

New Vocabulary

- completing the square

Let's check this out using algebra tiles!

Show the following quadratics with the tiles;

$$x^2 + 4x + 4$$

$$x^2 - 6x + 9$$

NOW, explore these two;

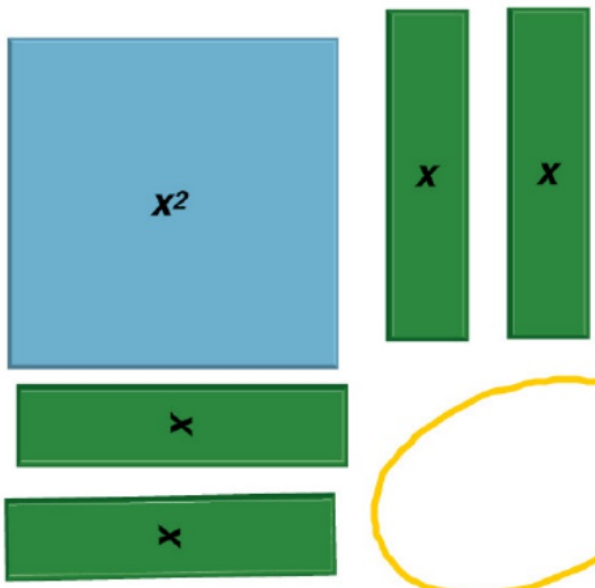
$$x^2 + 4x + 1 \quad \text{and} \quad x^2 - 6x + 6$$

Things to think about:

1) For ax^2+bx+c , is there a pattern between the "b" and "c" term?

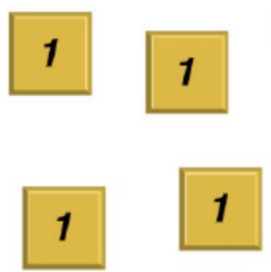
2) Is there a rule I can make with this pattern?

$$\begin{aligned}
 &(x+y)(x+y) \\
 &= x^2 + x + x + y^2 \\
 &= x^2 + 2xy + y^2 \\
 &ax^2 + bx + c
 \end{aligned}$$



"In order to complete the square, I need to add 4 more..."

$$x^2 + 4x$$



 **Key Concept** Completing the Square 

Words

To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

Step 1 Find one half of b , the coefficient of x .

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

Symbols

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

← divide
 $b \rightarrow 2$
← square
it!

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Example 1 Find the value of c that makes each trinomial a perfect square.

1. $x^2 - 18x + c$ **81**

2. $x^2 + 22x + c$ **121**


3. $x^2 + 9x + c$ **$\frac{81}{4}$**

4. $x^2 - 7x + c$ **$\frac{49}{4}$**

Handwritten work for problem 1:

$$\textcircled{1} \quad \frac{-18}{2} = (-9)^2 \quad \textcircled{3} \quad \left(\frac{9}{2}\right)^2$$
$$= 81$$

I will also be using this algebra trick...

 **Key Concept Square Root Property**

Words To solve a quadratic equation in the form $x^2 = n$, take the square root of each side.

Symbols For any number $n \geq 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example $x^2 = 25$
 $x = \pm\sqrt{25}$ or ± 5



Solve each equation. Check your solutions.

a. $(y - 6)^2 = 81$

$$(y - 6)^2 = 81$$

$$y - 6 = \pm\sqrt{81}$$

$$y - 6 = \pm 9$$

$$y = 6 \pm 9$$

$$y = 6 + 9 \quad \text{or} \quad y = 6 - 9$$

$$= 15 \qquad \qquad = -3$$

The roots are 15 and -3 .

Original equation

Square Root Property

$$81 = 9 \cdot 9$$

Add 6 to each side.

Separate into two equations.

Simplify.

Check in the original equation.

$$\begin{array}{r}
 2 \times 4 \\
 \times 2 \\
 \hline
 4
 \end{array}$$

(5) $x^2 + 4x + 4 = 6 + 4$

$(x+2)(x+2) \rightarrow (x+2)^2 = \sqrt{10}$

$x+2 = \pm \sqrt{10}$

$-2 \quad -2$

$x = -2 \pm \sqrt{10}$

Examples 2–3 Solve each equation by completing the square. Round to the nearest tenth if necessary.

5. $x^2 + 4x = 6$ **-5.2, 1.2**

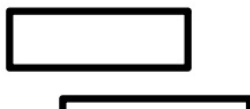
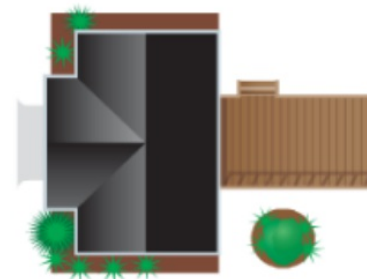
6. $x^2 - 8x = -9$ **1.4, 6.6**

7. $4x^2 + 9x - 1 = 0$ **-2.4, 0.1**

8. $-2x^2 + 10x + 22 = 4$ **-1.4, 6.4**

Example 4

9. **CCSS MODELING** Collin is building a deck on the back of his family’s house. He has enough lumber for the deck to be 144 square feet. The length should be 10 feet more than its width. What should the dimensions of the deck be? **8 ft by 18 ft**



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Practice and Problem Solving

Extra Practice is on page

Example 1 Find the value of c that makes each trinomial a perfect square.

10. $x^2 + 26x + c$ **169**

11. $x^2 - 24x + c$ **144**

12. $x^2 - 19x + c$ **$\frac{361}{4}$**

13. $x^2 + 17x + c$ **$\frac{289}{4}$**

14. $x^2 + 5x + c$ **$\frac{25}{4}$**

15. $x^2 - 13x + c$ **$\frac{169}{4}$**

16. $x^2 - 22x + c$ **121**

17. $x^2 - 15x + c$ **$\frac{225}{4}$**

18. $x^2 + 24x + c$ **144**

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Examples 2–3 Solve each equation by completing the square. Round to the nearest tenth if necessary.

19 $x^2 + 6x - 16 = 0$ **-8, 2**

21. $x^2 - 8x - 1 = 8$ **-1, 9**

23. $x^2 - 11x + 3 = 5$ **-0.2, 11.2**

25. $2x^2 - 2x + 7 = 5$ **∅**

27. $4x^2 + 6x = 12$ **-2.6, 1.1**

29. $-2x^2 + 10x = -14$ **-1.1, 6.1**

20. $x^2 - 2x - 14 = 0$ **-2.9, 4.9**

22. $x^2 + 3x + 21 = 22$ **-3.3, 0.3**

24. $5x^2 - 10x = 23$ **-1.4, 3.4**

26. $3x^2 + 12x + 81 = 15$ **∅**

28. $4x^2 + 5 = 10x$ **0.7, 1.8**

30. $-3x^2 - 12 = 14x$ **-3.5, -1.1**

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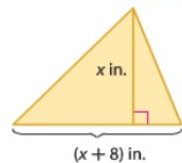
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- Example 4** 31. **FINANCIAL LITERACY** The price p in dollars for a particular stock can be modeled by the quadratic equation $p = 3.5t - 0.05t^2$, where t represents the number of days after the stock is purchased. When is the stock worth \$60? **on the 30th and 40th day after purchase**

GEOMETRY Find the value of x for each figure. Round to the nearest tenth if necessary.

32. $A = 45 \text{ in}^2$ **6.3**



33. $A = 110 \text{ ft}^2$ **5.3**



34. **NUMBER THEORY** The product of two consecutive even integers is 224. Find the integers. **14 and 16; -14 and -16**

35. **CCSS PRECISION** The product of two consecutive negative odd integers is 483. Find the integers. **-21 and -23**

36. **GEOMETRY** Find the area of the triangle below. **216 m²**

