$$(x+4)^2 \neq x^2 + 16$$

$$(x+4)^2 = x^2 + 8x + 16$$

## KeyConcept Factoring Perfect Square Trinomials

Symbols

$$a^2 + 2ab + b^2 = (a+b)(a+b) = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)(a-b) = (a-b)^2$$

Examples

$$x^2 + 8x + 16 = (x+4)(x+4) \text{ or } (x+4)^2$$

$$x^2 - 6x + 9 = (x-3)(x-3) \text{ or } (x-3)^2$$

-6

-3 ╳ -3

9

8

4 ╳ 4

16
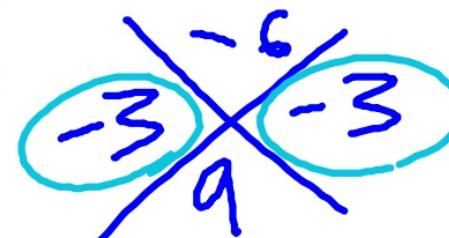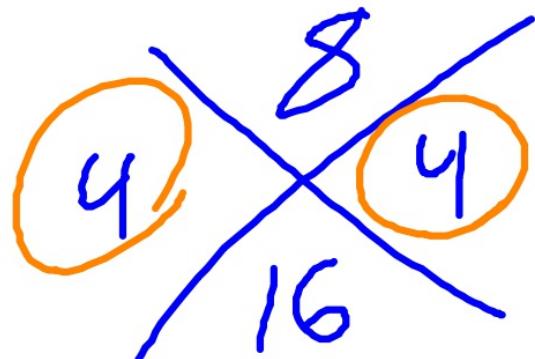
EXAMPLE 1 **Recognize and Factor Perfect Square Trinomials**

$5x \cdot 5x$ $3 \cdot 3$

A. Determine whether $25x^2 - 30x + 9$ is a perfect square trinomial. If so, factor it.

1. Is the first term a perfect square?   Yes, $25x^2 = (5x)^2$.

2. Is the last term a perfect square?   Yes, $9 = 3^2$.

3. Is the middle term equal to $2(5x)(3)$?

                                        Yes, $30x = 2(5x)(3)$.

Answer: $25x^2 - 30x + 9$ is a perfect square trinomial.

$(5x - 3)(5x - 3)$

$15x$

$15x$

Example 1 Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $25x^2 + 60x + 36$ **yes; $(5x + 6)^2$**

2. $6x^2 + 30x + 36$ **no**

5x·5x   *6·6*

Example 2 Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

3. $2x^2 - x - 28$ **$(x - 4)(2x + 7)$**

5. $4x^2 + 64$ **$4(x^2 + 16)$**

③ $7 \times \begin{matrix} -1 \\ -8 \end{matrix}$ -56

GCF

$$\begin{array}{c|c|c} & x & -4 \\ \hline 2x & 2x^2 & -9x \\ \hline 7 & 7x & -28 \end{array}$$

① $(5x + 6)(5x + 6)$

$25x^2 + 30x + 30x + 36$

Example 1 Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

1. $25x^2 + 60x + 36$ **yes; $(5x + 6)^2$**
2. $6x^2 + 30x + 36$ **no**

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

Example 2 Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

3. $2x^2 - x - 28$ **$(x - 4)(2x + 7)$**
4. $6x^2 - 34x + 48$ **$2(x - 3)(3x - 8)$**
5. $4x^2 + 64$ **$4(x^2 + 16)$**
6. $4x^2 + 9x - 16$ **prime**

$\frac{3}{4}$

Examples 3–4 Solve each equation. Confirm your answers using a graphing calculator.

7. $4x^2 = 36$ **± 3**
8. $25a^2 - 40a = -16$ $\frac{4}{5}$
9. $64y^2 - 48y + 18 = 9$ $\frac{3}{8}$
10. $(z + 5)^2 = 47$ **$-5 \pm \sqrt{47}$ or about -11.86 and 1.86**

⑦ $4x^2 = 36.$

$$\frac{4x^2}{4} - \frac{36}{4} = \frac{0}{4}$$

$$x^2 - 9 = 0$$

$\rightarrow (x - 3)(x + 3) = 0$

$x - 3 = 0 \qquad x + 3 = 0$

$x = 3 \qquad x = -3$

9. $64y^2 - 48y + 18 = 9$

$$\underbrace{}_{\displaystyle \frac{-9 - 9}{64y^2 - 48y + 9 = 0}}$$

$8y \quad 8y \qquad\qquad -3 \cdot -3$

$2^x \; \boxed{-24y}$

$(8y - 3)(8y - 3) = 0$

$8y - 3 = 0$

$\dfrac{+3 \quad +3}{\dfrac{8y}{8} = \dfrac{3}{8}}$

$y = \dfrac{3}{8}$

Example 1   Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*. If so, factor it.

13. $16x^2 - 56x + 49$ **yes; $(4x - 7)^2$**

15 $x^2 + 26x + 168$ **no**

Example 2   Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*. **28. $(x + 2y)(x - 2)(x + 2)$ 30. $(r - 6)(r + 6)(2r - 1)$ 32. $2cd(c^2 + d^2)(2c - 5)$**

16. $24d^2 + 39d - 18$ **$3(8d - 3)(d + 2)$**

17. $8x^2 + 10x - 21$ **prime**

18. $2b^2 + 12b - 24$ **$2(b^2 + 6b - 12)$**

19. $8y^2 - 200z^2$ **$8(y - 5z)(y + 5z)$**

20. $16a^2 - 121b^2$ **$(4a - 11b)(4a + 11b)$**

21. $12m^3 - 22m^2 - 70m$
$2m(2m - 7)(3m + 5)$

22. $8c^2 - 88c + 242$ **$2(2c - 11)^2$**

23. $12x^2 - 84x + 147$ **$3(2x - 7)^2$**

24. $w^4 - w^2$ **$w^2(w - 1)(w + 1)$**

25. $12p^3 - 3p$ **$3p(2p + 1)(2p - 1)$**

26. $16q^3 - 48q^2 + 36q$ **$4q(2q - 3)^2$**

27. $4t^3 + 10t^2 - 84t$ **$2t(t + 6)(2t - 7)$**

28. $x^3 + 2x^2y - 4x - 8y$

29. $2a^2b^2 - 2a^2 - 2ab^3 + 2ab$
$2a(a - b)(b + 1)(b - 1)$

30. $2r^3 - r^2 - 72r + 36$

31. $3k^3 - 24k^2 + 48k$ **$3k(k - 4)(k - 4)$**

32. $4c^4d - 10c^3d + 4c^2d^3 - 10cd^3$

33. $g^2 + 2g - 3h^2 + 4h$ **prime**

Examples 3–4 Solve each equation. Confirm your answers using a graphing calculator.

34. $4m^2 - 24m + 36 = 0$ **3**

35 $(y - 4)^2 = 7$ **$4 \pm \sqrt{7}$**

36. $a^2 + \frac{10}{7}a + \frac{25}{49} = 0$ **$-\frac{5}{7}$**

37. $x^2 - \frac{3}{2}x + \frac{9}{16} = 0$ **$\frac{3}{4}$**

38. $x^2 + 8x + 16 = 25$ **1, −9**

39. $5x^2 - 60x = -180$ **6**

40. $4x^2 = 80x - 400$ **10**

41. $9 - 54x = -81x^2$ **$\frac{1}{3}$**

42. $4c^2 + 4c + 1 = 15$ **$\dfrac{-1 \pm \sqrt{15}}{2}$**

43. $x^2 - 16x + 64 = 6$ **$8 \pm \sqrt{6}$**