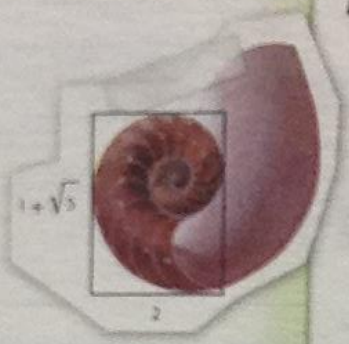
Example



4. **The *golden rectangle* is found frequently in the nautilus shell. The length of the longer side divided by the length of the shorter side is equal to $\frac{1+\sqrt{5}}{2}$. Estimate this value.**

First estimate the value of $\sqrt{5}$.

$4 < 5 < 9$ 4 and 9 are the closest perfect squares.

$2^2 < 5 < 3^2$ $4 = 2^2$ and $9 = 3^2$

$\sqrt{2^2} < \sqrt{5} < \sqrt{3^2}$ Find the square root of each number.

$2 < \sqrt{5} < 3$ Simplify.

Since 5 is closer to 4 than 9, the best integer estimate for $\sqrt{5}$ is 2. Use this value to evaluate the expression.

$\frac{1+\sqrt{5}}{2} \approx \frac{1+2}{2}$ or 1.5

Guided Practice

Estimate to the nearest integer. (Examples 1 and 2)

1. $\sqrt{28} \approx$ 5
2. $\sqrt{135} \approx$ 12
3. $\sqrt{38.7} \approx$ 6
4. $\sqrt{51} \approx$ 4
5. $\sqrt{200} \approx$ 6
6. $\sqrt{95} \approx$ 5

7. **STEM** Tobias dropped a tennis ball from a height of 60 meters. The time in seconds it takes for the ball to fall 60 feet is $0.25(\sqrt{60})$. Find three sets of approximations for the amount of time it will take. Then determine how long it will take for the ball to hit the ground. (Example 3)

Sample answer: between 1.75 and 2 seconds, between 1.925 and 1.95 seconds, between 1.935 and 1.9275 seconds; about 2 seconds

8. The number of swings back and forth of a pendulum of length L in inches each minute is $\frac{375}{\sqrt{L}}$. About how many swings will a 40-inch pendulum make each minute? (Example 4)

62.5 swings

9. **Building on the Essential Question** How can I estimate the square root of a non-perfect square?

Sample answer: Find the perfect squares less than the number and the perfect square greater than the number. Determine which one the original number is closer to and take the square root of that number.

Rate Yourself!

How confident are you about finding the square root of a non-perfect square? Color in the section that applies.

For more help, go online to access a Personal Tutor.

Name _____ My Homework _____

Independent Practice

Go online for Step-by-Step Solutions

Estimate to the nearest integer. (Examples 1 and 2)

1. $\sqrt{23} \approx$ 5
2. $\sqrt{197} \approx$ 14
3. $\sqrt{15.6} \approx$ 4
4. $\sqrt{85.1} \approx$ 9

Show your work.

5. $\sqrt[3]{22} \approx$ 3
6. $\sqrt[3]{34} \approx$ 3
7. $\sqrt[3]{989} \approx$ 10
8. $\sqrt[3]{250} \approx$ 6

9. The area of Kaitlyn's square garden is 345 square feet. One side of the garden is next to a shed. She wants to put a fence around the other three sides of the garden. Find three sets of approximations for the amount of fence it will take. Then determine how much fence she should buy.

(Example 3) Sample answer: 54 ft and 57 ft; 55.5 ft and 55.8 feet; 55.71 ft and 55.74 feet; 56 feet

10. In Little League, the bases are squares with sides of 14 inches. The expression $\sqrt{(s^2 + s^2)}$ represents the distance *diagonally across* a square of side length s. Estimate the diagonal distance across a base to the nearest inch. (Example 4) 20 in.

11 STEM The formula $t = \frac{\sqrt{h}}{4}$ represents the time t in seconds that it takes an object to fall from a height of h feet. If a rock falls from a height of 125 feet, estimate how long it will take to reach the ground. (Example 4)

about 2.75 seconds

Order each set of numbers from least to greatest.

12. $\{7, 9, \sqrt{50}, \sqrt{85}\}$ $7, \sqrt{50}, 9, \sqrt{85}$
13. $\{\sqrt[3]{105}, 7, 5, \sqrt{38}\}$ $\sqrt[3]{105}, 5, \sqrt{38}, 7$

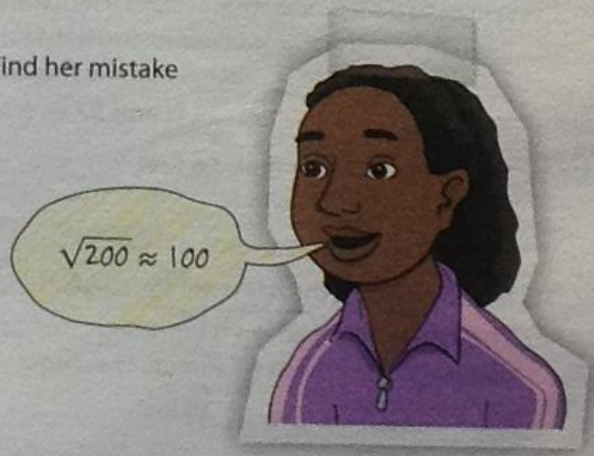
14. **Persevere with Problems** Amanda purchased a storage cube that has a volume of 4 cubic feet. She wants to put it on a bookshelf that is 12 inches tall. Will the cube fit? Explain. No; 12 inches = 1 foot and the cube root of 4 > 1.

15. Without a calculator, determine which is greater, $\sqrt{94}$ or 10. Explain your reasoning. 10; Since 94 is less than 100, $\sqrt{94}$ is less than 10.

H.O.T. Problems Higher Order Thinking

16. **Persevere with Problems** Find two numbers that have square roots between 7 and 8. One number should have a square root closer to 7 and the other number should have a square root closer to 8. Justify your answer.
Sample answers: 50; 60. Since $49 < 50 < 64$ and 50 is closer to 49 than to 64, $\sqrt{50}$ is closer to 7 than to 8. Since $49 < 60 < 64$ and 60 is closer to 64 than to 49, $\sqrt{60}$ is closer to 8 than to 7.

17. **Find the Error** Jasmine is estimating $\sqrt{200}$. Find her mistake and correct it. She incorrectly estimated. She found half of 200, not the square root. Since $196 < 200 < 225$, the square root of 200 is between 14 and 15. Since 200 is closer to 196, the square root of 200 is about 14.



18. **Construct an Argument** If $x^4 = y$, then x is the fourth root of y. Explain how to estimate the fourth root of 30. Find the fourth root of 30 to the nearest whole number.
Since $16 < 30 < 81$, the fourth root is between 2 and 3. Since 30 is closer to 16 than 81, the fourth root of 30 is about 2.

19. **Reason Inductively** Suppose x is a number between 1 and 10 and y is a number between 10 and 20. Determine whether the statement below is always, sometimes or never true. Explain your reasoning.

$$\sqrt{x} > \sqrt[3]{y}$$

sometimes; Sample answer: $\sqrt{9}$ is greater than $\sqrt[3]{18}$, but $\sqrt{4}$ is less than $\sqrt[3]{18}$.