

9-5 The Quadratic Formula

Key Concept The Quadratic Formula

The solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{ax^2 + bx + c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\frac{2/a \cdot b}{2/a \cdot b} = \left(\frac{b}{2a}\right)^2$$

$$= \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a} \cdot \frac{4a}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-b}{2a} \quad \frac{-b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 Use the Quadratic Formula

Solve $x^2 - 12x = -20$ by using the Quadratic Formula.

Step 1 Rewrite the equation in standard form.

$$x^2 - 12x = -20 \quad \text{Original equation}$$

$$x^2 - 12x + 20 = 0 \quad \text{Add 20 to each side.}$$

Step 2 Apply the Quadratic Formula.

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(20)}}{2(1)} \\&= \frac{12 \pm \sqrt{144 - 80}}{2} \\&= \frac{12 \pm \sqrt{64}}{2} \text{ or } \frac{12 \pm 8}{2} \\x &= \frac{12 - 8}{2} \text{ or } x = \frac{12 + 8}{2} \\&= 2 \qquad \qquad \qquad = 10\end{aligned}$$

$$(x - 10)(x - 2) = 0$$

$$\begin{array}{r}x - 10 = 0 \\+ 10 \quad + 10 \\ \hline\end{array}$$

$$\begin{array}{r}x - 2 = 0 \\+ 2 \quad + 2 \\ \hline\end{array}$$

Quadratic Formula

$a = 1$, $b = -12$, and $c = 20$

Multiply.

Subtract and take the square root.

Separate the solutions.

The solutions are 2 and 10.

Q: Is there an easier way to do this?

Example 2 Use the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

b. $10x^2 - 5x = 25$

Step 1 Rewrite the equation in standard form.

$$10x^2 - 5x = 25$$

Original equation

$$10x^2 - 5x - 25 = 0$$

Subtract 25 from each side.

Step 2 Apply the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(10)(-25)}}{2(10)}$$

$a = 10$, $b = -5$, and $c = -25$

$$= \frac{5 \pm \sqrt{25 + 1000}}{20}$$

Multiply.

$$= \frac{5 \pm \sqrt{1025}}{20}$$

Add.

$$= \frac{5 - \sqrt{1025}}{20} \text{ or } \frac{5 + \sqrt{1025}}{20}$$

Separate the solutions.

$$\approx -1.4 \quad \approx 1.9$$

Simplify.

I'm ok with
this as
your
answer for
now...

The solutions are about -1.4 and 1.9 .

Calculator use can get the final answer....

$$2x^2 - x - 5 = 0$$

Examples 1-2 Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 2x - 15 = 0$ **-3, 5** 2. $x^2 - 10x + 16 = 0$ **2, 8** 3. $x^2 - 8x = -10$ **6.4, 1.6**

4. $x^2 + 3x = 12$ **2.3, -5.3** 5. $10x^2 - 31x + 15 = 0$ **0.6, 2.5** 6. $5x^2 + 5 = -13x$ **-0.5, -2.1**

Example 3 Solve each equation. State which method you used. See student work for method.

7. $2x^2 + 11x - 6 = 0$ **$-6, \frac{1}{2}$**

8. $2x^2 - 3x - 6 = 0$ **2.6, -1.1**

9. $9x^2 = 25$ **$\pm \frac{5}{3}$**

10. $x^2 - 9x = -19$ **5.6, 3.4**

① $a = 1$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $b = -2$
 $c = -15$
 $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)}$
 $x = \frac{2 \pm \sqrt{4 + 60}}{2} = \frac{2 \pm \sqrt{64}}{2}$
 $x = \frac{2 \pm 8}{2} \Rightarrow \frac{10}{2}$ or $\frac{-6}{2}$

9. $9x^2 = 25 \pm \frac{5}{3}$

(a) $9x^2 = 25$
 $\frac{-25}{-25}$

$9x^2 - 25 = 0$

$a^2 - b^2$

$(3x + 5)(3x - 5) = 0$

$3x - 5 = 0$
 $+5$
 $3x = 5$
 $x = \frac{5}{3}$

$x = \frac{5}{3}$

$3x + 5 = 0$
 -5

$3x = -5$
 $x = -\frac{5}{3}$

$x = -\frac{5}{3}$

$$ax^2 + bx + c = 0$$

2. $x^2 - 10x + 16 = 0$ 2,8

$$a = 1$$

$$b = -10$$

$$c = 16$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{(100) - 64}}{2}$$

$$= \frac{10 \pm \sqrt{36}}{2} = \frac{10 \pm 6}{2}$$

3. $x^2 - 8x = -10$ 6.4, 1.6

$$x^2 - 8x + 10 = 0$$

$$a = 1$$

$$b = -8$$

$$c = 10$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

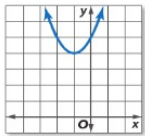
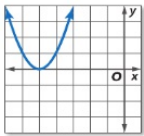
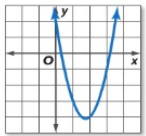
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(10)}}{2(1)}$$

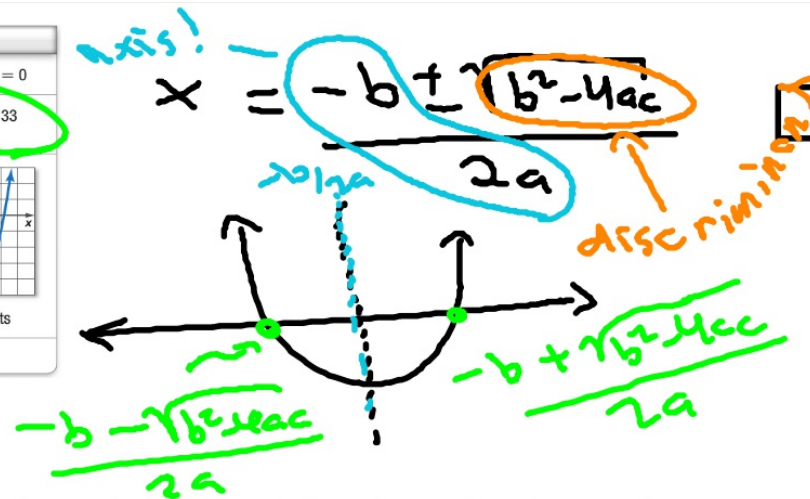
$$= \frac{8 \pm \sqrt{64 - 40}}{2} = \frac{8 \pm \sqrt{24}}{2}$$

$$= \frac{8 + \sqrt{24}}{2}, \frac{8 - \sqrt{24}}{2}$$

simplify 😊

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.0000000000
(8+√24)/2
6.449489743
(8-√24)/2
1.550510257
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KeyConcept Using the Discriminant			
Equation	$x^2 + 2x + 5 = 0$	$x^2 + 10x + 25 = 0$	$2x^2 - 7x + 2 = 0$
Discriminant	$b^2 - 4ac = -16$ negative	$b^2 - 4ac = 0$ zero	$b^2 - 4ac = 33$ positive
Graph of Related Function			
	0 x-intercepts	1 x-intercept	2 x-intercepts
Real Solutions	0	1	2



Example 4 State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

11. $x^2 - 9x + 21 = 0$ **-3; no real solutions** 12. $2x^2 - 11x + 10 = 0$ **12; 41; two real solutions**

13. $9x^2 + 24x = -16$ **0; one real solution** 14. $3x^2 - x - 8 = 0$ **97; two real solutions**

$9x^2 + 24x + 16 = 0$

(11) $b^2 - 4ac$
 $(-9)^2 - 4(1)(21)$
 $81 - 84 = -3$

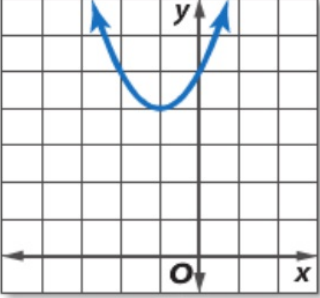
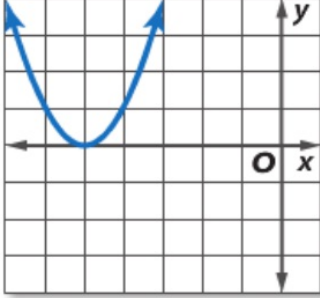
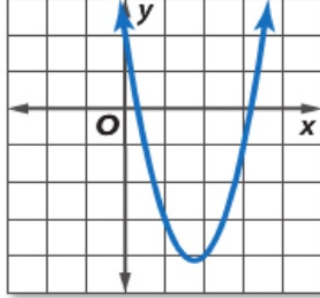
$3x^2 - x - 8 = 0$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(12) $b^2 - 4ac$
 $(-11)^2 - 4(2)(10)$
 $121 - 80 = 41$

(13) $b^2 - 4ac$
 $(24)^2 - 4(9)(16)$
 $576 - 576 = 0$



 **KeyConcept** Using the Discriminant

Equation	$x^2 + 2x + 5 = 0$	$x^2 + 10x + 25 = 0$	$2x^2 - 7x + 2 = 0$
Discriminant	$b^2 - 4ac = -16$ negative	$b^2 - 4ac = 0$ zero	$b^2 - 4ac = 33$ positive
Graph of Related Function	 <p>0 x-intercepts</p>	 <p>1 x-intercept</p>	 <p>2 x-intercepts</p>
Real Solutions	0	1	2

Examples 1–2 Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

16. $4x^2 + 5x - 6 = 0$ $-2, \frac{3}{4}$ 17. $x^2 + 16 = 0$ \emptyset 18. $6x^2 - 12x + 1 = 0$ $1.9, 0.1$

19. $5x^2 - 8x = 6$ $2.2, -0.6$ 20. $2x^2 - 5x = -7$ \emptyset 21. $5x^2 + 21x = -18$ $-3, -\frac{6}{5}$

22. $81x^2 = 9$ $\pm\frac{1}{3}$ 23. $8x^2 + 12x = 8$ $0.5, -2$ 24. $4x^2 = -16x - 16$ -2

25. $10x^2 = -7x + 6$ $0.5, -1.2$ 26. $-3x^2 = 8x - 12$ $1.1, -3.7$ 27. $2x^2 = 12x - 18$ 3

28. **AMUSEMENT PARKS** The Demon Drop at Cedar Point in Ohio takes riders to the top of a tower and drops them 60 feet. A function that approximates this ride is $h = -16t^2 + 64t - 60$, where h is the height in feet and t is the time in seconds. About how many seconds does it take for riders to drop 60 feet? **about 2.5 seconds**

Example 3 Solve each equation. State which method you used. **See student work for method.**

29. $2x^2 - 8x = 12$ $-1.2, 5.2$ 30. $3x^2 - 24x = -36$ $2, 6$ 31. $x^2 - 3x = 10$ $-2, 5$

32. $4x^2 + 100 = 0$ \emptyset 33. $x^2 = -7x - 5$ $-6.2, -0.8$ 34. $12 - 12x = -3x^2$ 2

17 $|x^2 + 16 = 0 \quad \emptyset$

$$ax^2 + bx + c = 0$$
$$1x^2 + 0x + 16 = 0$$

$$a = 1$$

$$b = 0$$

$$c = 16$$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{0 \pm \sqrt{-64}}{2} \leftarrow \text{!!!}$$