

LESSON 8-8 Differences of Squares

$$x^2 + 0x - 25$$

 Key Concept Difference of Squares

Symbols $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$

Examples $x^2 - 25 = (x + 5)(x - 5)$ or $(x - 5)(x + 5)$

$t^2 - 64 = (t + 8)(t - 8)$ or $(t - 8)(t + 8)$

~~$$\begin{array}{r} 0 \\ -8 \quad 8 \\ \hline -64 \end{array}$$~~

~~$$\begin{array}{r} 0 \\ 5 \quad -5 \\ \hline -25 \end{array}$$~~

	x	-5
x	x^2	$-5x$
5	$5x$	-25

EXAMPLE 1 Factor Differences of Squares**A. Factor $m^2 - 64$.**

$$\begin{aligned}m^2 - 64 &= m^2 - 8^2 \\ &= (m + 8)(m - 8)\end{aligned}$$

Write in the form $a^2 - b^2$.

$$(a + b)(a - b)$$

Factor the difference of squares.

Answer: $(m + 8)(m - 8)$

EXAMPLE 1 Factor Differences of Squares**B. Factor $16y^2 - 81z^2$.**

$$16y^2 - 81z^2 = (4y)^2 - (9z)^2$$

$$= (4y + 9z)(4y - 9z)$$

Write in the form
 $a^2 - b^2$.

Factor the difference
of squares.

Answer: $(4y + 9z)(4y - 9z)$

EXAMPLE 1 Factor Differences of Squares

C. Factor $3b^3 - 27b$.

If the terms of a binomial have a common factor, the GCF should be factored out first before trying to apply any other factoring technique.

$$3b^3 - 27b = 3b(b^2 - 9)$$

$$= 3b[(b)^2 - (3)^2]$$

$$= 3b(b + 3)(b - 3)$$

The GCF of $3b^3$ and $27b$ is $3b$.

Write in the form $a^2 - b^2$.

Factor the difference of

squares

Examples 1-3 Factor each polynomial.

1. $x^2 - 9$ $(x + 3)(x - 3)$

2. $4a^2 - 25$ $(2a + 5)(2a - 5)$

3. $9m^2 - 144$ $9(m + 4)(m - 4)$



5. $u^4 - 81$

7. $20r^4 - 45n^4$

9. $2c^3 + 3c^2 - 2c - 3$

③ $9(m^2 - 16)$

11. $3t^3 + 2t^2 - 48t - 32$

$(3m - 12)(3m + 12)$
 $3 \cdot 3 (m - 4)(m + 4)$

EXAMPLE 2**Apply a Technique More than Once****A. Factor $y^4 - 625$.**

EXAMPLE 2**Apply a Technique More than Once****B. Factor $256 - n^4$.**

Examples 1–3 Factor each polynomial.

1. $x^2 - 9$

3. $9m^2 - 144$



5. $u^4 - 81$

7. $20r^4 - 45n^4$

9. $2c^3 + 3c^2 - 2c - 3$

11. $3t^3 + 2t^2 - 48t - 32$

9. $(c + 1)(c - 1)(2c + 3)$ 11. $(t + 4)(t - 4)(3t + 2)$



EXAMPLE 3 Apply Different Techniques

A. Factor $9x^5 - 36x$.

$$\begin{aligned} 9x^5 - 36x &= 9x(x^4 - 4) \\ &= 9x[(x^2)^2 - 2^2] \\ &= 9x(x^2 - 2)(x^2 + 2) \end{aligned}$$

Factor out the GCF.

Write $x^2 - 4$ in $a^2 - b^2$ form.

Factor the difference of squares.

Answer: $9x(x^2 - 2)(x^2 + 2)$

EXAMPLE 3 Apply Different Techniques

B. Factor $6x^3 + 30x^2 - 24x - 120$.

$$6x^3 + 30x^2 - 24x - 120$$

Original polynomial

$$= 6(x^3 + 5x^2 - 4x - 20)$$

Factor out the GCF.

$$= 6[(x^3 - 4x) + (5x^2 - 20)]$$

Group terms with common factors.

$$= 6[x(x^2 - 4) + 5(x^2 - 4)]$$

Factor each grouping.

$$= 6(x^2 - 4)(x + 5)$$

$x^2 - 4$ is the common factor.

$$= 6(x + 2)(x - 2)(x + 5)$$

Factor the difference of squares.

Answer: $6(x + 2)(x - 2)(x + 5)$

Examples 1–3 Factor each polynomial.

1. $x^2 - 9$

3. $9m^2 - 144$

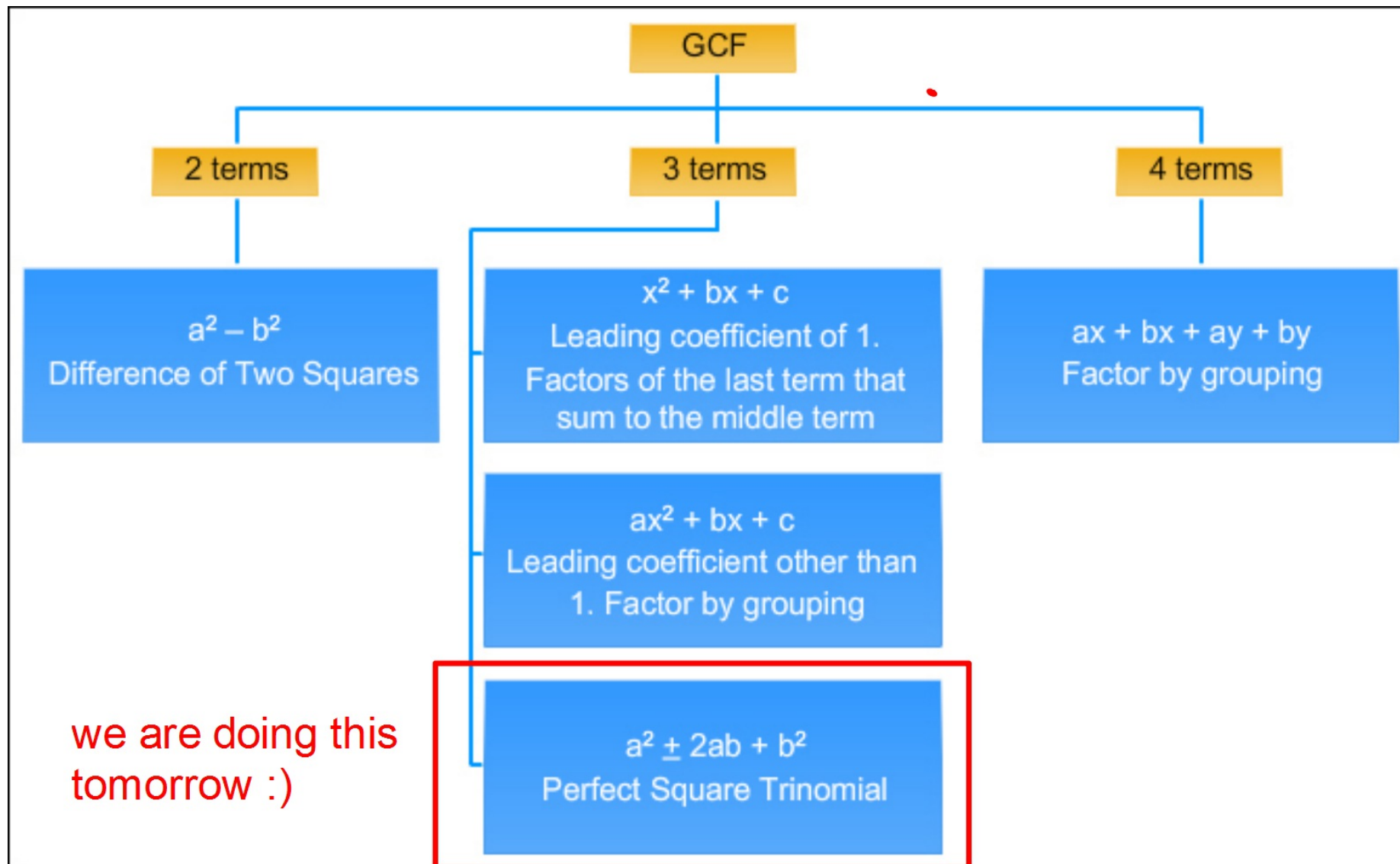
5. $u^4 - 81$

7. $20r^4 - 45n^4$ $5(2r^2 - 3n^2)(2r^2 + 3n^2)$

9. $2c^3 + 3c^2 - 2c - 3$

11. $3t^3 + 2t^2 - 48t - 32$

⑦ GCF: 5
 $5(4r^4 - 9n^4)$
 $5(2r^2 - 3n^2)(2r^2 + 3n^2)$



2. $4a^2 - 25$ $(2a + 5)(2a - 5)$

4. $2p^3 - 162p$ $2p(p + 9)(p - 9)$

6. $2d^4 - 32f^4$ $2(d^2 + 4f^2)(d + 2f)(d - 2f)$

8. $256n^4 - c^4$ $(16n^2 + c^2)(4n + c)(4n - c)$

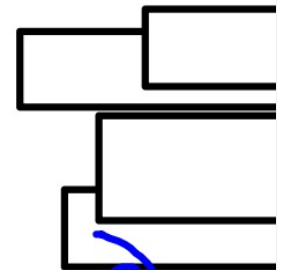
10. $f^3 - 4f^2 - 9f + 36$ $(f + 3)(f - 3)(f - 4)$

12. $w^3 - 3w^2 - 9w + 27$ $(w - 3)(w + 3)(w - 3)$

⑥ $2(d^4 - 16f^4)$

⑧ $(16n^2 + c^2)$
 $(16n^2 - c^2)$

④ GCF: $2p$
 $= 2p(p^2 - 81)$
 $= 2p(p - 9)(p + 9)$
 $= 2p(d^2 + 4f^2)(d^2 - 4f^2)$
⑥ $2(d^2 + 4f^2)(d + 2f)(d - 2f)$
 $2(d^2 + 4f^2)(d + 2f)(d - 2f)$



10. $f^3 - 4f^2 - 9f + 36$ $(f+3)(f-3)(f-4)$

12. $w^3 - 3w^2 - 9w + 27$ $(w-3)(w+3)(w-3)$

⑩ $f^3 - 4f^2 - 9f + 36$
 $= (f^3 - 4f^2) - (9f - 36)$
 $= f^2(f-4) - 9(f-4)$
 $= (f^2 - 9)(f-4)$
 $= (f+3)(f-3)(f-4)$

Factor by grouping,
Diff. of squares.

Examples 1–3 Factor each polynomial. **25–43.** See margin.

Additional Answers

25. $7(h^2 + p^2)(h + p)(h - p)$

26. $(c + 7)(c - 7)(3c + 2)$

27. $6k^2(h^2 + 3k)(h^2 - 3k)$

28. $5a(a + 2)(a - 2)$

29. $(f + 8)(f - 8)(f + 2)$

30. $3r(r + 8)(r - 8)$

31. $10q(q + 11)(q - 11)$

32. $3x(n^2 + 3x)(n^2 - 3x)$

33. $p^3r(r + 1)(r - 1)(r^2 + 1)$

34. $8c(c + 1)(c - 1)$

35. $(r + 10)(r - 10)(r - 5)$

36. $(t + 1)(t - 1)(3t - 7)$

37. $(a + 7)(a - 7)$

38. $(m + 3)(m - 3)(4m + 9)$

39. $3(m^4 + 81)$

40. $(x + 5)(x - 5)(3x + 1)$

41. $2(a + 4)(a - 4)(6a + 1)$

42. $x(x + 6)(x - 6)(x + 6)$

43. $3(m + 5)(m - 5)(5m + 4)$

15. $q^2 - 121$

$(q + 11)(q - 11)$

18. $w^4 - 625$

$(w^2 + 25)(w + 5)(w - 5)$

21. $h^3 - 100h$

$h(h + 10)(h - 10)$

24. $x^2 - 4y^2$

$(x + 2y)(x - 2y)$

27. $6k^2h^4 - 54k^4$

30. $3r^3 - 192r$

33. $p^3r^5 - p^3r$

36. $3t^3 - 7t^2 - 3t + 7$

38. $4m^3 + 9m^2 - 36m - 81$

40. $3x^3 + x^2 - 75x - 25$

42. $x^4 + 6x^3 - 36x^2 - 216x$

16. $r^4 - k^4$

$(r^2 + k^2)(r + k)(r - k)$

19. $r^2 - 9t^2$

$(r + 3t)(r - 3t)$

22. $h^4 - 256$

$(h^2 + 16)(h + 4)(h - 4)$

25. $7h^4 - 7p^4$

28. $5a^3 - 20a$

31. $10q^3 - 1210q$

34. $8c^3 - 8c$

37. $a^2 - 49$

39. $3m^4 + 243$

41. $12a^3 + 2a^2 - 192a - 32$

43. $15m^3 + 12m^2 - 375m - 300$

17. $6n^4 - 6$

$6(n^2 + 1)(n + 1)(n - 1)$

20. $2c^2 - 32d^2$

$2(c + 4d)(c - 4d)$

23. $2x^3 - x^2 - 162x + 81$

$(x + 9)(x - 9)(2x - 1)$

26. $3c^3 + 2c^2 - 147c - 98$

29. $f^3 + 2f^2 - 64f - 128$

32. $3xn^4 - 27x^3$

35. $r^3 - 5r^2 - 100r + 500$