9-7 Special Functions

2 Absolute Value Functions Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value parent function, written as $f(x) = |x|$, all of the values of the range are nonnegative.

KeyConcept Absolute Value Function

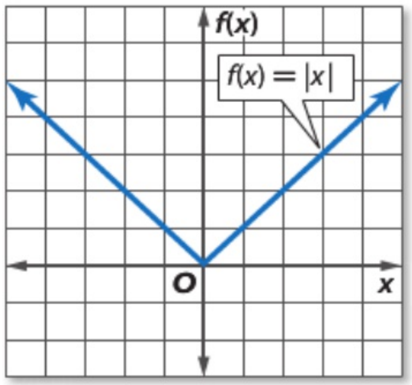
Parent function: $f(x) = |x|$, defined as

$$f(x) = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

Type of graph: V-shaped

Domain: all real numbers

Range: all nonnegative real numbers



The absolute value function is called a **piecewise-defined function** because it is defined using two or more expressions.

x	$\lvert ax+b \rvert + c$
−2	2
−1	1
0	0
1	1
2	2

-2 0 2

2 2

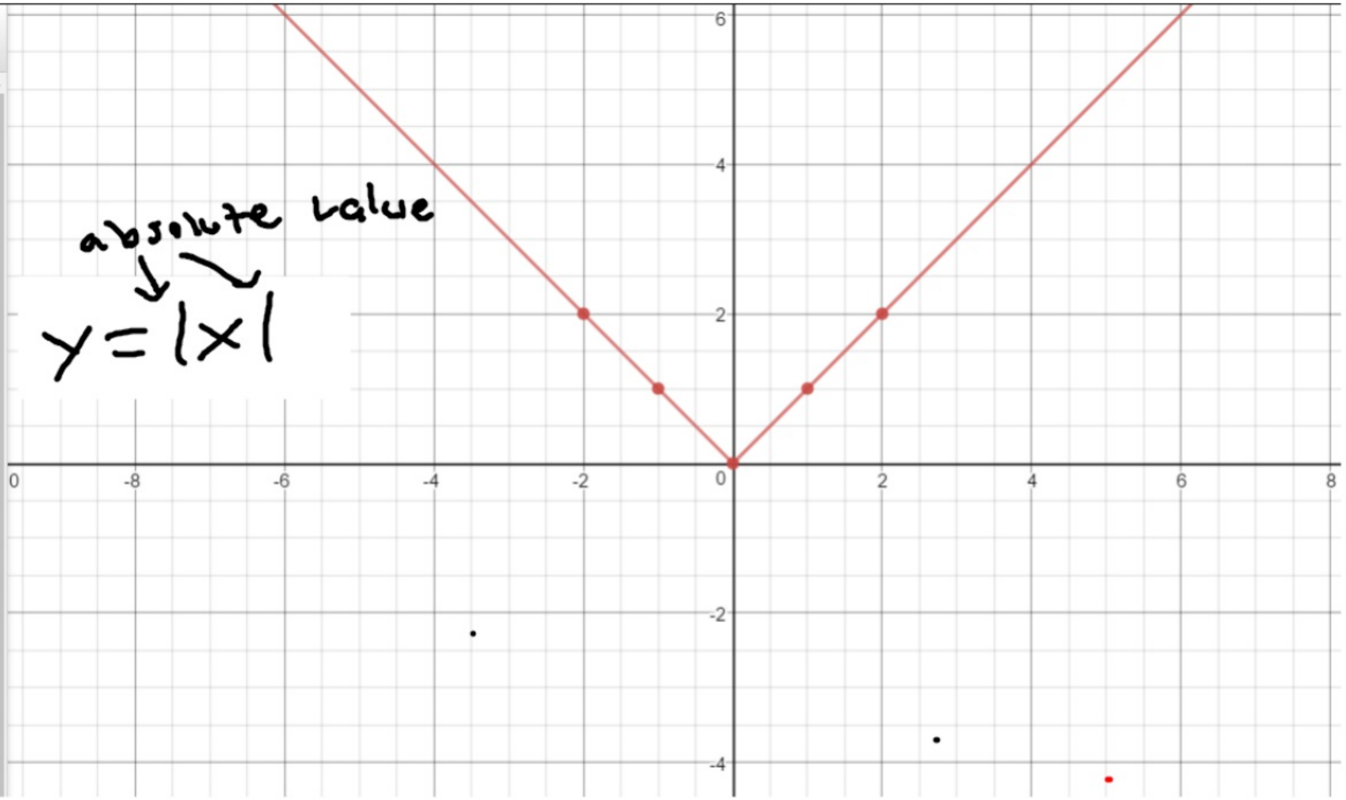
$a = 1$ (−10 to 10)

PEMAS

$b = 0$ (−10 to 10)

0

$c = 0$ (−10 to 10)



x	$\lvert x-3\rvert-5$
1	−3
2	−4
3	−5
4	−4
5	−3

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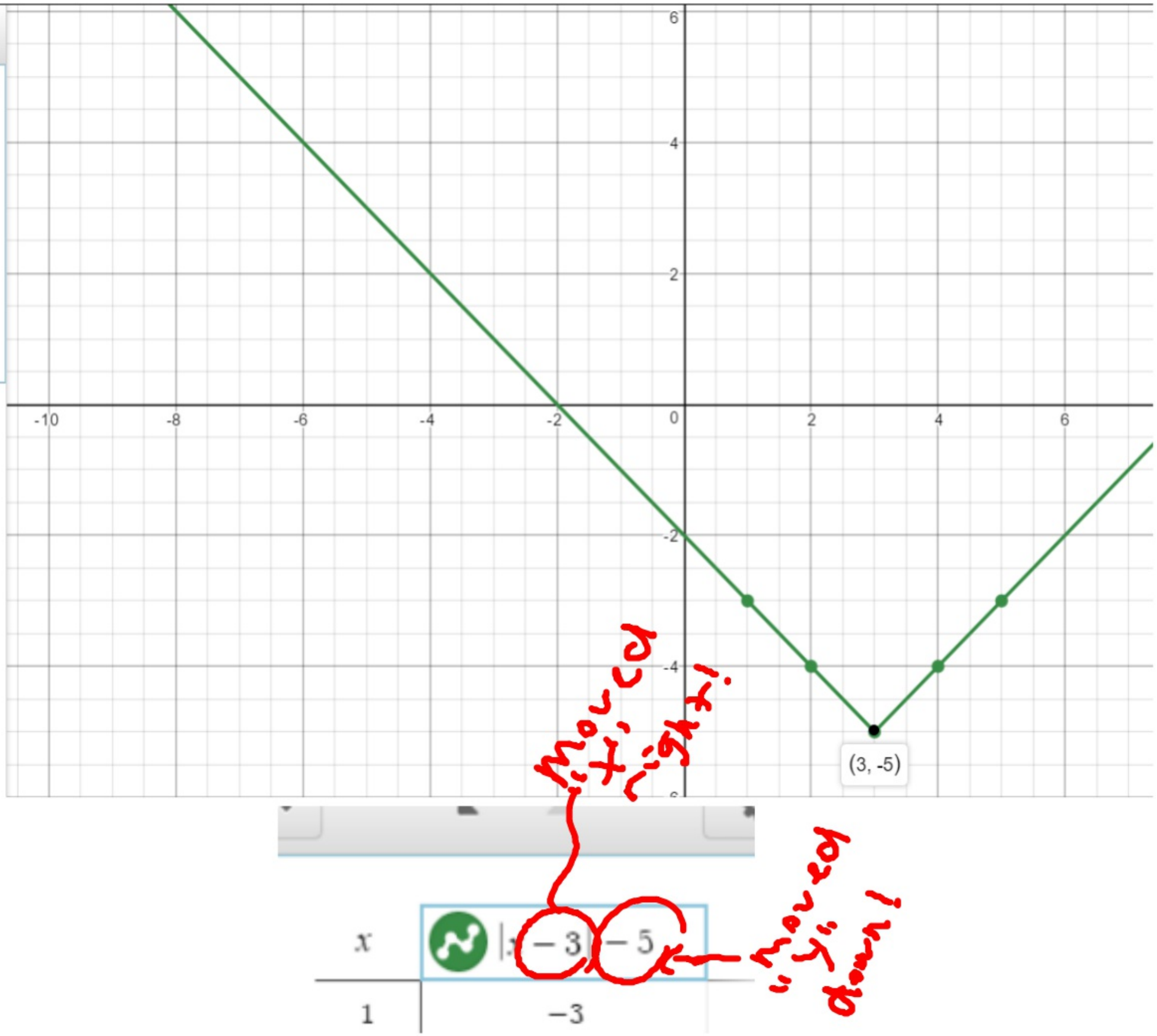


Example 3 Absolute Value Function

Graph $f(x) = |x - 4|$. State the domain and range.

Since $f(x)$ cannot be negative, the minimum point of the graph is where $f(x) = 0$.

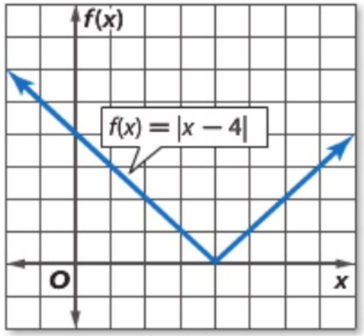
$f(x) = |x - 4|$ — Original function

$0 = x - 4$ — Replace $f(x)$ with 0 and $|x - 4|$ with $x - 4$.

$4 = x$ — Add 4 to each side.

Next make a table of values. Include values for $x > 4$ and $x < 4$.

$f(x) = \|x - 4\|$	
x	$f(x)$
−2	6
0	4
2	2
4	0
5	1
6	2
7	3
8	4



The domain is all real numbers. The range is all real numbers greater than or equal to 0. Note that this is the graph of $f(x) = |x|$ shifted 4 units to the right. Notice that the graph is symmetric about the line $x = 4$, and the minimum value of the function is 0 at $x = 4$. As x increases, $f(x)$ increases, and as x decreases, $f(x)$ increases.

"I can use any number."

Domain: "all real #'s"

Range: $y \geq 0$

"anything greater than or equal to zero.."

GuidedPractice

3. Graph $f(x) = |2x + 1|$. State the domain and range.

3 Graph $f(x) = |2x + 2|$. State the domain and range.

Examples 3–4 Graph each function. State the domain and range. **5–8. See C**

5. $f(x) = |x - 3|$

6. $g(x) = |2x + 4|$

(5)

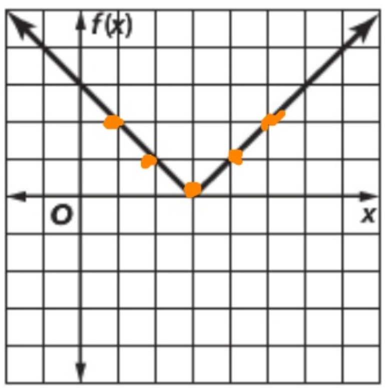
x	y
1	2
2	1
3	0
4	1
5	2

(6)

x	y
−4	4
−3	2
−2	0
−1	2
0	4

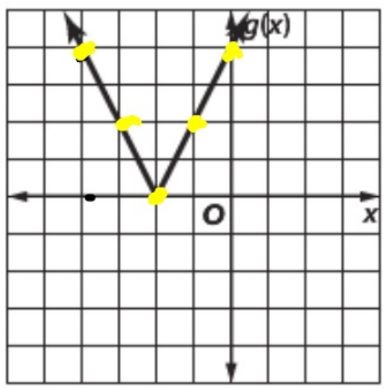
Lesson 9-7

5.



D = all real numbers,
R = $f(x) \geq 0$

6.



D = all real numbers,
R = $g(x) \geq 0$

mples 3–4 **Graph each function. State the domain and range.** **17–30. See Ch**

17. $f(x) = |2x - 1|$

18. $f(x) = |x + 5|$

19. $g(x) = |-3x - 5|$

20. $g(x) = |-x - 3|$

21. $f(x) = \left|\frac{1}{2}x - 2\right|$

22. $f(x) = \left|\frac{1}{3}x + 2\right|$

23. $g(x) = |x + 2| + 3$

24. $g(x) = |2x - 3| + 1$

24

$2x - 3 = 0$

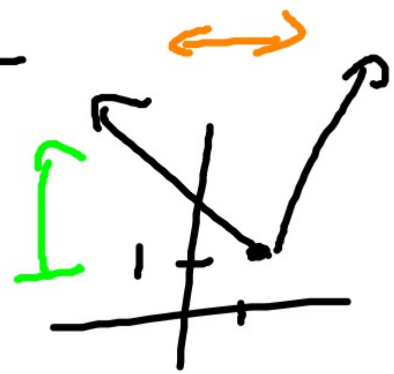
$+3 \quad +3$

$2x = 3$

$\frac{2x}{2} = \frac{3}{2}$

$x = \frac{3}{2}$

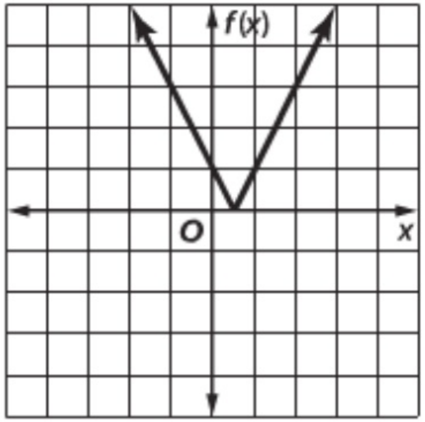
x	y
0	4
1	2
1.5	1
2	2
3	4



D: $\mathbb{R}$ ← all real numbers.

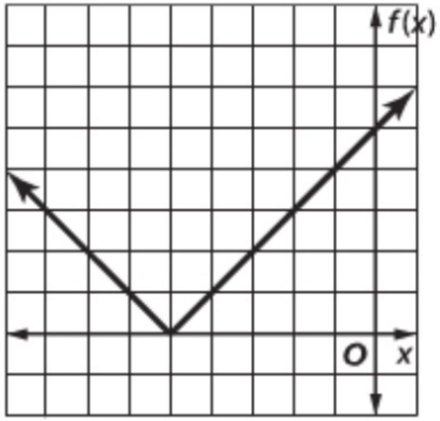
R: $y \geq 1$

17.



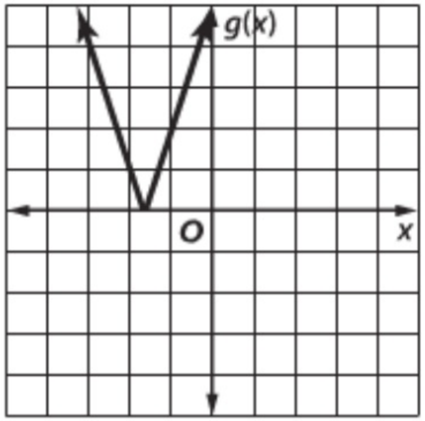
D = all real numbers,
R = $f(x) \geq 0$

18.



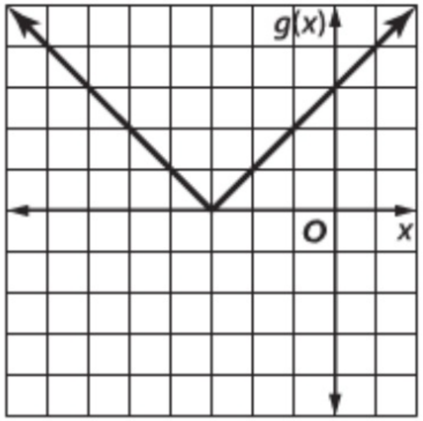
D = all real numbers,
R = $f(x) \geq 0$

19.



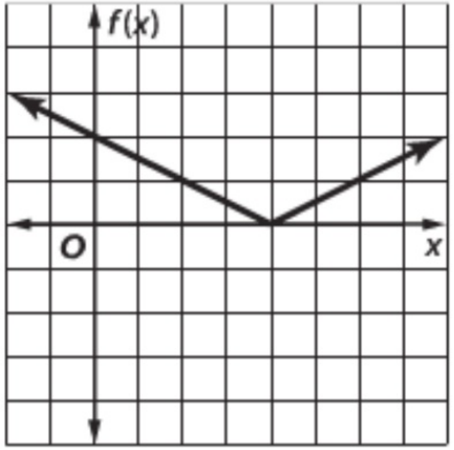
D = all real numbers,
R = $g(x) \geq 0$

20.



D = all real numbers,
R = $g(x) \geq 0$

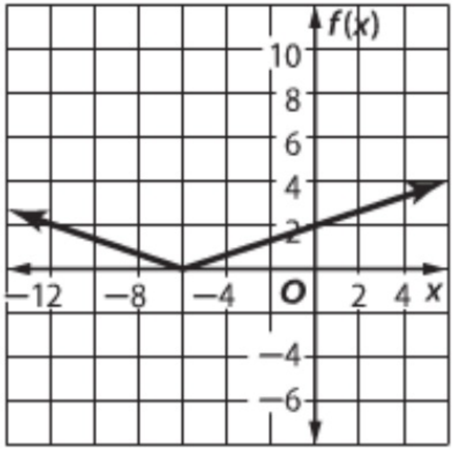
21.



D = all real numbers,

R = $f(x) \geq 0$

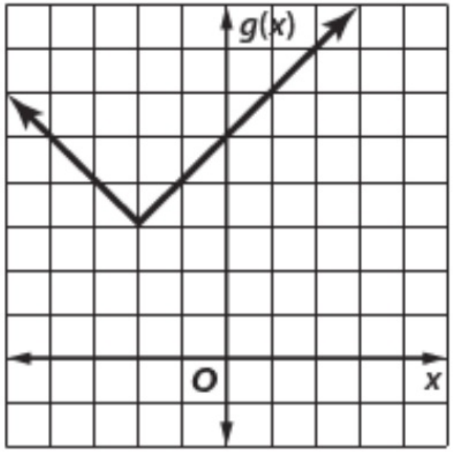
22.



D = all real numbers,

R = $f(x) \geq 0$

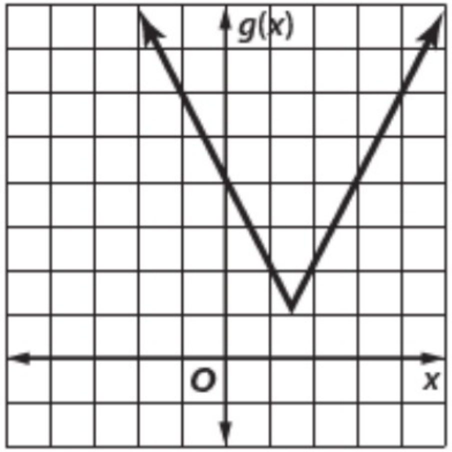
23.



D = all real numbers,

R = $g(x) \geq 3$

24.

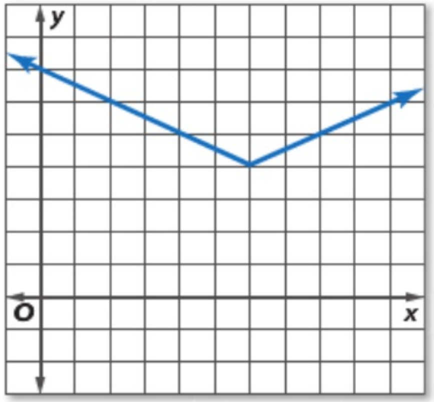


D = all real numbers,

R = $g(x) \geq 1$

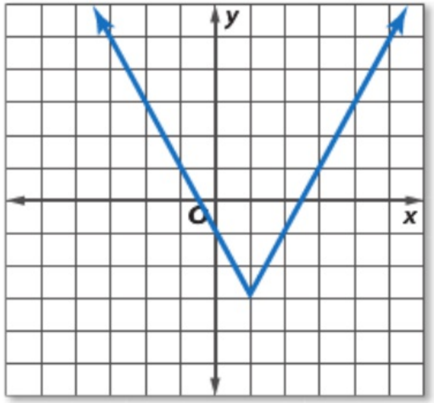
B **Determine the domain and range of each function.**

31



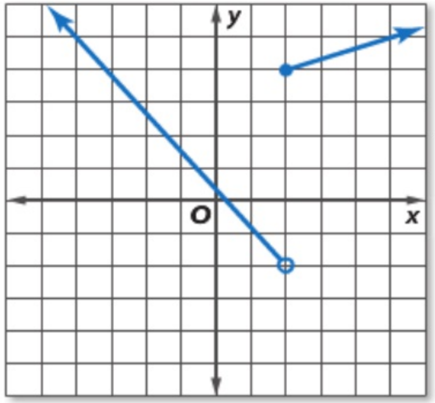
D = all real numbers;
R = $y \geq 4$

32.

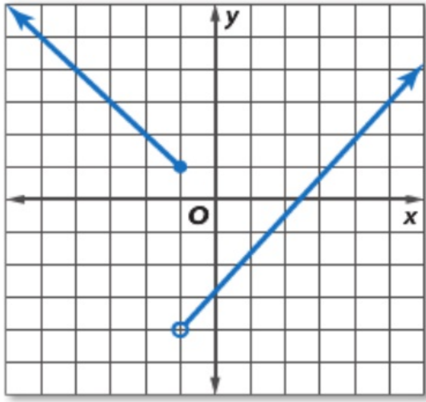


D = all real numbers;
R = $y \geq -3$

35.



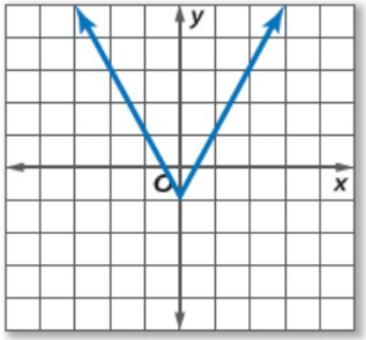
36.



For Exercises 38–41, match each graph to one of the following equations.

A	B	C	D
$y = 2x - 1$	$y = [\![2x]\!] - 1$	$y = \lvert 2x \rvert - 1$	$y = \begin{cases} 2x + 1 \text{ if } x > 0 \\ -2x + 1 \text{ if } x \leq 0 \end{cases}$

38.



C

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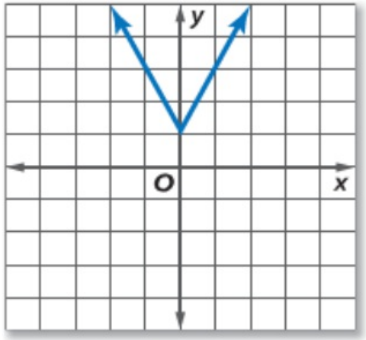
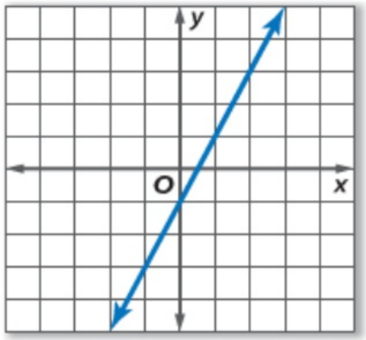


D

40.



41.