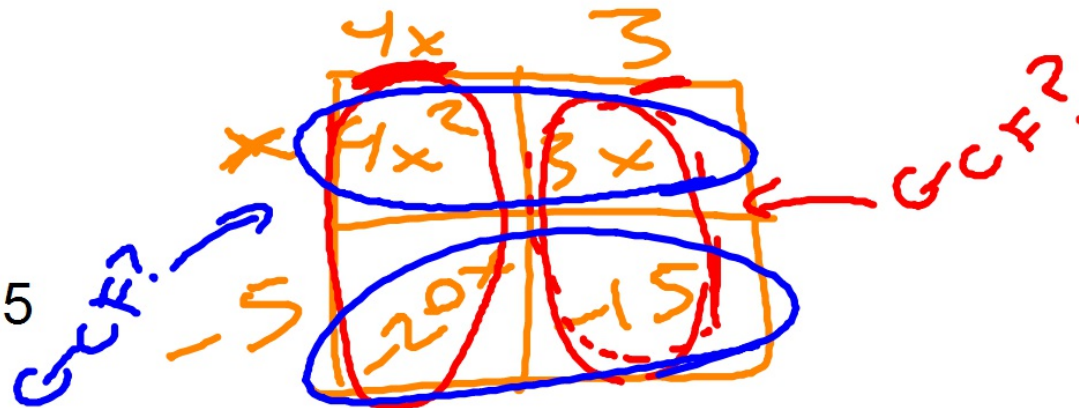
← This works, but let's try another way...

Challenge: how do we do this?

Let's try using the diamond (or x factor) method to approach this, BUT a little bit different....

$$(4x + 3)(x - 5)$$
$$4x^2 - 20x + 3x - 15$$
$$4x^2 - 17x - 15$$

first, let's multiply together....



...wouldn't it be nice to use the box method in reverse?

remember, to factor, we need to do all of this *in reverse*.

$$(4x + 3)(x - 5)$$

$$4x^2 - 20x + 3x - 15$$

$$4x^2 - 17x - 15$$

$$4x^2 - 17x - 15$$

NOW, let's try to factor...

1) Put the **middle number** on top.

2) Put the **PRODUCT** of the **first and last number** on the bottom.

$$\begin{array}{c} \text{ } \\ \diagdown \quad -17 \quad \diagup \\ \text{ } \\ \text{ } \\ \diagup \quad -20 \quad \diagdown \\ \text{ } \\ \text{ } \\ \diagdown \quad -60 \quad \diagup \\ \text{ } \\ \text{ } \\ \diagup \quad 3 \quad \diagdown \\ \text{ } \\ \text{ } \end{array}$$

"what two numbers multiply to -60 and adds up to -17?"

$$(4x + 3)(x - 5)$$

$$4x^2 - 20x + 3x - 15$$

$$4x^2 - 17x - 15$$

	$x - 5$	
$4x + 3$	$4x^2$	$-20x$
	$3x$	$-15$

NOW, let's use the box method in reverse!

- 1) Put the **first and last terms** diagonal to each other.
- 2) put **the middle terms** in the other two boxes.
- 3) look across the rows and columns to factor appropriately!



## Check Your Understanding



ns begin on page R13.



**Examples 1–3** Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1.  $3x^2 + 17x + 10$   $(3x + 2)(x + 5)$

2.  $2x^2 + 22x + 56$   $2(x + 4)(x + 7)$

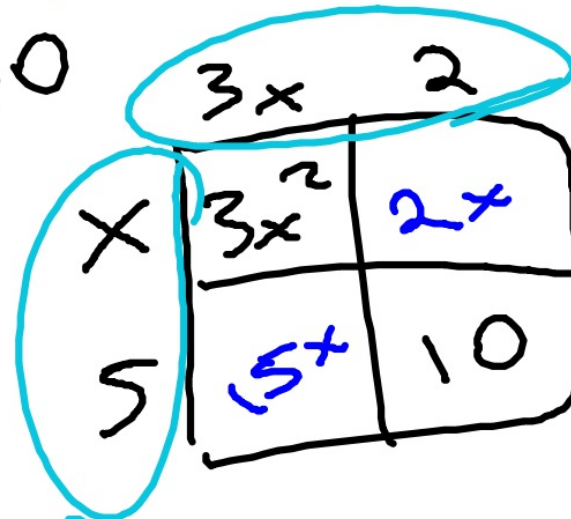
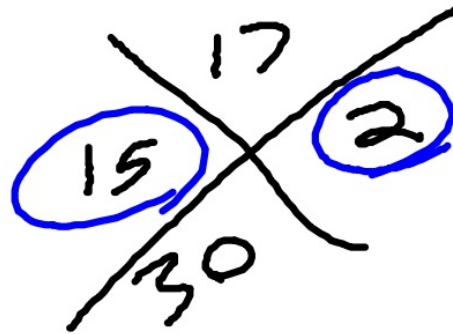
3.  $5x^2 - 3x + 4$

4.  $3x^2 - 11x - 20$   $(3x + 4)(x - 5)$

Check out the answer to number 2...they took out the GCF first!

①

$$3x^2 + 17x + 10$$



$$(3x^2 + 15x) + (2x + 10)$$
$$3x(x+5) + 2(x+5)$$

## Check Your Understanding

 = Step-by-Step Solutions begin on page R13.



**Examples 1–3** Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

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2.  $2x^2 + 22x + 56$   $2(x + 4)(x + 7)$

3.  $5x^2 - 3x + 4$  **prime**

4.  $3x^2 - 11x - 20$   $(3x + 4)(x - 5)$

②  $2(x^2 + 11x + 28)$   
 $2(x + 4)(x + 7)$

	$x$	$4$
$x$	$x^2$	$4x$
$7$	$7x$	$28$



## Check Your Understanding

 = Step-by-Step Solutions begin on page R13.

**Examples 1–3** Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1.  $3x^2 + 17x + 10$   $(3x + 2)(x + 5)$

2.  $2x^2 + 22x + 56$   $2(x + 4)(x + 7)$

3.  $5x^2 - 3x + 4$  **prime**

4.  $3x^2 - 11x - 20$   $(3x + 4)(x - 5)$

$4$

$\begin{array}{r} 4 \overline{) -11} \\ \underline{-60} \\ -20 \end{array}$   
 $(3)(-20)$

$3x^2 + 4x - 15x - 20$

$3x^2$	$4x$
$-15x$	$-20$

$x$   
 $-5$

**Example 4**

Solve each equation. Confirm your answers using a graphing calculator.

5.  $2x^2 + 9x + 9 = 0$   $-\frac{3}{2}, -3$

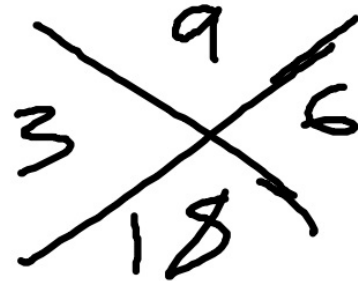
6.  $3x^2 + 17x + 20 = 0$   $-\frac{5}{3}, -4$

7.  $3x^2 - 10x + 8 = 0$   $\frac{4}{3}, 2$

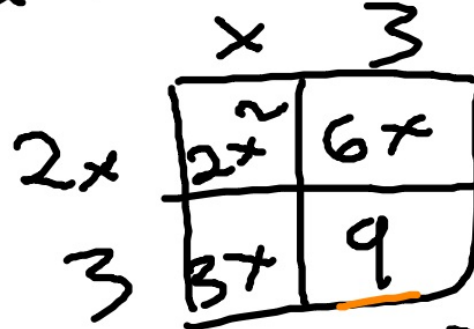
8.  $2x^2 - 17x + 30 = 0$   $\frac{5}{2}, 6$

1) factor

5



$2x^2 + 6x + 3x + 9$



$(2x+3)(x+3) = 0$

2) zero product property!

$2x+3=0$   
 $-3 \quad -3$

$x+3=0$   
 $-3 \quad -3$

$\frac{2x}{2} = \frac{-3}{2}$   
 $x = -\frac{3}{2}$

$x = -3$

**Example 4**

Solve each equation. Confirm your answers using a graphing calculator.

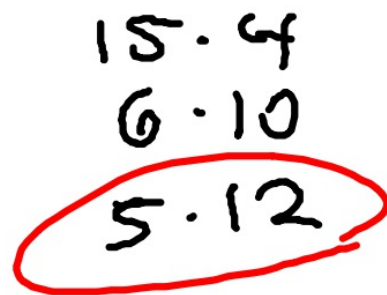
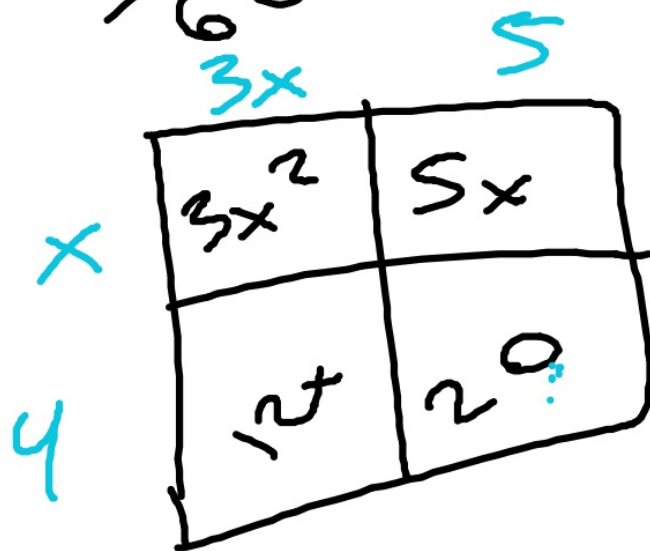
5.  $2x^2 + 9x + 9 = 0$   $-\frac{3}{2}, -3$

7.  $3x^2 - 10x + 8 = 0$   $\frac{4}{3}, 2$

6.  $3x^2 + 17x + 20 = 0$   $-\frac{5}{3}, -4$

8.  $2x^2 - 17x + 30 = 0$   $\frac{5}{2}, 6$

6



$(3x + 5)(x + 4) = 0$

$3x + 5 = 0$   
 $-5$   


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 $x + 4 = 0$   
 $-4$

$x = -4$

$3x + 5 = 0$   
 $-5$   


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 $x = -\frac{5}{3}$

**Example 4**

Solve each equation. Confirm your answers using a graphing calculator.

5.  $2x^2 + 9x + 9 = 0$   $-\frac{3}{2}, -3$

6.  $3x^2 + 17x + 20 = 0$   $-\frac{5}{3}, -4$

7.  $3x^2 - 10x + 8 = 0$   $\frac{4}{3}, 2$

8.  $2x^2 - 17x + 30 = 0$   $\frac{5}{2}, 6$

8

$$\begin{array}{r} -5 \quad -17 \\ \hline 60 \quad -12 \end{array}$$

	$x - 6$
$2x^2$	$-12x$
$-5x$	$30$

$$(2x - 5)(x - 6) = 0$$

$$\begin{array}{r} 2x - 5 = 0 \\ +5 \\ \hline 2x = 5 \\ x = \frac{5}{2} \end{array} \quad \begin{array}{r} x - 6 = 0 \\ +6 \\ \hline x = 6 \end{array}$$

$x = 6$

$x = \frac{5}{2}$

11.  $(2x + 3)(x + 8)$     13.  $2(2x + 5)(x + 7)$

**Practice and Problem Solving**

Extra Practice is on page R8.

**Examples 1–3** Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

10.  $5x^2 + 34x + 24$

11.  $2x^2 + 19x + 24$

12.  $4x^2 + 22x + 10$      $2(2x + 1)(x + 5)$

13.  $4x^2 + 38x + 70$

14.  $2x^2 - 3x - 9$

15.  $4x^2 - 13x + 10$      $(4x - 5)(x - 2)$

16.  $2x^2 + 3x + 6$     **prime**

17.  $5x^2 + 3x + 4$     **prime**

18.  $12x^2 + 69x + 45$      $3(4x + 3)(x + 5)$

19.  $4x^2 - 5x + 7$     **prime**

20.  $5x^2 + 23x + 24$   
 $(5x + 8)(x + 3)$

21.  $3x^2 - 8x + 15$     **prime**

Handwritten work for problem 13:

⑬    ②  $(2x^2 + 14x + 35)$

~~ACROSS~~

<del>2x</del>	<del>5</del>
2x <sup>2</sup>	5x
14x	<del>35</del>

$2(2x + 5)(x + 7)$



10.  $(5x + 4)(x + 6)$  11.  $(2x + 3)(x + 8)$  13.  $2(2x + 5)(x + 7)$  14.  $(2x + 3)(x - 3)$

**Practice and Problem Solving**

Extra Practice is on page R8.

**Examples 1–3** Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

10.  $5x^2 + 34x + 24$

13.  $4x^2 + 38x + 70$

16.  $2x^2 + 3x + 6$  **prime**

19.  $4x^2 - 5x + 7$  **prime**

11.  $2x^2 + 19x + 24$

14.  $2x^2 - 3x - 9$

17.  $5x^2 + 3x + 4$  **prime**

20.  $5x^2 + 23x + 24$   
 **$(5x + 8)(x + 3)$**

12.  $4x^2 + 22x + 10$   **$2(2x + 1)(x + 5)$**

15.  $4x^2 - 13x + 10$   **$(4x - 5)(x - 2)$**

18.  $12x^2 + 69x + 45$   **$3(4x + 3)(x + 5)$**

21.  $3x^2 - 8x + 15$  **prime**

(21)

~~$\frac{-8}{45}$~~       $-1 \cdot 45$   
 $-9 \cdot 5$   
 $-3 \cdot 15$

prime

20wE  
 add  
 sp to  
 -8...



15.  $4x^2 - 13x + 10$   $(4x - 5)$  $(x - 2)$

~~$\begin{array}{r} -13 \\ -5 \quad -8 \\ \hline 40 \end{array}$~~

$4x^2 - 5x - 8x + 10$

	$4x$	$-5$
$x$	$4x^2$	$-5x$
$-2$	$-8x$	$10$

Solve each equation. Confirm your answers using a graphing calculator.

23.  $2x^2 + 9x - 18 = 0$   $\frac{3}{2}, -6$

24.  $4x^2 + 17x + 15 = 0$   $-\frac{5}{4}, -3$

25.  $-3x^2 + 26x = 16$   $\frac{2}{3}, 8$

26.  $-2x^2 + 13x = 15$   $\frac{3}{2}, 5$

27.  $-3x^2 + 5x = -2$   $-\frac{1}{3}, 2$

28.  $-4x^2 + 19x = -30$   $-\frac{5}{4}, 6$

**25**  $-3x^2 + 26x = 16$   
 $\quad\quad\quad -16 \quad -16$   


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$-(-3x^2 + 26x - 16) = 0$

$3x^2 - 26x + 16 = 0$

~~$\begin{array}{r} -26 \\ -24 \quad -2 \\ \hline 48 \end{array}$~~

$3x^2 - 24x - 2x + 48 = 0$   
 $\quad\quad\quad x \quad -8$

$3x^2$	$-24x$
$-2x$	$48$

$(x - 24)(3x - 2) = 0$

~~$x - 24 = 0$~~

~~$x = 24$~~

$3x - 2 = 0$   
 $\quad\quad +2 \quad +2$   


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$\frac{3x}{3} = \frac{2}{3}$   
 $x = \frac{2}{3}$